ANALYSIS FOR ALIGNMENT OF OPTICAL SYSTEMS

by

Régis Tessieres

———————————————

A Thesis Submitted to the Faculty of the

DEPARTMENT OF OPTICAL SCIENCES

In Partial Fulfillment of the Requirements
For the Degree of

MASTER OF SCIENCE

In the Graduate College

THE UNIVERSITY OF ARIZONA

2 0 0 3

Statement by Author

This thesis has been submitted in partial fulfillment of requirements for an advanced degree at The University of Arizona and is deposited in the University Library to be made available to borrowers under rules of the Library.

Brief quotations from this thesis are allowable without special permission, provided that accurate acknowledgment of source is made. Requests for permission for extended quotation from or reproduction of this manuscript in whole or in part may be granted by the head of the major department or the Dean of the Graduate College when in his or her judgment the proposed use of the material is in the interests of scholarship. In all other instances, however, permission must be obtained from the author.

Signed: _____

Approval By Thesis Director

This thesis has been approved on the date shown below:

_____   _____
James H. Burge   Date
Pofessor of Optical Sciences

Acknowledgments

I would like to thank my committee for their feedback and specially my advisor Professor Jim Burge for his constant support and to have provided me with such an interesting topic and Professor Roland Shack to have introduced me to the theory of tilted and decentered systems. It would have been difficult to achieve this work without their help and support.

Also, I would like to thank my family and Mélanie Laurent for their moral support.

Table of Contents

Table of Contents—*Continued*

Table of Contents—*Continued*

List of Figures

LIST OF FIGURES—*Continued*

List of Tables

ABSTRACT

The alignment process of an optical system can be more complicated than its original design. For example a misaligned multi-element system will suffer from imaging aberrations that vary in a complex way over the field of view. Aberration fields for this kind of system have been well described by perturbing the well known aberrations for axisymmetric systems. These have been written in terms of polynomial expansions in pupil and field space, which are useful for understanding the phenomena, but are difficult to apply. This thesis will show how to express these relations in terms of Zernike polynomials. It will be seen that they are convenient to decompose each aberrations in terms of their field dependencies. And that they can be easily used to perform a least squares fit on the data coming from a wavefront sensor or a raytracing program to retrieve the perturbations present in a system. It is shown on two different systems, that the relations are an efficient tool that give insight during the alignment process. They allow to minimize the noise and to get faster to the solution.

ANALYSIS FOR ALIGNMENT OF OPTICAL SYSTEMS

Régis Tessieres, M.S.
The University of Arizona, 2003

Director: James H. Burge

The alignment process of an optical system can be more complicated than its original design. For example a misaligned multi-element system will suffer from imaging aberrations that vary in a complex way over the field of view. Aberration fields for this kind of system have been well described by perturbing the well known aberrations for axisymmetric systems. These have been written in terms of polynomial expansions in pupil and field space, which are useful for understanding the phenomena, but are difficult to apply. This thesis will show how to express these relations in terms of Zernike polynomials. It will be seen that they are convenient to decompose each aberrations in terms of their field dependencies. And that they can be easily used to perform a least squares fit on the data coming from a wavefront sensor or a raytracing program to retrieve the perturbations present in a system. It is shown on two different systems, that the relations are an efficient tool that give insight during the alignment process. They allow to minimize the noise and to get faster to the solution.

Chapter 1

INTRODUCTION

The computation of aberrations in a misaligned optical system has been well described in the past. These aberrations can be used to analyze the effects of tilt and decenter in terms of aberration fields in aligned systems and to perform a tolerancing analysis [1, 2, 3, 4, 5].

Also, some specific publications on astronomical optics [6, 7, 8, 9, 10, 11, 12, 13, 14] describe for two-mirror telescopes the effects of misalignments on the image quality and present a way to align such a telescope by relating aberration coefficients (Zernike coefficients for some of them) and the tilts and decenters in the system.

The publications [1, 2, 3, 4, 5] can be generally used in all systems to evaluate the image degradation due to misalignments but are not easy to use to align an optical system by relating aberration coefficients and perturbations. On the other hand the publications such as [9, 10, 12] on telescope alignment have useful relations between Zernike coefficients and field dependencies which can be used to retrieve the misalignments in a two-mirror telescope. But these publications are only valid for this type of system and to align a given optical system with several degrees of freedom, general relations valid for any system and developed at least for fifth order aberrations are needed.

The purpose of this thesis is to give general equations which are valid for any type of system and which relate Zernike coefficients (they can be easily computed with any optical software) with their field dependencies. And to use these relations for two different systems that are modelled with optical software so as to find the relationships between the Zernike coefficients and the tilts and decenters of each element in the system. Then by measuring the Zernike coefficients at different field
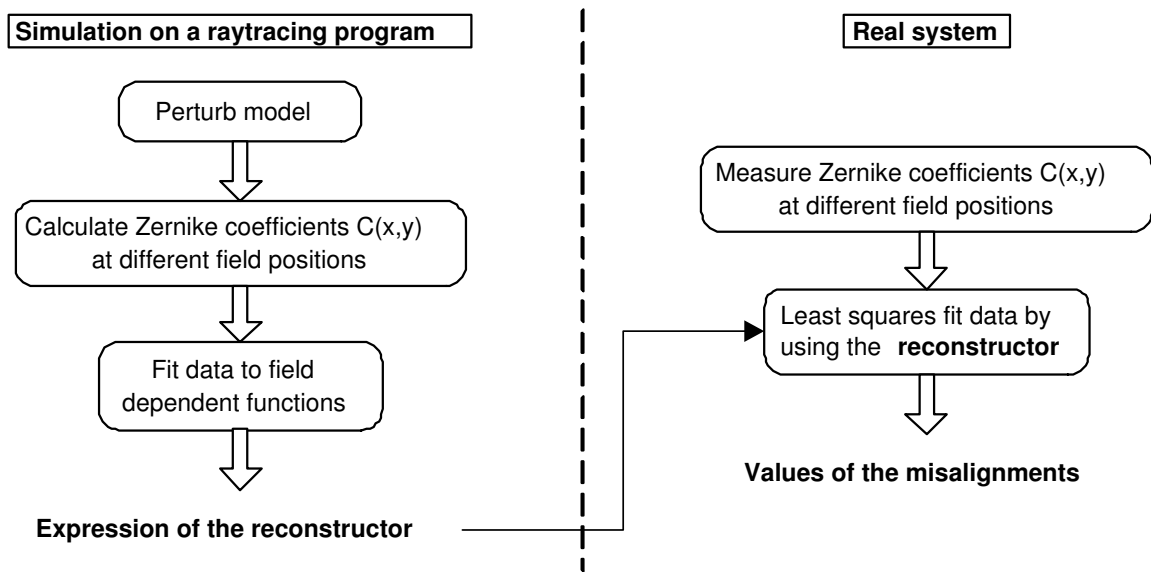
FIGURE 1.1. Block diagram of the general procedure

positions it will be possible to retrieve the misalignments present in the system. The general procedure that will be followed in this thesis to find the perturbations in a system is represented by a block diagram in figure 1.1.

This thesis will be based on the theory of misaligned system developed at the university of Arizona. The development of aberrations for third order was first developed by R.A. Buchroeder in his PhD dissertation by using the vector formulation of the wavefront expansion introduced by R.V. Shack. Then his work was carried on by K.P. Thompson who extended the theory of R.A. Buchroeder to the fifth order. After that, several people including J.R. Rogers used this theory to design unobstructed telescopes.

In chapter 2, the theory of aberrations in misaligned systems through fifth order developed at the University of Arizona will be introduced. The important results needed for the thesis will be underlined.

In chapter 3, the relations shown in chapter 2 will be transformed so as to get expressions of the Zernike coefficients in misaligned systems which are more convenient to use with raytracing programs and wavefront sensors.

In chapter 4, the expressions in terms of Zernike coefficients and field dependencies will be used on a two element telescope so as to find the relations between the Zernike coefficients and the perturbations of the system.

In chapter 5, the same procedure that was applied in chapter 4 will used. But a more complicated telescope (the LSST) will be considered which has a lot of degrees of freedom.

Chapter 2

Background

The analysis of the perturbations in this thesis is based on the vector formulation of the wavefront expansion developed by R.V. Shack in the 1970's. It will be seen that this is a convenient way to express the aberrations and to account for perturbations. And it can be applied to any type of system. It does not depend on the number or on the type of surfaces.

In a first section, the vector formulation of the wavefront expansion will be introduced for an aligned system. A second section, will show how to account for the perturbations in the expression of the wavefront expansion. Finally a third section will analyze the effects of perturbations such as decenter and tilt on the expressions of the aberrations.

2.1 Development of the wavefront expansion

2.1.1 Conventions

Before starting to express the wavefront expansion, the conventions which will be used need to be defined. The conventions used to express the wavefront expansion will be the same as the ones used in the dissertation of K.P. Thompson [3] and are represented in figure 2.1.

$\vec{H}$ represents the field position in the image plane with x and y its components along $\vec{x}$ and $\vec{y}$ respectively as shown in figure 2.1. $\vec{\rho}$ represents the pupil position with $\rho \sin \phi$ and $\rho \cos \phi$ its components along $\vec{x}$ and $\vec{y}$ as shown on figure 2.1. Therefore they are expressed by:

$$\vec{H} = H\, e^{i\theta} \qquad \text{and} \qquad \vec{\rho} = \rho\, e^{i\phi}$$
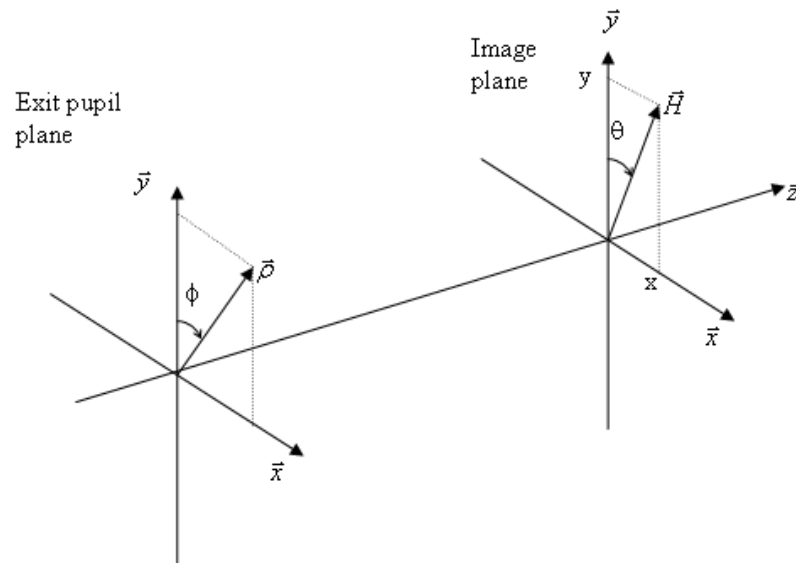
FIGURE 2.1. Conventions

Where the left-handed convention is used whereas most optical software use a right handed convention. Thus the equations that will be developed in section 3 with optical softwares will have to be used with some precautions.

2.1.2 Scalar formulation of the wavefront expansion

A common way to describe aberrations in a centered, rotationally symmetric optical system is to use the wave aberration expansion shown in equation 2.1.

$$W = \sum_j \sum_p^\infty \sum_n^\infty \sum_m^\infty (W_{klm})_j \, H^k \rho^l \cos^m \phi \,, \quad \text{where} \begin{cases} k = 2p + m \\ l = 2n + m \end{cases} \tag{2.1}$$

The total wave aberration at the image plane of a rotational symmetric optical system is simply the sum of the surfaces contributions:

$$W = \sum_j W_j$$

The development through fifth order excluding the first order terms of equation 2.1 is:

$$W = \sum_j W_{040j}\rho^4 + \sum_j W_{131j}H\rho^3\cos\phi + \sum_j W_{220j}H^2\rho^2 + \sum_j W_{222j}H^2\rho^2\cos^2\phi$$

$$+ \sum_j W_{311j}H^3\rho\cos\phi + \sum_j W_{060j}\rho^6 + \sum_j W_{151j}H\rho^5\cos\phi + \sum_j W_{420j}H^4\rho^2$$

$$+ \sum_j W_{422j}H^4\rho^2\cos^2\phi + \sum_j W_{511j}H^5\rho\cos\phi + \sum_j W_{240j}H^2\rho^4$$

$$+ \sum_j W_{242j}H^2\rho^4\cos^2\phi + \sum_j W_{331j}H^3\rho^3\cos\phi + \sum_j W_{333j}H^3\rho^3\cos^3\phi$$

$$(2.2)$$

Since piston represents a constant phase change that does not degrade the image, it has been left over in equation 2.2.

2.1.3 Vector formulation of the wavefront expansion

When it is necessary to consider tilts and decenters in a system it is more convenient to express the wavefront expansion with a vector formulation rather than the scalar one. The vector formulation of equation 2.1 is:

$$W = W[(\vec{H}\cdot\vec{H}), (\vec{H}\cdot\vec{\rho}), (\vec{\rho}\cdot\vec{\rho})]$$
$$= \sum_j \sum_p^\infty \sum_n^\infty \sum_m^\infty (W_{klm})_j \left(\vec{H}\cdot\vec{H}\right)^p \left(\vec{\rho}\cdot\vec{\rho}\right)^n \left(\vec{H}\cdot\vec{\rho}\right)^m \qquad (2.3)$$

Equation 2.3 can be developed as:

$$W = \sum_j \sum_p^\infty \sum_n^\infty \sum_m^\infty (W_{klm})_j \, H^{2p}\,\rho^{2n}\,H^m\rho^m\cos\left(\theta - \phi\right) \qquad (2.4)$$

Thus for a rotational symmetric system equation 2.4 is the same as equation 2.1 by taking $\theta = 0$.

