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# Parametric Circular Aperture Segmentation Formalism 

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#### Abstract

To minimize the artifacts in Point Spread Functions (PSFs) of space telescopes such as the 6 major spikes shown in the star image observed by the James Webb Space Telescope, which are caused by the primary mirror outer boundary and the gaps between segments, an alternative method to segmentize large circular telescope mirror is proposed. From the fact that the strong spikes are due to the parallel-ness of all linear boundary edges and gaps in the current hexagonal segmentation approach, the proposed segmentation methodology minimizes such parallel-ness. The proposed method comes with a set of formulas that can assist the design process including the systematic calculation of number of segments and the size of each segment to cover the entire mirror pupil area. Some example PSFs of a few possible segmented mirrors with the proposed formalism will be presented.


Keywords: telescope mirrors, segmentation, Point Spread Function, segmented mirror

## 1. INTRODUCTION

The James Webb Space Telescope (JWST) is redefining the history of astronomy and astrophysics with its superb optical performance imaging the deep space objects since its successful launch in 2021. While it will serve the international astronomical science community for upcoming decades further enhancements with potential optical solutions may benefit the next generation community following the JWST in the future. The 6 major spikes shown in the telescope alignment evaluation image of the James Webb Space Telescope (JWST) deployed in space [1] clearly indicate both the diffraction by the telescope's primary mirror boundary outline composed of hexagonal segment mirrors and the additional diffraction from the finite gaps between hexagonal mirror segments. The mirrors' boundary with gaps acts like multiple linear gratings in 3 different directions and creates the unique PSF spikes, each of which is perpendicular to the corresponding edges of the hexagonal mirrors [2].

Breckinridge et al. [3, 4] investigated various shapes and diffraction effects of segments for large optical telescopes including the traditional hexagonal shapes and demonstrated that the distinct spikes shown in Fig. 1 could be suppressed with the segment topology optimization by distributing the diffracted light energy in axially symmetrical pattern. In the numerical study, the sub-total number of segmented mirrors in each group (as a function of the distance from the center of pupil) increases in area as the segments are positioned further away from the center, which requires a systematic formalism to control the segmentation numbers and sizes during numerical optimization process.

In this work, we present a concise and complete mathematical formula for keystone shape segmentation achieving the equal area and equal height in segmentation rings (i.e., subgroup of segments).


Fig. 1 Alignment evaluation image of the James Webb Space Telescope showing the excellent Point Spread Function mainly caused by the segmented telescope primary pupil topology as theoretically predicted. (Image credit: NASA/STScl)

## 2. KEYSTONE SHAPE SEGMENTATION

Let us consider a circle of radius R and it is divided in n ring subgroups. Each ring has equal width $\Delta$ and is divided in a number of Keystone shape pieces, as shown in Fig. 2.


Fig. 2 Various topological possibilities for the proposed equal-area Keystone shape segmentation for a circular aperture.

In order to acquire the equal area condition for every segment, we set the area as $\pi r_{0}{ }^{2}$, where $r_{0}$ is the radius of the circular segment (or center hole) located at the center, as denoted in solid black in Fig. 2. Then we sought the solution that each ring has area equal to $k_{j} \pi r_{0}^{2}$, where $k_{j}$ is the number of Keystone pieces in the $\mathrm{j}^{\text {th }}$ ring and the integer j goes from 1 through n . Multiple segmentation solutions can be found as depicted in Fig. 2 and they are summarized in a set of simple equations as followings.

The width is related to the radius R as

$$
\begin{equation*}
R=(n \delta+1) r_{0} \tag{1}
\end{equation*}
$$

where $\delta$ is one of two parameters to decide final segmentation topology and is an integer multiplied to $r_{0}$ to set the width of the ring as

$$
\begin{equation*}
\Delta=\delta r_{0} \tag{2}
\end{equation*}
$$

Thus, the final value of each mirror segment's area can be analytically determined by combination of the two design parameters, n (the number of rings) and $\delta$ (shown in Eq. 1) for a given circular aperture with keystone segmentation approach.
The width of each ring $\Delta$ is determined and, naturally, the number of segments in the $\mathrm{j}^{\text {th }}$ ring subgroup can be also calculated as

$$
\begin{equation*}
k_{j}=k_{1}+p(j-1) \tag{3}
\end{equation*}
$$

where

$$
\left\{\begin{array}{c}
k_{1}=\delta(\delta+2)  \tag{4}\\
p=2 \delta^{2}
\end{array}\right.
$$

Finally, the total number of segments is expressed as

$$
\begin{equation*}
(n \delta+1)^{2} \tag{5}
\end{equation*}
$$

## 3. PSF SIMULATIONS

The proposed Keystone shape segmentation includes 2 different types of edge geometries, curved circle in the outer boundary and the radial lines between segment gaps. We computed the PSF (Point Spread Function) utilizing the proposed segmentation formalism for 3 different case study examples as shown in Fig. 3.

The PSFs shown in Fig. 3 clearly show that the diffraction patterns corresponding the linear radial edges are not amplified causing distinct spikes but spread out more evenly due to the rotating orientations of the radial (yet linear) edges.


Fig. 3 Three Point Spread Functions (bottom row) of three different Keystone segmentation example cases (top row) utilizing proposed equal-area-equal-width segmentation formalism. (Note: x and y dimensions are arbitrary and can be scaled according to the actual optical system's pupil sizes and focal lengths.)

## 4. CONCLUSIONS

Based on the proposed mathematical pupil segmentation formalism, we summarize following 4 implications:

1. A systematic mathematical formalism is proposed for Keystone segmentation for circular pupil optical system applications.
2. Each Keystone segment has equal area and equal height (i.e., width) as a design constraint.
3. The total number of segments as well as the area and height are mathematically defined and adjustable as pupil design optimization parameters.
4. The segmentation formalism can be used as a numerical PSF or system optimization loop utilizing merit functions based on the science goal and/or PSF requirements of the optical systems.

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