The development through fifth order for equation 2.3, excluding first order terms, becomes:

$$W = \sum_j W_{040j}(\vec{\rho} \cdot \vec{\rho})^2 + \sum_j W_{131j}(\vec{H} \cdot \vec{\rho})(\vec{\rho} \cdot \vec{\rho}) + \sum_j W_{220_{Mj}}(\vec{H} \cdot \vec{H})(\vec{\rho} \cdot \vec{\rho})$$

$$+ \frac{1}{2}\sum_j W_{222j}(\vec{H}^2 \cdot \vec{\rho}^2) + \sum_j W_{311j}(\vec{H} \cdot \vec{H})(\vec{H} \cdot \vec{\rho}) + \sum_j W_{060j}(\vec{\rho} \cdot \vec{\rho})^3$$

$$+ \sum_j W_{151j}(\vec{H} \cdot \vec{\rho})(\vec{\rho} \cdot \vec{\rho})^2 + \sum_j W_{420_{Mj}}(\vec{H} \cdot \vec{H})^2(\vec{\rho} \cdot \vec{\rho})$$

$$+ \frac{1}{2}\sum_j W_{422j}(\vec{H} \cdot \vec{H})(\vec{H}^2 \cdot \vec{\rho}^2) + \sum_j W_{511j}(\vec{H} \cdot \vec{H})^2(\vec{H} \cdot \vec{\rho}) \tag{2.5}$$

$$+ \sum_j W_{240_{Mj}}(\vec{H} \cdot \vec{H})(\vec{\rho} \cdot \vec{\rho})^2 + \frac{1}{2}\sum_j W_{242j}(\vec{H}^2 \cdot \vec{\rho}^2)(\vec{\rho} \cdot \vec{\rho})$$

$$+ \sum_j W_{331_{Mj}}(\vec{H} \cdot \vec{H})(\vec{H} \cdot \vec{\rho})(\vec{\rho} \cdot \vec{\rho}) + \frac{1}{4}\sum_j W_{333j}(\vec{H}^3 \cdot \vec{\rho}^3)$$

Note that the astigmatism is defined relative to the medial focus and not to the sagittal focus as it is usually done. Therefore, there is a factor $\frac{1}{2}$ for the astigmatism and to show that the field curvature is defined for the medial plane, the subscript "$_M$" is used. The details of the calculations which lead to these results can be found in Appendix A.

For W_{331} the subscript $_M$ is used to account for the conversion of $\cos^3(\theta - \phi)$ to $\cos\big(3(\theta - \phi)\big)$ for the elliptical coma, W_{333}. This is done by using $W_{331_M} = W_{331} + \frac{3}{4}W_{333}$ and the trigonometric identity $cos^3\theta = \frac{1}{4}\cos 3\theta + \frac{3}{4}\cos\theta$.

Some quick comments will be done on these aberrations to have a better understanding later on the effects of misalignments. But the aberrations will not be

explained in detail because it is beyond the scope of this thesis. For more complete discussion on aberration theory see [17, 18, 19, 20]. Only the useful comments for later will be underlined.

2.1.4   Comments on third order aberrations

The third order aberrations represent the main contribution to an aberrated image for an optical system. They are a good approximation for systems which contain a limited number of elements. Note that third order aberrations refer to third order in transverse ray aberration which corresponds to fourth order in wavefront aberration.

- Spherical aberration

  W_{040} represents spherical aberration. As it can be seen in figure 2.2 it is independent of the field H. Thus, it will be constant over the field at the image plane.

- Coma

  W_{131} represents coma. It has a comatic shape which increases linearly in the field. This can be seen in figure 2.3 where a 2D view of the image plane is shown.

- Field curvature

  W_{220} represents field curvature. It has a quadratic dependence with the field representing the shape of the Petzval curvature. See figure 2.4 for its representation.

- Astigmatism

  W_{222} represents astigmatism. It has a quadratic dependence with the field as shown in figure 2.5 where a 2D view is represented.

- Distortion

  W_{311} represents distortion. It has a cubic dependence with the field.
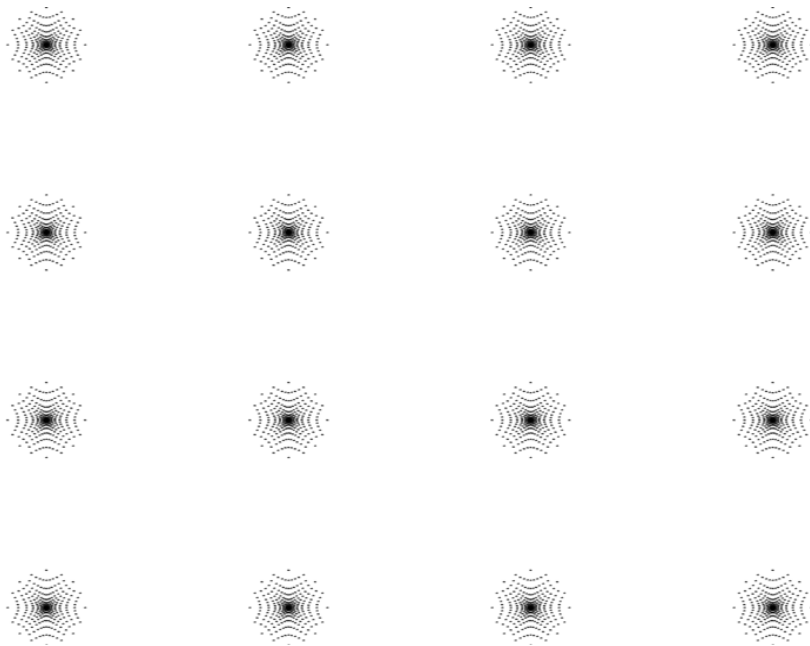
FIGURE 2.2. Representation of the third order spherical aberration
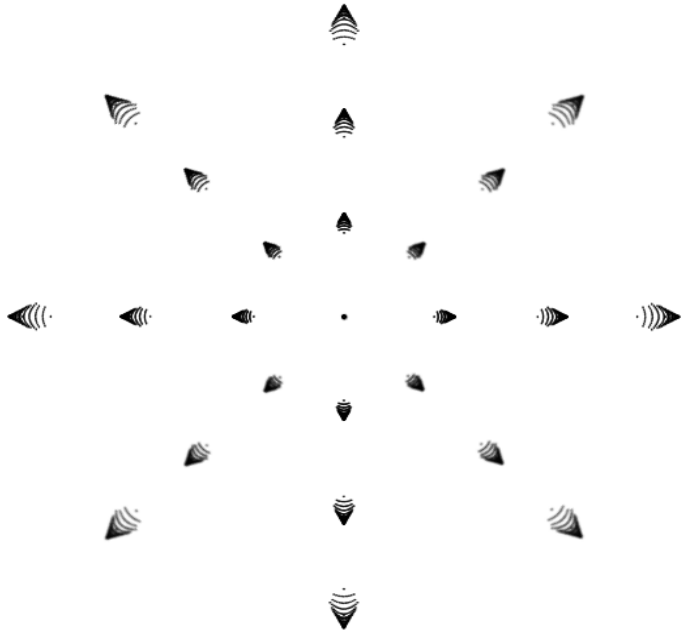
FIGURE 2.3. Representation of the third order coma
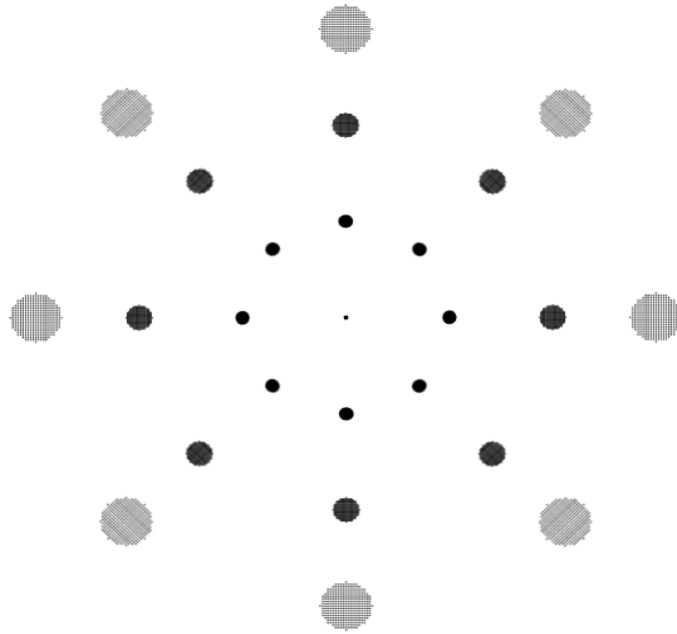
FIGURE 2.4. Representation of the third order field curvature
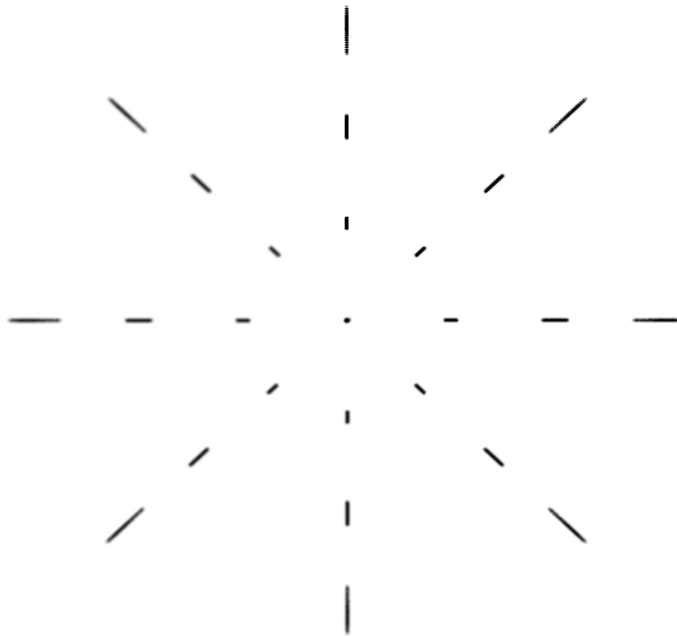
FIGURE 2.5. Representation of the third order astigmatism

2.1.5 Comments on fifth order aberrations

When the number of elements in a system increases, the third order aberrations are not always enough to represent a system correctly. Then, the development of the wavefront expansion is to be carried out through the fifth order.

- Fifth order spherical aberration

  W_{060} represents fifth order spherical aberration. Like W_{040} it is independent of the field and will be constant over the field as shown in figure 2.6.

- Coma

  W_{151} represents fifth order coma. It has the same field dependence as third order coma. The linear dependence can be seen in figure 2.7.

- Field curvature for fifth order astigmatism

  W_{420} represents field curvature for fifth order astigmatism. It behaves as third order field curvature except that it has a quartic dependence instead of a quadratic one as shown in figure 2.8.

- Fifth order astigmatism

  W_{422} represents fifth order astigmatism. It behaves as third order astigmatism except that its dependence is quartic instead of quadratic as shown in figure 2.9.

- Distortion

  W_{511} represents distortion. It is like third order distortion except that it grows as a power of five with the field.

- Field curvature for oblique spherical aberration

  W_{240} represents the field curvature for oblique spherical aberration. Like for third order field curvature it has a quadratic field dependence as shown in figure 2.10.

- Oblique spherical aberration

  W_{242} represents oblique spherical aberration. It is a combination of spherical aberration and astigmatism. The field dependence is also quadratic as shown in figure 2.11.

- Field cubed coma

  W_{331} represents field cubed coma. It behaves as third order coma except that the field dependence is cubic. Its spot diagram representation in the field is shown in figure 2.12.

- Elliptical coma (Trefoil)

  W_{333} represents elliptical coma. It can be compared to third order coma except that the field dependence is cubic and that the end of the comatic shape is elliptic instead of round. It is represented in figure 2.13.
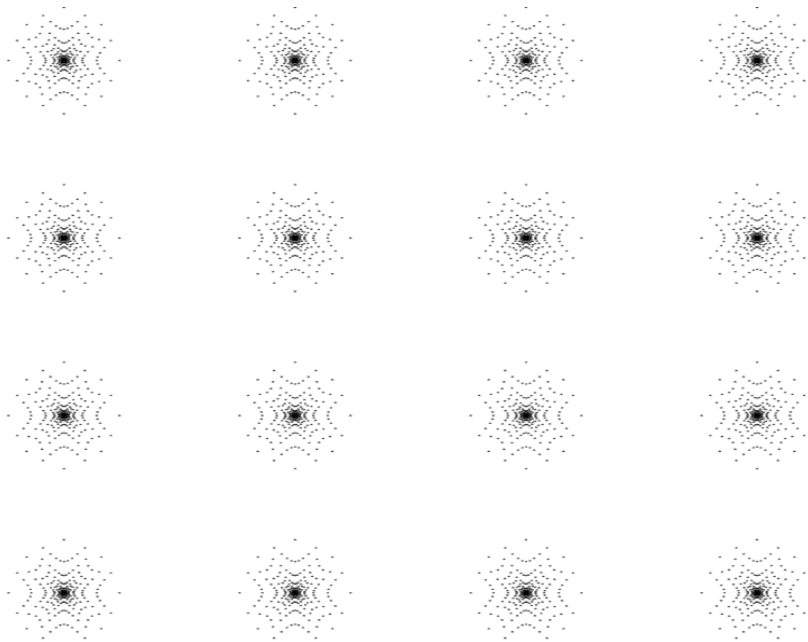
FIGURE 2.6. Representation of the fifth order spherical aberration
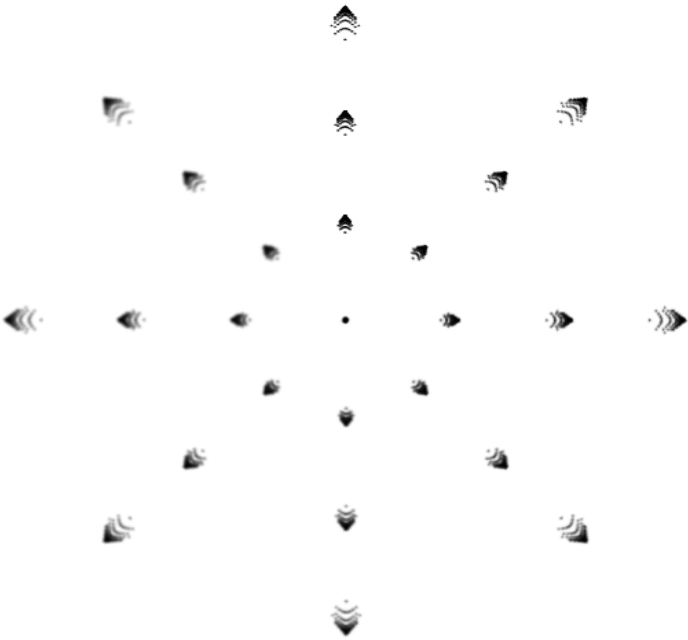
Figure 2.7. Representation of fifth order coma
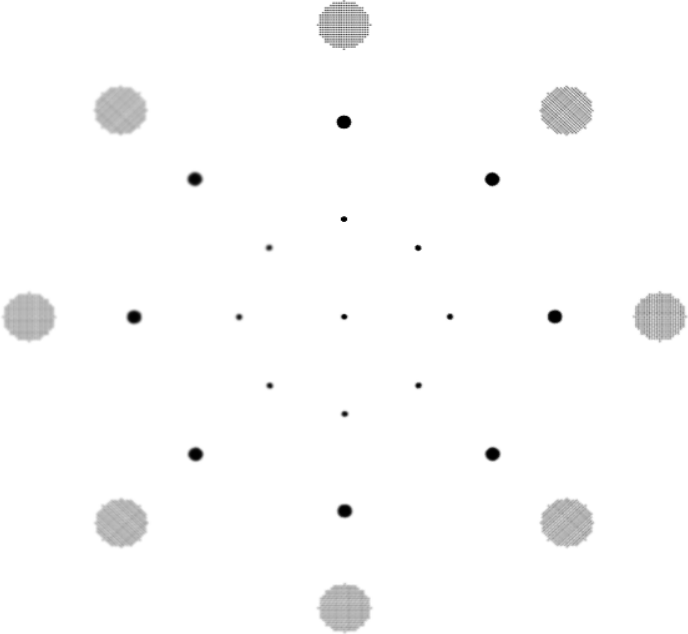
Figure 2.8. Representation of the field curvature for fifth order astigmatism
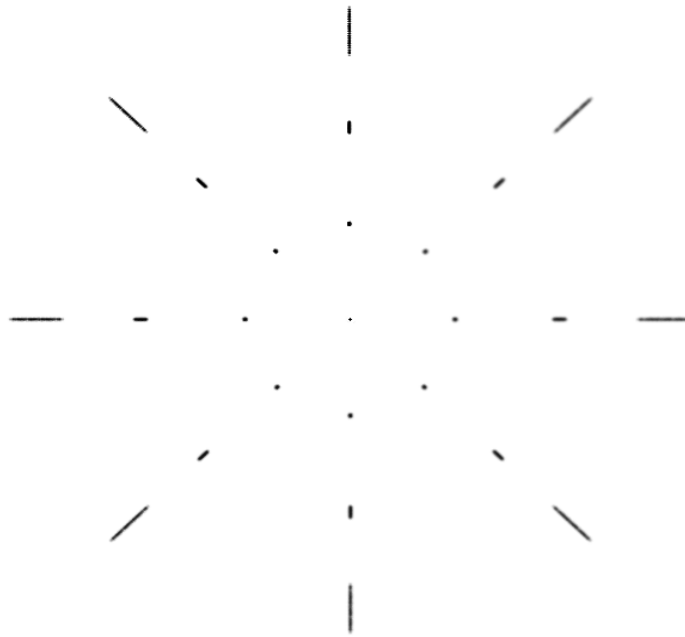
FIGURE 2.9. Representation of the fifth order astigmatism
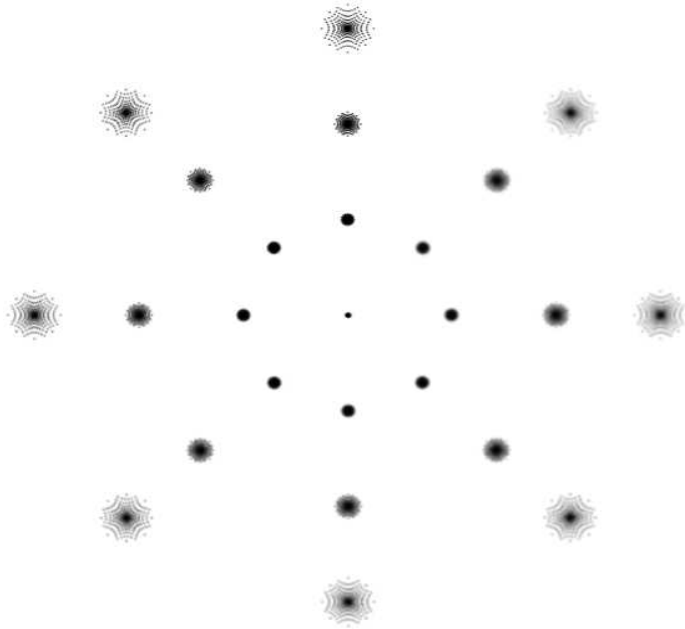
FIGURE 2.10. Representation of the field curvature for oblique spherical aberration
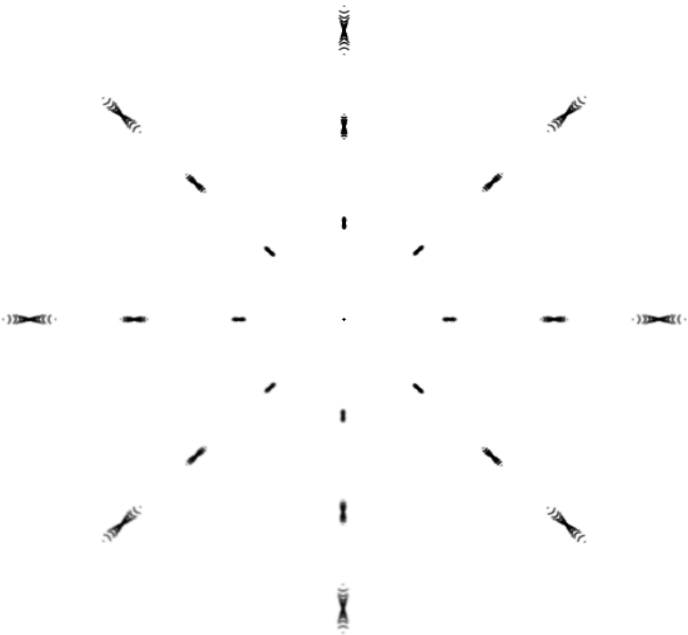
FIGURE 2.11. Representation of the oblique spherical aberration
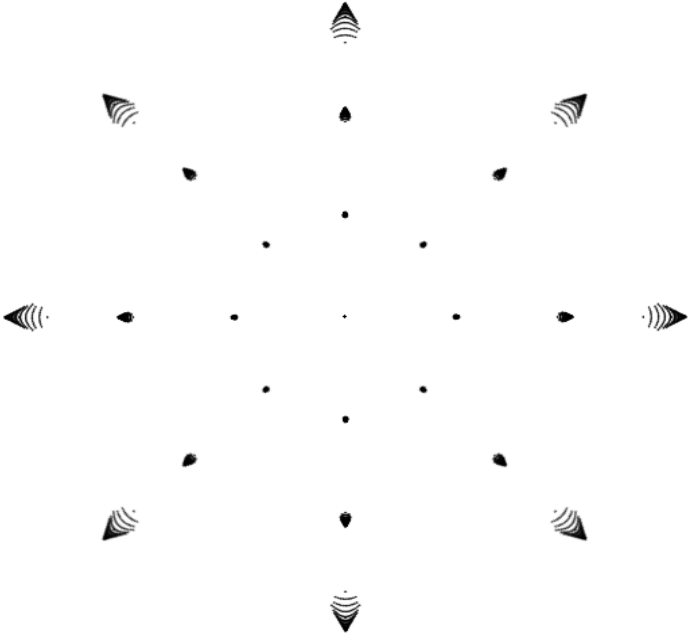
FIGURE 2.12. Representation of the field cubed coma
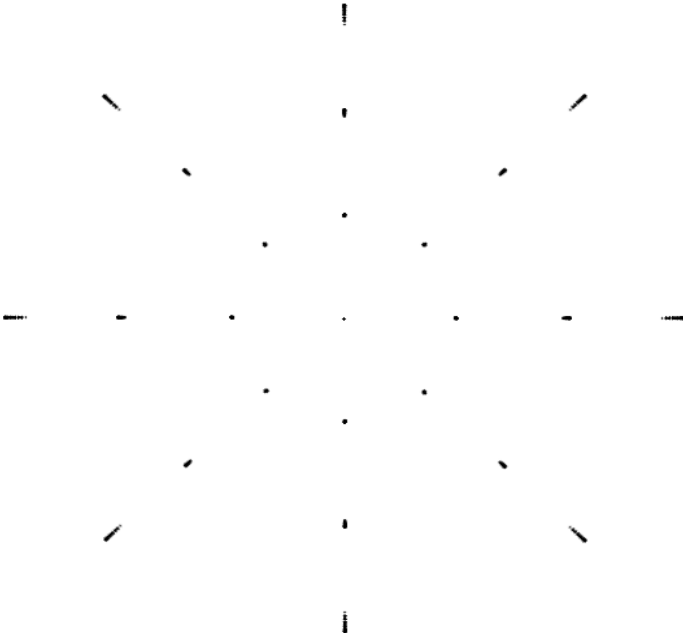
FIGURE 2.13. Representation of trefoil

2.2 Wavefront expansion in a misaligned system

The behavior of the aberration field in a misaligned system is based on two properties. First, the aberration field at the image plane is still the sum of individual surface contributions. Secondly, the aberration field contribution of a surface is centered along the line connecting the centers of the pupils for the surface and the center of curvature of the surface. When a system is perturbed, no new aberration will be created but the behavior of the aberration field at the image plane will be modified. The aberrations expected will still be the same (spherical aberration, coma,...) but they will have different field dependencies compared to a centered system.

The vector formulation is convenient to express this situation. It allows the treatment of nonmeridional and noncoplanar tilts and decenters by introducing a vector $\vec{\sigma}_j$ which represents the decentration of the center of the aberration field W_j with respect to the unperturbed field center (center of the Gaussian image plane which is located by the optical axis ray (OAR)[1]). Thus, a new vector is to be defined to represent a position in the field. This is done by introducing the effective height $\vec{H}_{Aj}$ which is defined in figure 2.14 as:

$$\vec{H}_{Aj} = \vec{H} - \vec{\sigma}_j \tag{2.6}$$

Therefore, the expression of the wavefront expansion in a perturbed system is defined by replacing $\vec{H}$ by $\vec{H}_{Aj}$ in equation 2.3:

$$
\begin{aligned}
W &= \sum_j \sum_p^\infty \sum_n^\infty \sum_m^\infty (W_{klm})_j \left(\vec{H}_{Aj} \cdot \vec{H}_{Aj}\right)^p \left(\vec{\rho} \cdot \vec{\rho}\right)^n \left(\vec{H}_{Aj} \cdot \vec{\rho}\right)^m \\
&= \sum_j \sum_p^\infty \sum_n^\infty \sum_m^\infty (W_{klm})_j \left((\vec{H} - \vec{\sigma}_j) \cdot (\vec{H} - \vec{\sigma}_j)\right)^p \left(\vec{\rho} \cdot \vec{\rho}\right)^n \left((\vec{H} - \vec{\sigma}_j) \cdot \vec{\rho}\right)^m
\end{aligned}
\tag{2.7}
$$

---

[1] The OAR is the ray connecting the centers of the pupils with the object and the images created by all the surfaces.
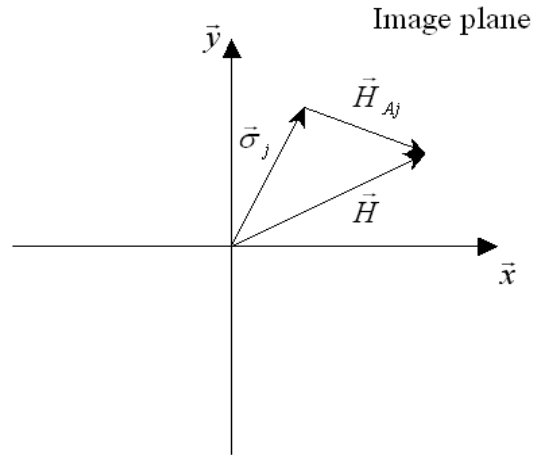
FIGURE 2.14. Representation of the effective field height

The use of the vector formulation for the wavefront expansion for a misaligned system was made possible by the multiplication of vectors. It is different from a dot or a cross product as will be shown in chapter 3. This formulation accounts for tilts and decenters.

2.3 Expressions of the third and fifth order aberrations in a perturbed system

The following sections will give the expressions of the wavefront expansion in a perturbed system for the third and the fifth order aberrations. The expressions come from the derivation of equation 2.7 for each aberration. The full derivations from equation 2.7 to the equations presented in this section will not be done since it is not the purpose of this thesis and they can be found in the dissertation of Kevin P. Thompson [3].

The equations which will be presented in this section use vectors and scalars that have been introduced in the dissertation of K.P. Thompson and can be found in table 2.1 at the end of this section.

2.3.1 Third order aberrations

- Spherical aberration

  Since spherical aberration is independent of the field, it is unchanged when the system is perturbed. Its expression remains the same.

- Third order astigmatism

  The Wavefront expansion for the astigmatism using equation 2.7 is:

$$W = \frac{1}{2}\left(W_{222}\vec{H}^2 - 2\vec{H}\vec{A}_{222} + \vec{B}_{222}^2\right) \cdot \vec{\rho}^2 = W_{quadratic} + W_{linear} + W_{constant} \quad (2.8)$$

Where $\vec{A}_{222}$ and $\vec{B}_{222}$ are defined in Table 2.1. Again all the steps between equation 2.7 and equation 2.8 can be found in [3].

The first term, $W_{quadratic}$, represents the term that is present in an aligned system. It is the conventional quadratic field dependence which was present in equation 2.5 and represented in figure 2.15.

Besides the quadratic dependence with the field, there is now a linear and a constant dependencies. So even if a system is corrected for astigmatism, when it is misaligned some linear and/or constant astigmatism can appear in the system. The graphical representation of the different type of astigmatism are shown in figures 2.15, 2.16 and 2.17.

An interesting case arises when the system is not corrected from astigmatism in the nominal design and when it is misaligned. The combination of the different types of astigmatism will produce a "binodal astigmatism" which is represented in figure 2.18. Roland V. Shack was the first person to discover and understand this type of astigmatism. The name comes from the fact that the aberration has two nodes[2]. As seen on figure 2.18 the aberration is not null at

---

[2]A node is the position where the aberration is equal to zero

(a) Sagittal focus   (b) Tangential focus
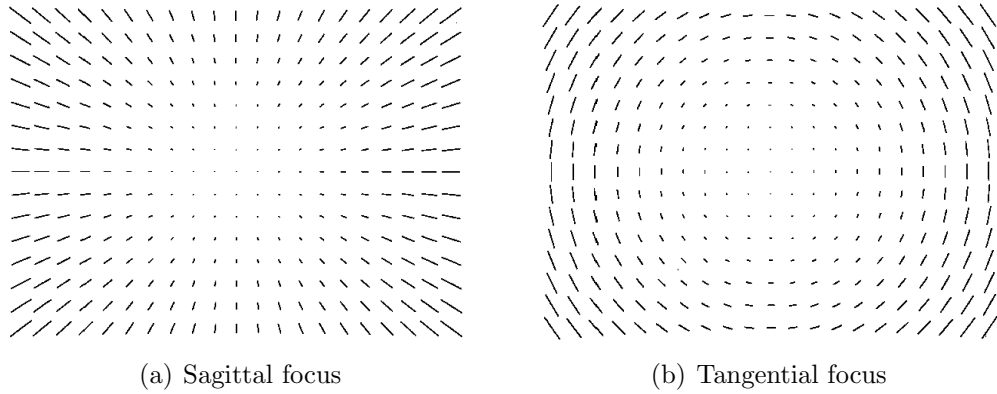
FIGURE 2.15. Spot diagrams of the quadratic astigmatism

(a) Sagittal focus   (b) Tangential focus
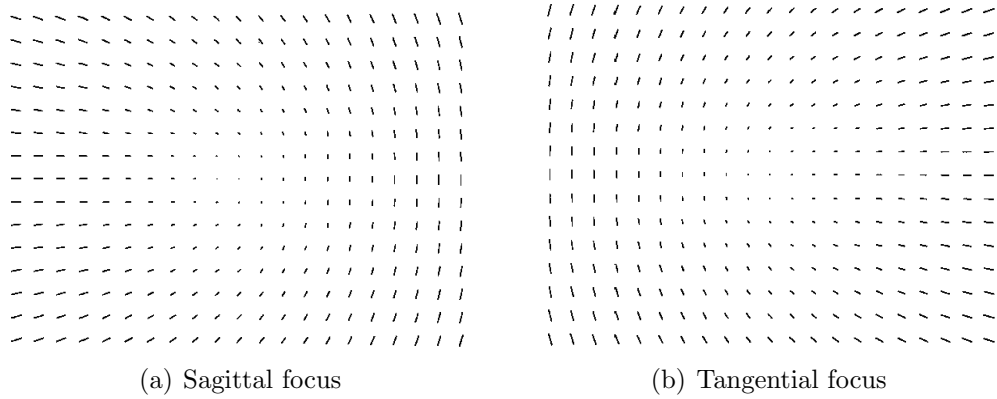
FIGURE 2.16. Spot diagrams of the linear astigmatism

(a) Sagittal focus   (b) Tangential focus
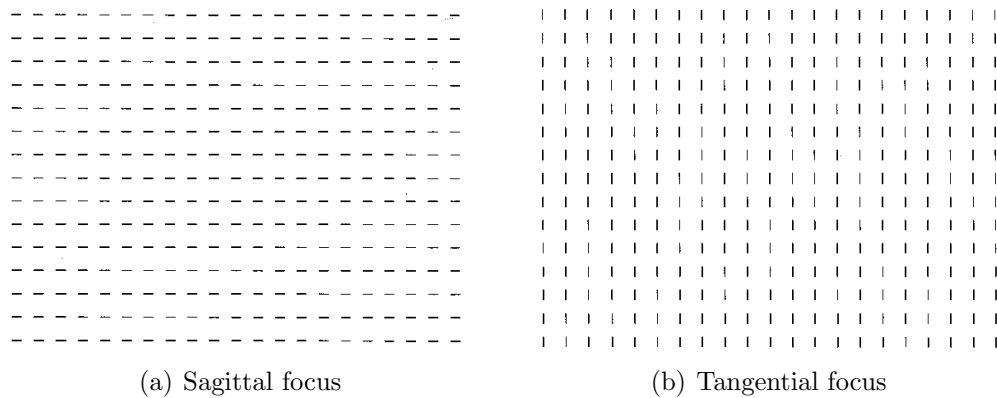
FIGURE 2.17. Spot diagrams of the constant astigmatism

(a) Sagittal focus   (b) Tangential focus
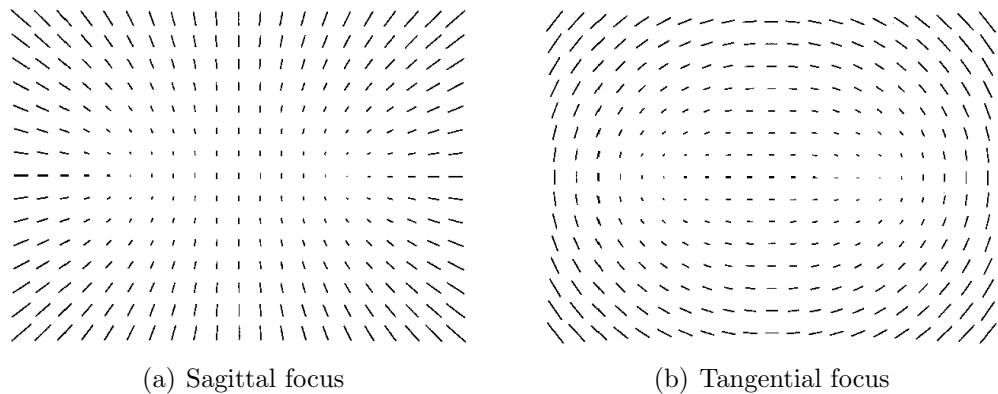
FIGURE 2.18. Spot diagrams of the binodal astigmatism

the center of the field as it was the case for the usual quadratic astigmatism but now it is null in the field and for two positions.

An other way to consider binodal astigmatism is to consider a two element system which contains astigmatism. In this case the astigmatism of each element increases quadratically from the center of the field. But when the system is misaligned the astigmatism contribution of each element is no longer centered at the center of the Gaussian image plane but shifted in the field as shown in figure 2.19. Thus the node for each aberration contribution does not longer coincide and two nodes appear in the field. To have a better visualization of the phenomenon a 3D representation of the astigmatism of the two surfaces is shown in figure 2.20. Due to the fact that the astigmatism is quadratic with field, each astigmatism contribution was represented with a paraboloid. Since each aberration contribution is not the same, one surface will be wider than the other one. On this figure, the nodes of each contribution do not coincide and for two positions where the two suraces intersect to each other, the total astigmatism will be zero. Then there will be two nodes instead of one. A representation in 2D is also shown in figure 2.21. To get more details on the graphical view of the aberration summation and on the position of the nodes
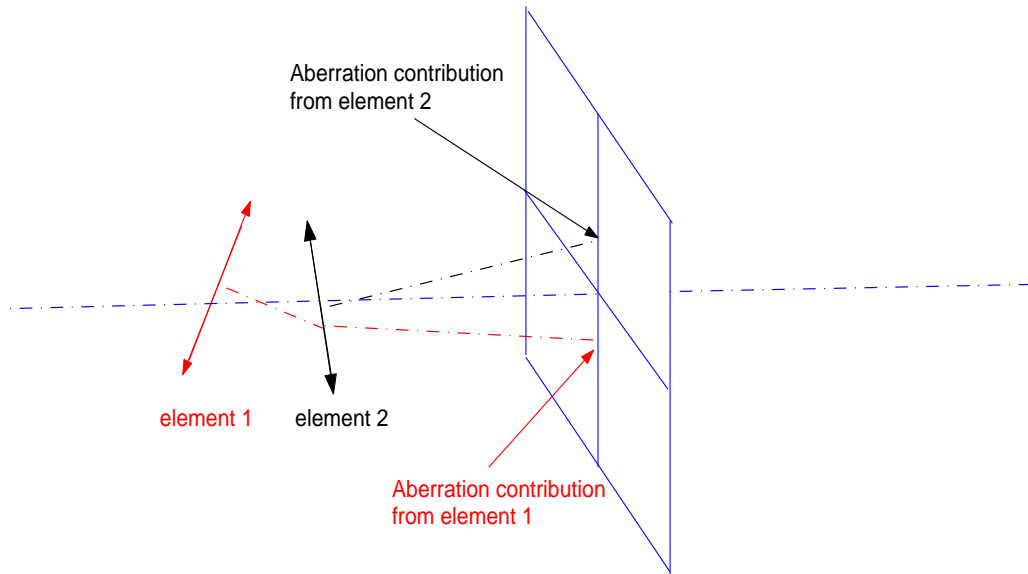
FIGURE 2.19. Schematic view of a two element misaligned system

see [16].

It was pointed out in [16] that for a two element misaligned system, a binodal astigmatism can exist but that is also true for a multiple element system. And it was demonstrated in the dissertation of K.P. Thompson [3] that there can never be more than 2 nodes for astigmatism. Moreover, it was shown in the dissertation that the maximum number of nodes is equal to the field power of the aberration. Here for astigmatism, the maximum number of nodes is two because astigmatism is quadratic with field.

- Third order coma

  The Wavefront expansion for coma using equation 2.7 is:

$$W = \big((W_{131}\vec{H} - \vec{A}_{131}) \cdot \vec{\rho}\big)\rho^2 = W_{linear} + W_{constant} \qquad (2.9)$$

Where $\vec{A}_{131}$ is defined in table 2.1.

The first term, W_{linear}, represents the term that is present in an aligned system. It is the conventional linear dependence with field for coma. But now when the
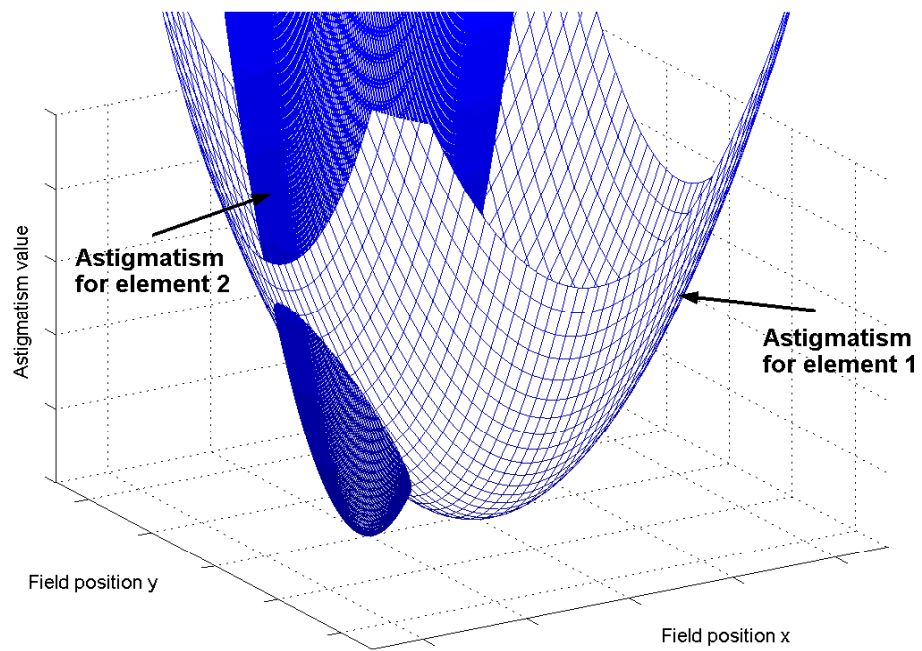
FIGURE 2.20. Representation of the astigmatism contribution for each element in 3D. The node for each aberration contribution is displaced in the field (not centered anymore on axis) because of the perturbations. Since astigmatism is quadratic with field, each astigmatism contribution was represented with a paraboiloid.
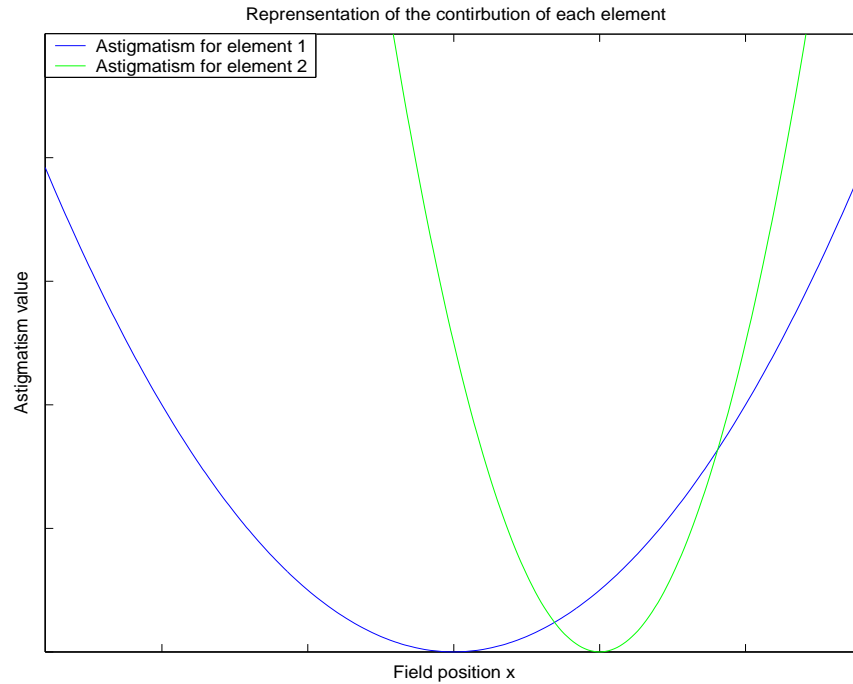
FIGURE 2.21. Representation of the astigmatism contribution for each element in 2D

system is misaligned, a new constant term will appear.

To have a better understanding, a centered two-element system is considered again but now it will contain coma. In this case the coma of each element increases linearly from the center of the Gaussian image plane (GIP). But when the system is misaligned as shown in figure 2.19, the coma contribution of each element is no longer centered at the center of the GIP but shifted in the field. The representation of the coma contributions for the two element system was done in figure 2.22. Since coma is linear with field, the shape in a three dimensional view is a cone. As for astigmatism, the coma contribution of one element is larger than the other one. Thus, one surface is wider than the other one as shown in figure 2.22. There is a position on the intersection curve of the two surfaces where the total coma is zero.
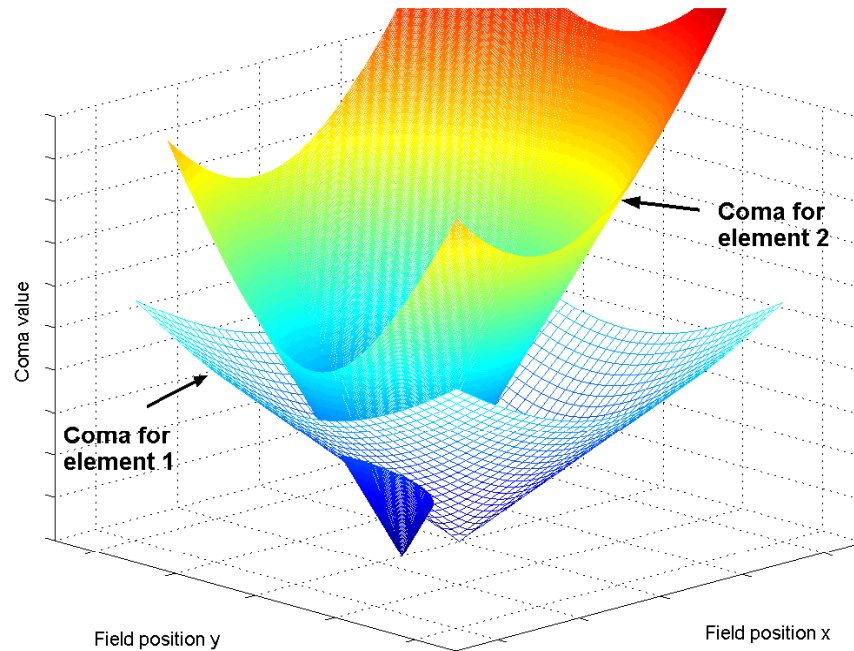
FIGURE 2.22. Representation of the coma contribution for each element in 3D. The perturbations shift the node of each contribution from the center of the Gaussian image plane. Since coma is linear with field, each coma contribution was represented with a cone.

If the nominal design of a system has some original coma and is misaligned, coma will still increase linearly but its node will no longer be centered at the center of the Gaussian image plane. According to 2.9, the node will be located by $\vec{A}_{131}/W_{131}$. On the other hand, if the original system is corrected for coma, some coma can appear at the image plane if the system is misaligned. But this time coma will be constant over the field.

Again as it was said before the maximum possible number of nodes is equal to the field power of the aberration. Thus for coma, there can only be a maximum of one node.

- Third order field curvature

The Wavefront expansion for field curvature using equation 2.7 is:

$$W = \left(W_{220M}(\vec{H}\cdot\vec{H}) - 2\vec{H}\cdot\vec{A}_{220M} + B_{220M}\right)\left(\vec{\rho}\cdot\vec{\rho}\right) = W_{quadratic} + W_{linear} + W_{constant} \tag{2.10}$$

Where $\vec{A}_{220M}$ and B_{220M} are defined in table 2.1.

Like for astigmatism, field curvature can exhibit a linear and a constant dependence in a misaligned system.

- Third order distortion

   The Wavefront expansion for third order distortion using equation 2.7 is:

$$
\begin{aligned}
W = {} & W_{311}(\vec{H}\cdot\vec{H})\vec{H}\cdot\vec{\rho} & & (W_{cubic}) \\
& - 2(\vec{H}\cdot\vec{A}_{311})\vec{H}\cdot\vec{\rho} & & (W_{quadratic\#1}) \\
& + 2B_{311}\vec{H}\cdot\vec{\rho} & & (W_{linear\#1}) \\
& - (\vec{H}\cdot\vec{H})\vec{A}_{311}\cdot\vec{\rho} & & (W_{quadratic\#2}) \\
& + (\vec{B}_{311}^2\vec{H}^*)\cdot\vec{\rho} & & (W_{linear\#2}) \\
& - \vec{C}_{311}\cdot\vec{\rho} & & (W_{constant})
\end{aligned}
\tag{2.11}
$$

Where $\vec{A}_{311}$, B_{311}, $\vec{B}_{311}^2$ and $\vec{C}_{311}$ are defined in table 2.1. W_{cubic} represents the distortion of an aligned system and the other terms are the terms that can appear when a system is misaligned.

Note that there can be more than one term for the same field dependence.

2.3.2 Fifth order aberrations

- Fifth order spherical aberration

   Like for third order spherical aberration, there is no dependence with the field. Thus the expression of fifth order spherical aberration in a misaligned system remains the same.

- Field cubed coma

  The Wavefront expansion for field cubed coma using equation 2.7 is:

$$
\begin{aligned}
W = {}& W_{331_M}\big(\vec{H} \cdot \vec{H}\big)\vec{H} \cdot \vec{\rho}\,\big(\vec{\rho} \cdot \vec{\rho}\big) && (W_{cubic}) \\
& - 2\big(\vec{H} \cdot \vec{A}_{331_M}\big)\vec{H} \cdot \vec{\rho}\,\big(\vec{\rho} \cdot \vec{\rho}\big) && (W_{quadratic\#1}) \\
& + 2B_{331_M}\big(\vec{H} \cdot \vec{\rho}\big)\big(\vec{\rho} \cdot \vec{\rho}\big) && (W_{linear\#1}) \\
& - \big(\vec{H} \cdot \vec{H}\big)\vec{A}_{331_M} \cdot \vec{\rho}\,\big(\vec{\rho} \cdot \vec{\rho}\big) && (W_{quadratic\#2}) \\
& + \big(\vec{B}^{\,2}_{331_M}\vec{H}^{*}\big) \cdot \vec{\rho}\,\big(\vec{\rho} \cdot \vec{\rho}\big) && (W_{linear\#2}) \\
& - \big(\vec{C}_{331_M} \cdot \vec{\rho}\big)\big(\vec{\rho} \cdot \vec{\rho}\big) && (W_{constant})
\end{aligned}
\tag{2.12}
$$

  Where $\vec{A}_{331_M}$, B_{331_M}, $\vec{B}^{2}_{331_M}$ and $\vec{C}_{331_M}$ are defined in table 2.1.

- Elliptical coma

  The Wavefront expansion for elliptical coma using equation 2.7 is:

$$
\begin{aligned}
W = \Big(\ & \frac{1}{4}W_{333}\vec{H}^{3} \cdot \vec{\rho}^{\,3} && (W_{cubic}) \\
& - \frac{3}{4}(\vec{H}^{2}\vec{A}_{333}) \cdot \vec{\rho}^{\,3} && (W_{quadratic}) \\
& + \frac{3}{4}(\vec{H}\vec{B}^{2}_{333}) \cdot \vec{\rho}^{\,3} && (W_{linear}) \\
& - \frac{1}{4}\vec{C}^{3}_{333} \cdot \vec{\rho}^{\,3}\Big) && (W_{constant})
\end{aligned}
\tag{2.13}
$$

  Where $\vec{A}_{333}$, $\vec{B}^{2}_{333}$ and $\vec{C}^{3}_{333}$ are defined in table 2.1.

- Quartic field curvature

  The Wavefront expansion for quartic field curvature using equation 2.7 is:

$$
\begin{aligned}
W = {}& \Big(W_{420M}\big(\vec{H} \cdot \vec{H}\big) \cdot \big(\vec{H} \cdot \vec{H}\big)\Big)\big(\vec{\rho} \cdot \vec{\rho}\big) && (W_{quartic}) \\
& - 4\Big(\big(\vec{H} \cdot \vec{H}\big) \cdot \big(\vec{H} \cdot \vec{A}_{420M}\big)\Big)\big(\vec{\rho} \cdot \vec{\rho}\big) && (W_{cubic}) \\
& + B_{420M}\big(\vec{H} \cdot \vec{H}\big)\big(\vec{\rho} \cdot \vec{\rho}\big) && (W_{quadratic\#1}) \\
& + 2\big(\vec{H}^{2} \cdot \vec{B}^{2}_{420M}\big)\big(\vec{\rho} \cdot \vec{\rho}\big) && (W_{quadratic\#2}) \\
& - 4\big(\vec{H} \cdot \vec{C}_{420M}\big)\big(\vec{\rho} \cdot \vec{\rho}\big) && (W_{linear}) \\
& + D_{420M}\big(\vec{\rho} \cdot \vec{\rho}\big) && (W_{constant})
\end{aligned}
\tag{2.14}
$$

Where $\vec{A}_{420_M}$, B_{420_M}, $\vec{B}^2_{420_M}$, $\vec{C}_{420_M}$ and $\vec{D}_{420_M}$ are defined in table 2.1.

- Quartic astigmatism

  The Wavefront expansion for quartic astigmatism using equation 2.7 is:

$$
\begin{aligned}
W = {} & \frac{1}{2}W_{422}\big(\vec{H}\cdot\vec{H}\big)\vec{H}^2\cdot\vec{\rho}^2 && (W_{quartic}) \\
& -\big(\vec{H}\cdot\vec{H}\big)\big(\vec{H}\vec{A}_{422}\big)\cdot\vec{\rho}^2 && (W_{cubic\#1}) \\
& +\frac{3}{2}\big(\vec{H}\cdot\vec{H}\big)\vec{B}^2_{422}\cdot\vec{\rho}^2 && (W_{quadratic\#1}) \\
& -\big(\vec{H}\cdot\vec{A}_{422}\big)\vec{H}^2\cdot\vec{\rho}^2 && (W_{cubic\#2}) \\
& -\big(\vec{C}^3_{422}\vec{H}^*\big)\cdot\vec{\rho}^2 && (W_{linear\#1}) \\
& +\frac{3}{2}B_{422}\vec{H}^2\cdot\vec{\rho}^2 && (W_{quadratic\#2}) \\
& -\frac{3}{2}\big(\vec{H}\vec{C}_{422}\big)\cdot\vec{\rho}^2 && (W_{linear\#2}) \\
& +\frac{1}{2}\vec{D}^2_{422}\cdot\vec{\rho}^2 && (W_{constant})
\end{aligned}
\tag{2.15}
$$

  Where $\vec{A}_{422}$, B_{422}, $\vec{B}^2_{422}$, $\vec{C}_{422}$, $\vec{C}^3_{422}$ and $\vec{D}^2_{422}$ are defined in table 2.1.

- Fifth order coma

  The Wavefront expansion for fifth order coma using equation 2.7 is:

$$
W = \big((W_{151}\vec{H}-\vec{A}_{151})\cdot\vec{\rho}\big)\big(\vec{\rho}\cdot\vec{\rho}\big)^2 = W_{linear}+W_{constant}
\tag{2.16}
$$

  Where $\vec{A}_{151}$ is defined in table 2.1.

  There is a linear and a constant part as third order coma.

- Oblique spherical aberration

  The Wavefront expansion for oblique spherical aberration using equation 2.7 is:

$$
W = \frac{1}{2}\Big(\big(W_{242}\vec{H}^2-2\vec{H}\vec{A}_{242}+\vec{B}^2_{242}\big)\cdot\vec{\rho}^{\,2}\Big)\rho^2 = W_{quadratic}+W_{linear}+W_{constant}
\tag{2.17}
$$

  Where $\vec{A}_{242}$ and $\vec{B}^2_{242}$ are defined in table 2.1.

- Field curvature for oblique spherical aberration

  The Wavefront expansion for field curvature for oblique spherical aberration using equation 2.7 is:

  $$W = \left(W_{240M}(\vec{H}\cdot\vec{H}) - 2\vec{H}\cdot\vec{A}_{240M} + B_{240M}\right)\left(\vec{\rho}\cdot\vec{\rho}\right)^2 = W_{quadratic} + W_{linear} + W_{constant}$$

  (2.18)

  Where $\vec{A}_{240M}$ and B_{240M} are defined in table 2.1.

- Fifth order distortion

  The Wavefront expansion for fifth order distortion using equation 2.7 is:

  $$
  \begin{aligned}
  W = \; & W_{511}(\vec{H}\cdot\vec{H})(\vec{H}\cdot\vec{H})\vec{H}\cdot\vec{\rho} && (W_{fifth}) \\
  & - 4(\vec{H}\cdot\vec{H})(\vec{H}\cdot\vec{A}_{511})\vec{H}\cdot\vec{\rho} && (W_{quartic\#1}) \\
  & + 6B_{511}(\vec{H}\cdot\vec{H})\vec{H}\cdot\vec{\rho} && (W_{cubic\#1}) \\
  & + 2(\vec{H}^2\cdot\vec{B}_{511}^2)\vec{H}\cdot\vec{\rho} && (W_{cubic\#2}) \\
  & - 4(\vec{H}\cdot\vec{C}_{511})\vec{H}\cdot\vec{\rho} && (W_{quadratic\#1}) \\
  & + 3D_{511}\vec{H}\cdot\vec{\rho} && (W_{linear\#1}) \\
  & - (\vec{H}\cdot\vec{H})(\vec{H}\cdot\vec{H})\vec{A}_{511}\cdot\vec{\rho} && (W_{quartic\#2}) \\
  & + 2(\vec{H}\cdot\vec{H})(\vec{B}_{511}^2\vec{H}^*) && (W_{cubic\#3}) \\
  & - 4(\vec{H}\cdot\vec{H})\vec{C}_{511}\cdot\vec{\rho} && (W_{quadratic\#2}) \\
  & - (\vec{H}^2\vec{C}_{511}^*)\cdot\vec{\rho} && (W_{quadratic\#3}) \\
  & - (\vec{C}_{511}^3\vec{H}^{2*})\cdot\vec{\rho} && (W_{quadratic\#4}) \\
  & + 2(\vec{D}_{511}^2\vec{H}^*)\cdot\vec{\rho} && (W_{linear\#2}) \\
  & - \vec{E}_{511}\cdot\vec{\rho} && (W_{constant})
  \end{aligned}
  $$

  (2.19)

  Where $\vec{A}_{511}$, B_{511}, $\vec{B}_{511}^2$, $\vec{C}_{511}^*$, $\vec{C}_{511}$, $\vec{C}_{511}^3$, D_{511}, $\vec{D}_{511}^2$ and $\vec{E}_{511}$ are defined in table 2.1.

At this point, the expressions and the behavior of the aberrations in a misaligned system are known. But the wavefront expansion used for these theoretical derivations

will not be convenient to use with optical software or with wavefront sensors during the process of alignment. The Zernike polynomials would be more convenient. Thus the next chapter will show how to relate the wavefront expansion of the different aberrations to the Zernike polynomials and their field dependencies.

$$W_{klm} = \sum_j W_{klm_j}$$

$$\vec{A}_{klm} = \sum_j W_{klm_j}\, \vec{\sigma}_j$$

$$B_{klm} = \sum_j W_{klm_j} (\vec{\sigma}_j \cdot \vec{\sigma}_j)$$

$$\vec{B}^2_{klm} = \sum_j W_{klm_j}\, \vec{\sigma}_j^2$$

$$\vec{C}_{klm} = \sum_j W_{klm_j} (\vec{\sigma}_j \cdot \vec{\sigma}_j)\, \vec{\sigma}_j$$

$$\vec{C}^3_{klm} = \sum_j W_{klm_j}\, \vec{\sigma}_j^3$$

$$D_{klm} = \sum_j W_{klm_j} (\vec{\sigma}_j \cdot \vec{\sigma}_j)^2$$

$$\vec{D}^2_{klm} = \sum_j W_{klm_j} (\vec{\sigma}_j \cdot \vec{\sigma}_j)\, \vec{\sigma}_j^2$$

$$\vec{E}_{klm} = \sum_j W_{klm_j} (\vec{\sigma}_j \cdot \vec{\sigma}_j)^2\, \vec{\sigma}_j$$

TABLE 2.1. Vectors and scalars used in the expression of the wavefront expansion

Chapter 3

EXPRESSIONS OF THE FIELD DEPENDENCIES OF THE ZERNIKE POLYNOMIALS IN A PERTURBED OPTICAL SYSTEM

Chapter 2 has proved that the common aberrations in a misaligned system can be expressed. It was shown that no new aberrations were created but the field dependencies of each aberration were modified. Moreover, it has been seen that the new field dependencies were always a power smaller than the original aberration. For example, third order astigmatism which has a quadratic field dependence was exhibiting linear and constant dependencies in a misaligned system.

The results shown in chapter 2 and which were derived in great detail in the dissertation of K.P. Thompson give an insight in the behavior of a misaligned system. It is a convenient tool to make a sensitivity analysis for example. But it would be easier to retrieve the perturbations in a misaligned system using Zernike polynomials which are more familar functions than the formulation in terms of wavefront expansions. The reason is that the wavefront at the exit of an optical system is generally fitted with the Zernike coefficients. Also, they are commonly used in optical software.

The issue with Zernike polynomials is that they do not explicitly give the information about the field dependencies. They are expressed in terms of ρ and ϕ and the information on the field dependencies is contained in the Zernike coefficients. Thus, distortion will be represented like tilt with Z_2 and Z_3[1] since distortion behaves like tilt except that its field dependence is cubic instead of being linear. It is the same thing for third order field curvature and defocus, they are both represented by Z_4[1] since field curvature is no more than defocus with a quadratic dependence.

---

[1] For the standard Zernike polynomials: $Z_2 = \sqrt{4}\rho\cos\phi$, $Z_3 = \sqrt{4}\rho\sin\phi$ and $Z_4 = \sqrt{3}(2\rho^2 - 1)$

The previous chapter has shown that tilts and decenters will modify the field dependencies of the aberrations. Thus, the information on the field for the Zernike polynomials is necessary to decompose the aberrations in terms of centered and misaligned aberrations.

In this chapter, it will be shown how the equations of chapter 2 relate to the Zernike polynomials. The Zernike coefficients will be explicitly expressed in terms of the field dependencies so as to use them to fit the data from a ray tracing program or a wavefront sensor. Therefore, the equations in this section will appear complicated but they will be convenient for fit purposes. This set of equations represents a powerful tool to align an optical system because they are general and work for any type of system as it will be shown in chapter 4 and 5.

First, the wavefront expansion and the Zernike coefficients will be related using the vector multiplication. Then, the equations which give the field dependencies for the Zernike polynomials for the third and fifth order will be given. And finally, the useful equations will be compiled to summarize this chapter.

3.1 Relations between the wavefront expansion and the Zernike polynomials by using the vector multiplication

To relate the vectorial form of the wavefront expansion to the Zernike polynomials, the vector multiplication will be used. Thus in this section, the vector multiplication will be explained in detail and the required equations to express the Zernike coefficients in the following sections will be underlined.

A multiplication between vectors gives an other vector coplanar with the other two. To be able to multiply two vectors, a vector needs to be considered as a phasor. If we consider two vectors $\vec{A}$ and $\vec{B}$ and express them as:

$$\vec{A} = ae^{i\alpha} = a_x\hat{i} + a_y\hat{j} \qquad a_x = a\sin\alpha,\, a_y = a\cos\alpha$$

$$\vec{B} = be^{i\beta} = b_x\hat{i} + b_y\hat{j} \qquad b_x = b\sin\beta,\, b_y = b\cos\beta$$

Then the multiplication between these two vectors is defined as:

$$\vec{A}\vec{B} = ab\,e^{i(\alpha+\beta)} = (a_y b_x + a_x b_y)\,\hat{i} + (a_y b_y - a_x b_x)\,\hat{j}$$
$$= ab\sin(\alpha+\beta)\,\hat{i} \;+\; ab\cos(\alpha+\beta)\,\hat{j} \tag{3.1}$$

By contrast to the dot product which is defined as follows:

$$\vec{A}\cdot\vec{B} = ab\,cos(\alpha-\beta) = a_x b_x + a_y b_y \tag{3.2}$$

The multiplication produces an other vector and the dot product a scalar. Thus, when a vector is squared or cubed, the results is not simply the magnitude of the vector squared or cubed as shown in equations 3.3 and 3.4.

$$\vec{A}^2 = a^2 e^{i2\alpha} = 2a_x a_y\,\hat{i} \;+\; (a_y^2 - a_x^2)\,\hat{j}$$
$$= a^2\sin 2\alpha\,\hat{i} \;+\; a^2\cos 2\alpha\,\hat{j} \tag{3.3}$$

$$\vec{A}^3 = a^3 e^{i3\alpha} = (3a_y^2 a_x - a_x^3)\,\hat{i} \;+\; (a_y^3 - 3a_x^2 a_y)\,\hat{j}$$
$$= a^3\sin 3\alpha\,\hat{i} \;+\; a^3\cos 3\alpha\,\hat{j} \tag{3.4}$$

The dissertation of Kevin P. Thompson also introduces conjugate vectors to preserve pupil dependence. In the expression of the vector, this implies a sign change in the exponent. The conjugate of $\vec{A}$ is expressed as:

$$\vec{A}^* = ae^{-i\alpha} = -a_x\hat{i} + a_y\hat{j} \tag{3.5}$$

Thus, the vector product with vector conjugate is defined as:

$$\vec{A}\vec{B}^* = ab\,e^{i(\alpha-\beta)} = (a_x b_y - a_y b_x)\,\hat{i} + (a_y b_y + a_x b_x)\,\hat{j} \tag{3.6}$$

Based on equations 3.3 through 3.5, the subsequent equations for $\vec{H}$ and $\vec{\rho}$ are derived.

For $\vec{H}$, the derivations are:

$$\vec{H}^2 \;=\; 2xy\,\hat{i} + (y^2 - x^2)\hat{j} \tag{3.7}$$

$$\vec{H}^3 \;=\; (3y^2 x - x^3)\,\hat{i} + (y^3 - 3x^2 y)\hat{j} \tag{3.8}$$

$$\vec{H}^* \;=\; -x\,\hat{i} + y\,\hat{j} \tag{3.9}$$

For $\vec{\rho}$, the derivations are:

$$\vec{\rho}^2 \; = \; \rho^2 \sin 2\phi \; \hat{i} + \rho^2 \cos 2\phi \, \hat{j} \tag{3.10}$$

$$\vec{\rho}^3 \; = \; \rho^3 \sin 3\phi \; \hat{i} + \rho^3 \cos 3\phi \, \hat{j} \tag{3.11}$$

$$\vec{\rho}^* \; = \; -\rho \sin \phi \; \hat{i} + \rho \cos \phi \, \hat{j} \tag{3.12}$$

Then by using equations 3.2, 3.1, 3.6, 3.7-3.12 in 2.8-2.19 the equations of the Zernike coefficients in the following sections 3.2 and 3.3 will be derived.

3.2 Field dependencies of the Zernike polynomials for the third order aberrations

In this section, only the third order aberrations for the Zernike polynomials will be considered. In this thesis, the standard Zernike polynomials are used and the ones which are considered in this section are represented in table 3.1.

Term	Orthonormal Zernike Polynomial $Z(\rho,\phi)$	Aberration name
2	$\sqrt{4}\rho \cos \phi$	Tilt X
3	$\sqrt{4}\rho \sin \phi$	Tilt Y
4	$\sqrt{3}(2\rho^2 - 1)$	Defocus
5	$\sqrt{6}(\rho^2 \sin 2\phi)$	45° 3rd order astigmatism
6	$\sqrt{6}(\rho^2 \cos 2\phi)$	0° 3rd order astigmatism
7	$\sqrt{8}(3\rho^3 - 2\rho) \sin \phi$	90° 3rd order coma
8	$\sqrt{8}(3\rho^3 - 2\rho) \cos \phi$	0° 3rd order coma

TABLE 3.1. Zernike polynomials for the third order aberrations

As it was said before, the Zernike polynomials do not explicitly include the information on the field dependencies, they are only expressed with ρ and ϕ. This section and the following one will show how to express explicitly the Zernike coefficients in terms of the field components x and y. It will be shown that the wavefront expansion

can be decomposed as:

$$W = \sum_i \left(\left(\sum_j C_i^j(x,y) \right) \cdot Z_i(\rho,\phi) \right) \quad \text{where} \quad \begin{cases} Z_i \text{ are the Zernike polynomials} \\ C_i^j \text{ are the Zernike coefficients which} \\ \text{contain the field dependencies of the } Z_i \\ i \text{ is the number of a zernike term} \\ j \text{ is the power of the field dependence} \end{cases}$$

(3.13)

By having the expressions of the coefficients C_i^j for the different zernike terms, it will be possible to fit the zernike coefficients coming from a raytracing program or a wavefront sensor for different field positions. The purpose of this section and the following one, is to find the expressions of these $C_i^j(x,y)$ for all the aberrations from third to fifth order. Note that sometimes in the literature the Z_i represent the coefficients instead of the polynomials.

- Third order astigmatism

  The different components of equation 2.8 can be expressed in terms of the field components. Again, the components of $\vec{H}$ along $\vec{x}$ and $\vec{y}$ are x and y respectively. And for $\vec{\rho}$ the components are $\rho \sin\phi$ and $\rho \cos\phi$. This is emphasized, because in the literature x and y sometimes represent the pupil components instead of the field components.

  For the quadratic component, the derivations are:

$$\begin{aligned} W_{quadratic} &= \frac{1}{2} W_{222} \vec{H}^2 \cdot \vec{\rho}^2 \\ &= \frac{1}{2} W_{222} \left(2\,x\,y\rho^2 \sin 2\phi + (y^2 - x^2)\rho^2 \cos 2\phi \right) \end{aligned}$$

(3.14)

Since $\frac{1}{2}W_{222}$ is constant, the constant α_0 which is equal to this term is introduced, then equation 3.14 becomes:

$$W_{quadratic} = \left(2\,\alpha_0 x\,y \right) \rho^2 \sin 2\phi + \left(\alpha_0(y^2 - x^2) \right) \rho^2 \cos 2\phi \tag{3.15}$$

$\sqrt{6}\rho^2 \sin 2\phi$ and $\sqrt{6}\rho^2 \cos 2\phi$ represent the standard Zernike terms Z_5 and Z_6 as shown in table 3.1. Thus, equation 3.15 can be rewritten as:

$$W_{quadratic} = \frac{2\,\alpha_0 x\,y}{\sqrt{6}} \cdot Z_5 + \frac{\alpha_0(y^2 - x^2)}{\sqrt{6}} \cdot Z_6$$
$$= C_5^2 \cdot Z_5 + C_6^2 \cdot Z_6 \tag{3.16}$$

Then, the constant α_0 can be redefined (by including $\sqrt{6}$ in it) so as to get the simplified terms:

$$\begin{cases} C_5^2 &= 2\,\alpha_0 x\,y \\ C_6^2 &= \alpha_0(y^2 - x^2) \end{cases} \tag{3.17}$$

As it was said before, this quadratic part represents the astigmatism of a centered system. The subscript $_0$ for the constant was used to express this aberration. For the following aberrations the subscript $_0$ will be used again to express the aberrations of a centered system.

For the linear component, the derivations are:

$$W_{linear} = -\left(\vec{H}\vec{A}_{222}\right) \cdot \vec{\rho}^2$$
$$= -\left((A_{222x}y + A_{222y}x)\rho^2 \sin 2\phi + (-A_{222x}x + A_{222y}y)\rho^2 \cos 2\phi\right) \tag{3.18}$$

Equation 3.18 represents the linear part of the astigmatism, where A_{222x} and A_{222y} are the $\vec{x}$ and $\vec{y}$ components of $\vec{A}_{222}$. Since these two components are constant, an expression of the linear astigmatism with two new constants α_1 and α_2 can be found as it was done before for the quadratic term:

$$\begin{cases} C_5^1 &= \alpha_1 y + \alpha_2 x \\ C_6^1 &= -\alpha_1 x + \alpha_2 y \end{cases} \tag{3.19}$$

For the constant part the derivations are:

$$W_{constant} = \frac{1}{2}\vec{B}_{222}^2 \cdot \vec{\rho}^2$$
$$= \left(2B_{222x}B_{222y}\rho^2 \sin 2\phi + (B_{222y}^2 - B_{222x}^2)\rho^2 \cos 2\phi\right) \tag{3.20}$$

Equation 3.20 represents the constant part of the astigmatism, where B_{222x} and B_{222y} are the x and y components of $\vec{B}_{222}$. As for the linear component, an

expression of the constant astigmatism with Z_5 and Z_6 with two new constants α_3 and α_4 can be found since B_{222x} and B_{222y} are two constants:

$$\begin{cases} C_5^0 & = \alpha_3 \\ C_6^0 & = \alpha_4 \end{cases} \tag{3.21}$$

Considering all the components of the astigmatism in a misaligned system, C_5 and C_6 can be defined as:

$$\begin{aligned} C_5 = \sum C_5^j = 2\,\alpha_0 x\,y + \alpha_1 y + \alpha_2 x + \alpha_3 \\ C_6 = \sum C_6^j = \alpha_0(y^2 - x^2) - \alpha_1 x + \alpha_2 y + \alpha_4 \end{aligned} \tag{3.22}$$

The same technique is used to find the subsequent expressions for the other aberrations with the conventions defined in figure 2.1. The independent constants β_j, ξ_j, μ_j, χ_j, κ_j, γ_j, δ_j, ν_j, ζ_j, η_j ($\forall$j) are used to express the other aberrations. The subscript $_0$ for the constants will be used to express the aberrations of a centered system and the following numbers will be used to express the aberrations that can possibly arise in a misaligned system.

- Third order coma

  As it has been shown for astigmatism, the different components of coma in equation 2.9 can be expressed in terms of the field components x and y as shown in equation 3.23 and 3.25. And then the expressions of the field dependencies of the Zernike coefficients can be derived.

$$W_{linear} = W_{131}(\vec{H} \cdot \vec{\rho})\rho^2 = W_{131}(x\rho^3 \sin\phi + y\rho^3 \cos\phi) \tag{3.23}$$

$$\implies \begin{cases} C_7^1 & = \beta_0 x \\ C_8^1 & = \beta_0 y \end{cases} \tag{3.24}$$

$$W_{constant} = -(\vec{A}_{131} \cdot \vec{\rho}) = -(A_{131x}\rho^3 \sin\phi + A_{131y}\rho^3 \cos\phi) \tag{3.25}$$

$$\implies \quad \begin{cases} C_7^0 & = \beta_1 \\ C_8^0 & = \beta_2 \end{cases} \tag{3.26}$$

Considering all the components of coma in a misaligned system, C_7 and C_8 can be defined as:

$$\begin{aligned} C_7 &= \sum C_7^j = \beta_0 x + \beta_1 \\ C_8 &= \sum C_8^j = \beta_0 y + \beta_2 \end{aligned} \tag{3.27}$$

- Third order field curvature

  C_4 can be derived using the same method as before: express the different components of third order field curvature in equation 2.10 in terms of the field components x and y and then derive C_4.

$$W_{quadratic} = W_{220M}(x^2 + y^2)\rho^2 \tag{3.28}$$

$$\implies \quad C_4^2 = \gamma_0(x^2 + y^2) \tag{3.29}$$

$$W_{linear} = -2(A_{220Mx}x + A_{220My}y)\rho^2 \tag{3.30}$$

$$\implies \quad C_4^1 = \gamma_1 x + \gamma_2 y \tag{3.31}$$

$$W_{constant} = B_{220M}\rho^2 \tag{3.32}$$

$$\implies \quad C_4^0 = \gamma_3 \tag{3.33}$$

Considering all the components of field curvature in a misaligned system, C_4 can be expressed as:

$$C_4 = \sum C_4^j = \gamma_0(x^2 + y^2) + \gamma_1 x + \gamma_2 y + \gamma_3 \tag{3.34}$$

- Third order distortion

  The different components of third order distortion in equation 2.11 can be express in terms of the field components x and y and then the expressions of C_2 and C_3 can be derived.

$$W_{cubic} = W_{311}(x^2 + y^2)\vec{H} \cdot (\rho \sin \phi \, \hat{i} + \rho \cos \phi \hat{j})$$

$$= W_{311}\big((x^3 + xy^2)\rho \sin \phi + (x^2 y + y^3)\rho \cos \phi\big) \tag{3.35}$$

$$\implies \begin{cases} C_2^3 & = \nu_0(x^2 y + y^3) \\ C_3^3 & = \nu_0(x^3 + xy^2) \end{cases} \tag{3.36}$$

$$W_{quadratic\#1} = -2(xA_{311x} + yA_{311y})(x\rho \sin \phi + y\rho \cos \phi)$$

$$= -2\big((x^2 A_{311x} + xyA_{311y})\rho \sin \phi + (xyA_{311x} + y^2 A_{311y})\rho \cos \phi\big) \tag{3.37}$$

$$\implies \begin{cases} C_2^2 & = \nu_1 xy + \nu_2 y^2 \\ C_3^2 & = \nu_1 x^2 + \nu_2 xy \end{cases} \tag{3.38}$$

$$W_{linear\#1} = 2B_{311}(x\rho \sin \phi + y\rho \cos \phi) \tag{3.39}$$

$$\implies \begin{cases} C_2^1 & = \nu_3 y \\ C_3^1 & = \nu_3 x \end{cases} \tag{3.40}$$

Since third order distortion has two quadratic and two linear dependencies, the prime sign will be used to differentiate the components.

$$W_{quadratic\#2} = -(x^2 + y^2)(A_{311x}\rho \sin \phi + A_{311y}\rho \cos \phi) \tag{3.41}$$

$$\implies \begin{cases} C_2^{2\prime} & = \nu_4(x^2 + y^2) \\ C_3^{2\prime} & = \nu_5(x^2 + y^2) \end{cases} \tag{3.42}$$

$$W_{linear\#2} = (\vec{B}_{311}^2 \vec{H}^*) \cdot (\rho \sin \phi \, \hat{i} + \rho \cos \phi \hat{j})$$

$$= \big((-(B_{311y}^2 - B_{311x}^2)x + 2B_{311x}B_{311y}y)\rho \sin \phi \tag{3.43}$$

$$+ (B_{311y}^2 - B_{311x}^2)y + 2B_{311x}B_{311y}x)\rho \cos \phi\big)$$

$$\implies \quad \begin{cases} C_2^{1'} &= \nu_6 y + \nu_7 x \\ C_3^{1'} &= -\nu_6 x + \nu_7 y \end{cases} \tag{3.44}$$

$$W_{constant} = -(C_{311x}\rho\sin\phi + C_{311y}\rho\cos\phi) \tag{3.45}$$

$$\implies \quad \begin{cases} C_2^0 &= \nu_8 \\ C_3^0 &= \nu_9 \end{cases} \tag{3.46}$$

Considering all the components of third order distortion in a misaligned system, C_2 and C_3 can be defined as:

$$C_2 = \sum C_2^j = \nu_0(x^2 y + y^3) + \nu_1 xy + \nu_2 y^2 + \nu_3 y + \nu_4(x^2 + y^2) + \nu_6 y + \nu_7 x + \nu_8$$

$$C_3 = \sum C_3^j = \nu_0(x^3 + xy^2) + \nu_1 x^2 + \nu_2 xy + \nu_3 x + \nu_5(x^2 + y^2) - \nu_6 x + \nu_7 y + \nu_9 \tag{3.47}$$

3.3 Field dependencies of the Zernike polynomials for the fifth order aberrations

Fifth order aberrations will introduce some new Zernike coefficients but will also reintroduce the Zernike coefficients that were seen for third order. That means that there can be some coupling between third and fifth order aberrations. This section will consider only the Zernike coefficients of table 3.2. The derivations of the C_i^j for this section are found by following the same procedure as for the third order aberrations.

- Field cubed coma

  The different components of field cubed coma in equation 2.12 can be expressed in terms of the field components x and y which leads to the expressions of C_7 and C_8. To differentiate the components that have the same field dependence, the prime sign will be used.

Term	Orthonormal Zernike Polynomial $Z(\rho,\phi)$	Aberration name
2	$\sqrt{4}\rho\cos\phi$	Tilt X
3	$\sqrt{4}\rho\sin\phi$	Tilt Y
4	$\sqrt{3}(2\rho^2-1)$	Defocus
5	$\sqrt{6}(\rho^2\sin 2\phi)$	$45°$ 3rd order astigmatism
6	$\sqrt{6}(\rho^2\cos 2\phi)$	$0°$ 3rd order astigmatism
7	$\sqrt{8}(3\rho^3-2\rho)\sin\phi$	$90°$ 3rd order coma
8	$\sqrt{8}(3\rho^3-2\rho)\cos\phi$	$0°$ 3rd order coma
9	$\sqrt{8}\rho^3\sin 3\phi$	$30°$ trefoil
10	$\sqrt{8}\rho^3\cos 3\phi$	$0°$ trefoil
11	$\sqrt{5}(6\rho^4-6\rho^2+1)$	3rd order spherical aberration
12	$\sqrt{10}(4\rho^4-3\rho^2)\cos 2\phi$	$0°$ 5th order astigmatism
13	$\sqrt{10}(4\rho^4-3\rho^2)\sin 2\phi$	$45°$ 5th order astigmatism
16	$\sqrt{12}(10\rho^5-12\rho^3+3\rho)\cos\phi$	$0°$ 5th order coma
17	$\sqrt{12}(10\rho^5-12\rho^3+3\rho)\sin\phi$	$90°5^{th}$ order coma

TABLE 3.2. Zernike polynomials for the fifth order aberrations

$$
\begin{aligned}
W_{cubic} &= W_{331_M}\left((x^2+y^2)\vec{H}\cdot\vec{\rho}\right)\rho^2 \\
&= W_{331_M}\left((x^3+xy^2)\rho^3\sin\phi + (y^3+x^2y)\rho^3\cos\phi\right)
\end{aligned}
\tag{3.48}
$$

$$
\implies \begin{cases} C_7^3 &= \xi_0(x^3+xy^2) \\ C_8^3 &= \xi_0(y^3+x^2y) \end{cases}
\tag{3.49}
$$

$$
\begin{aligned}
W_{quadratic\#1} &= -2\left(\left(xA_{331_{Mx}}+yA_{331_{My}}\right)\vec{H}\cdot\rho^2\vec{\rho}\right) \\
&= -2\left(\left(A_{331_{Mx}}x^2+A_{331_{My}}xy\right)\rho^3\sin\phi \right. \\
&\quad \left. + \left(A_{331_{Mx}}xy+A_{331_{My}}y^2\right)\rho^3\cos\phi\right)
\end{aligned}
\tag{3.50}
$$

$$
\implies \begin{cases} C_7^2 &= \xi_1\,x^2 + \xi_2\,xy \\ C_8^2 &= \xi_1\,xy + \xi_2\,y^2 \end{cases}
\tag{3.51}
$$

$$
\begin{aligned}
W_{linear\#1} &= 2B_{331_M}\vec{H}\cdot\rho^2\vec{\rho} \\
&= 2B_{331_M}\left(x\rho^3\sin\phi + y\rho^3\cos\phi\right)
\end{aligned}
\tag{3.52}
$$

$$\implies \begin{cases} C_7^1 & = \xi_3 \, x \\ C_8^1 & = \xi_3 \, y \end{cases} \tag{3.53}$$

$$\begin{aligned} W_{quadratic\#2} &= -\left((x^2 + y^2) \vec{A}_{331_M} \cdot \rho^2 \vec{\rho} \right) \\ &= -\left(A_{331_{Mx}} (x^2 + y^2) \rho^3 \sin\phi + A_{331_{My}} (x^2 + y^2) \rho^3 \cos\phi \right) \end{aligned} \tag{3.54}$$

$$\implies \begin{cases} C_7^{2'} & = \xi_4 \, (x^2 + y^2) \\ C_8^{2'} & = \xi_5 \, (x^2 + y^2) \end{cases} \tag{3.55}$$

$$\begin{aligned} W_{linear\#2} &= \left(\vec{B}_{331_M}^2 \vec{H}^* \right) \cdot \rho^2 \vec{\rho} \\ &= \left(\left((B_{331_{Mx}}^2 - B_{331_{My}}^2)x + (2B_{331_{Mx}} B_{331_{My}})y \right) \hat{i} \right. \\ &\quad + \left. \left((B_{331_{My}}^2 - B_{331_{Mx}}^2)y + (2B_{331_{Mx}} B_{331_{My}})x \right) \hat{j} \right) \left(\rho^3 \sin\phi \, \hat{i} + \rho^3 \cos\phi \, \hat{j} \right) \\ &= \left(2B_{331_{Mx}} B_{331_{My}} y + \left(B_{331_{Mx}}^2 - B_{331_{My}}^2 \right)x \right) \rho^3 \sin\phi \\ &\quad + \left(2B_{331_{Mx}} B_{331_{My}} x - \left(B_{331_{Mx}}^2 - B_{331_{My}}^2 \right)y \right) \rho^3 \cos\phi \end{aligned} \tag{3.56}$$

$$\implies \begin{cases} C_7^{1'} & = \xi_6 \, y + \xi_7 \, x \\ C_8^{1'} & = \xi_6 \, x - \xi_7 \, y \end{cases} \tag{3.57}$$

$$\begin{aligned} W_{constant} &= -\left(\vec{C}_{331_M} \cdot \vec{\rho} \right) \rho^2 \\ &= -C_{331_{Mx}} \rho^3 \sin\phi - C_{331_{My}} \rho^3 \cos\phi \end{aligned} \tag{3.58}$$

$$\implies \begin{cases} C_7^0 & = \xi_8 \\ C_8^0 & = \xi_9 \end{cases} \tag{3.59}$$

Considering all the components of field cubed coma in a misaligned system, C_7 and C_8 can be defined as:

$$\begin{aligned} C_7 = \sum C_7^j &= \xi_0(x^3 + xy^2) + \xi_1 \, x^2 + \xi_2 \, xy + \xi_3 \, x \\ &\quad + \xi_4 \, (x^2 + y^2) + \xi_6 \, y + \xi_7 \, x + \xi_8 \\ C_8 = \sum C_8^j &= \xi_0(y^3 + x^2 y) + \xi_1 \, xy + \xi_2 \, y^2 + \xi_3 \, y \\ &\quad + \xi_5 \, (x^2 + y^2) + \xi_6 \, x - \xi_7 \, y + \xi_9 \end{aligned} \tag{3.60}$$

- Elliptical coma (Trefoil)

  The derivations of C_9 and C_{10} is done with the same method as before by expressing the different components of elliptical coma in equation 2.13 in terms of the field components x and y.

$$W_{cubic} = \frac{1}{4} W_{333} \left((3y^2 x - x^3)\rho^3 \sin 3\phi + (y^3 - 3x^2 y)\rho^3 \cos 3\phi \right) \tag{3.61}$$

$$\implies \begin{cases} C_9^3 &= \mu_0(3y^2 x - x^3) \\ C_{10}^3 &= \mu_0(y^3 - 3x^2 y) \end{cases} \tag{3.62}$$

$$W_{quadratic} = -\frac{3}{4} \left(\left(A_{333x}(-x^2 + y^2) + 2A_{333y}xy \right)\hat{i} \right.$$
$$\left. + \left((A_{333x}(-2xy) + A_{333y}(y^2 - x^2)) \right)\hat{j} \right) \cdot \left(\rho^3 \sin 3\phi\,\hat{i} + \rho^3 \cos 3\phi\,\hat{j} \right) \tag{3.63}$$

$$\implies \begin{cases} C_9^2 &= \mu_1(y^2 - x^2) + \mu_2 2xy \\ C_{10}^2 &= -2\mu_1 xy + \mu_2(y^2 - x^2) \end{cases} \tag{3.64}$$

$$W_{linear} = \frac{3}{4} \left(\left(x(B_{333y}^2 - B_{333x}^2) + y2B_{333x}B_{333y} \right)\hat{i} \right.$$
$$\left. + \left(y(B_{333y}^2 - B_{333x}^2) - x2B_{333x}B_{333y} \right)\hat{j} \right) \cdot \left(\rho^3 \sin 3\phi\,\hat{i} + \rho^3 \cos 3\phi\,\hat{j} \right) \tag{3.65}$$

$$\implies \begin{cases} C_9^1 &= \mu_3 x + \mu_4 y \\ C_{10}^1 &= \mu_3 y - \mu_4 x \end{cases} \tag{3.66}$$

$$W_{constant} = -\frac{1}{4} \left((3C_{333y}^2 C_{333x} - C_{333x}^3)\rho^3 \sin 3\phi + (C_{333y}^3 - 3C_{333x}^2 C_{333y})\rho^3 \cos 3\phi \right) \tag{3.67}$$

$$\implies \begin{cases} C_9^0 &= \mu_5 \\ C_{10}^0 &= \mu_6 \end{cases} \tag{3.68}$$

Considering all the components of elliptical coma in a misaligned system, C_9 and C_{10} can be defined as:

$$C_9 = \sum C_9^j = \mu_0(3y^2x - x^3) + \mu_1(y^2 - x^2) + \mu_2 2xy + \mu_3 x + \mu_4 y + \mu_5$$
$$C_{10} = \sum C_{10}^j = \mu_0(y^3 - 3x^2y) - 2\mu_1 xy + \mu_2(y^2 - x^2) + \mu_3 y - \mu_4 x + \mu_6$$
$$(3.69)$$

- Quartic field curvature

  C_4 can be derived using the same method as before: express the different components of quartic field curvature in equation 2.14 in terms of the field components x and y and then derive C_4.

$$W_{quartic} = W_{420M}(x^2 + y^2)(x^2 + y^2)\rho^2 \qquad (3.70)$$

$$\implies \qquad C_4^4 = \psi_0(x^4 + y^4 + 2x^2y^2) \qquad (3.71)$$

$$W_{cubic} = -4(x^2 + y^2)(A_{420Mx}x + A_{420My}y)\rho^2 \qquad (3.72)$$

$$\implies \qquad C_4^3 = \psi_1(x^3 + xy^2) + \psi_2(x^2y + y^3) \qquad (3.73)$$

$$W_{quadratic\#1} = B_{420M}(x^2 + y^2)\rho^2 \qquad (3.74)$$

$$\implies \qquad C_4^2 = \psi_3(x^2 + y^2) \qquad (3.75)$$

$$W_{quadratic\#2} = 2\big(2B_{420Mx}B_{420My} \cdot 2xy + (B_{420My}^2 - B_{420Mx}^2)(y^2 - x^2)\big)\rho^2 \qquad (3.76)$$

$$\implies \qquad C_4^{2'} = \psi_4 xy + \psi_5(y^2 - x^2) \qquad (3.77)$$

$$W_{linear} = -4(C_{420Mx}x + C_{420My}y)\rho^2 \qquad (3.78)$$

$$\implies \quad C_4^1 = \psi_6 x + \psi_7 y \tag{3.79}$$

$$W_{constant} = D_{420M}\rho^2 \tag{3.80}$$

$$\implies \quad C_4^0 = \psi_8 \tag{3.81}$$

Considering all the components of quartic field curvature in a misaligned system, C_4 can be expressed as:

$$
\begin{aligned}
C_4 = \sum C_4^j =& \psi_0(x^4 + y^4 + 2x^2y^2) + \psi_1(x^3 + xy^2) + \psi_2(x^2y + y^3) \\
& + \psi_3(x^2 + y^2) + \psi_4 xy + \psi_5(y^2 - x^2) + \psi_6 x + \psi_7 y + \psi_8
\end{aligned}
\tag{3.82}
$$

- Quartic astigmatism

  The different components of quartic astigmatism in equation 2.15 can be expressed in terms of the field components x and y and then the expressions of C_5 and C_6 can be derived. Again, to differentiate the components that have the same field dependence, the prime sign will be used.

$$
\begin{aligned}
W_{quartic} &= \frac{1}{2}W_{422}(x^2 + y^2)(2xy\rho^2 \sin 2\phi + (y^2 - x^2)\rho^2 \cos 2\phi) \\
&= \frac{1}{2}W_{422}((2x^3y + 2xy^3)\rho^2 \sin 2\phi + (y^4 - x^4)\rho^2 \cos 2\phi)
\end{aligned}
\tag{3.83}
$$

$$
\implies \quad
\begin{cases}
C_5^4 &= 2\chi_0(x^3y + xy^3) \\
C_6^4 &= \chi_0(y^4 - x^4)
\end{cases}
\tag{3.84}
$$

$$
\begin{aligned}
W_{cubic\#1} &= -(x^2 + y^2)((xA_{422y} + yA_{422x})\rho^2 \sin 2\phi + (yA_{422y} - xA_{422x})\rho^2 \cos 2\phi) \\
&= -(A_{422x}(x^2y + y^3) + A_{422y}(x^3 + xy^2))\rho^2 \sin 2\phi \\
&\quad + (A_{422y}(x^2y + y^3) - A_{422x}(x^3 + xy^2))\rho^2 \cos 2\phi
\end{aligned}
\tag{3.85}
$$

$$
\implies \quad
\begin{cases}
C_5^3 &= \chi_1(x^2y + y^3) + \chi_2(x^3 + xy^2) \\
C_6^3 &= -\chi_1(x^3 + xy^2) + \chi_2(x^2y + y^3)
\end{cases}
\tag{3.86}
$$

$$W_{quadratic\#1} = \frac{3}{2}\big((x^2+y^2)2B_{422x}B_{422y}\rho^2 \sin 2\phi + (x^2+y^2)(B_{422y}^2 - B_{422x}^2)\rho^2 \cos 2\phi\big)$$

(3.87)

$$\implies \quad \begin{cases} C_5^2 &= \chi_3(x^2 + y^2) \\ C_6^2 &= \chi_4(x^2 + y^2) \end{cases}$$

(3.88)

$$W_{cubic\#2} = -\Big(\big(A_{422x}(xy^2 - x^3) + A_{422y}(2xy^2)\big)\rho^2 \sin 2\phi$$
$$+ \big(-A_{422x}(2x^2y) + A_{422y}(y^3 - x^2y)\big)\rho^2 \cos 2\phi\Big)$$

(3.89)

$$\implies \quad \begin{cases} C_5^{3'} &= \chi_5(xy^2 - x^3) + \chi_6(2xy^2) \\ C_6^{3'} &= -\chi_5(2x^2y) + \chi_6(y^3 - x^2y) \end{cases}$$

(3.90)

$$W_{linear\#1} = -\frac{1}{2}\Big(\big(3C_{422y}^2C_{422x} - C_{422x}^3)y - (C_{422y}^3 - 3C_{422x}^2C_{422y})x\big)\rho^2 \sin 2\phi$$
$$+ \big((3C_{422y}^2C_{422x} - C_{422x}^3)x + (C_{422y}^3 - 3C_{422x}^2C_{422y})y\big)\rho^2 \cos 2\phi\Big)$$

(3.91)

$$\implies \quad \begin{cases} C_5^1 &= \chi_7 y - \chi_8 x \\ C_6^1 &= \chi_7 x + \chi_8 y \end{cases}$$

(3.92)

$$W_{quadratic\#2} = \frac{3}{2}B_{422}\big(2xy\,\rho^2 \sin 2\phi + (y^2 - x^2)\rho^2 \cos 2\phi\big)$$

(3.93)

$$\implies \quad \begin{cases} C_5^{2'} &= \chi_9 2xy \\ C_6^{2'} &= \chi_9(-x^2 + y^2) \end{cases}$$

(3.94)

$$W_{linear\#2} = -\frac{3}{2}\big((C_{422y}x + C_{422x}y)\rho^2 \sin 2\phi + (C_{422y}y - C_{422x}x)\rho^2 \cos 2\phi\big)$$

(3.95)

$$\implies \quad \begin{cases} C_5^{1'} &= \chi_{10}x + \chi_{11}y \\ C_6^{1'} &= \chi_{10}y - \chi_{11}x \end{cases}$$

(3.96)

$$W_{constant} = \frac{1}{2}\big(2D_{422x}D_{422y}\rho^2 \sin 2\phi + (D_{422y}^2 - D_{422x}^2)\rho^2 \cos 2\phi\big)$$

(3.97)

$$\implies \begin{cases} C_5^0 & = \chi_{12} \\ C_6^0 & = \chi_{13} \end{cases} \tag{3.98}$$

Considering all the components of quartic astigmatism in a misaligned system, C_5 and C_6 can be defined as:

$$C_5 = \sum C_5^j = 2\chi_0(x^3y + xy^3) + \chi_1(x^2y + y^3) + \chi_2(x^3 + xy^2) + \chi_3(x^2 + y^2)$$
$$+ \chi_5(xy^2 - x^3) + \chi_6(2xy^2) + \chi_7y - \chi_8x + \chi_9 2xy$$
$$+ \chi_{10}x + \chi_{11}y + \chi_{12}$$

$$C_6 = \sum C_6^j = \chi_0(y^4 - x^4) - \chi_1(x^3 + xy^2) + \chi_2(x^2y + y^3) + \chi_4(x^2 + y^2)$$
$$- \chi_5(2x^2y) + \chi_6(y^3 - x^2y) + \chi_7x + \chi_8y + \chi_9(-x^2 + y^2)$$
$$+ \chi_{10}y - \chi_{11}x + \chi_{13}$$

$$\tag{3.99}$$

- Fifth order coma

  As before, the expression of the different components of fifth order coma in equation 2.16 in terms of the field components x and y leads to the derivation of Z_{16} and Z_{17}.

$$W_{linear} = \sum_j W_{151j}\left(x\rho^5 \sin\phi + y\rho^5 \cos\phi\right) \tag{3.100}$$

$$\implies \begin{cases} C_{16}^1 & = \kappa_0 y \\ C_{17}^1 & = \kappa_0 x \end{cases} \tag{3.101}$$

$$W_{constant} = -\left(A_{151x}\rho^5 \sin\phi + A_{151y}\rho^5 \cos\phi\right) \tag{3.102}$$

$$\implies \begin{cases} C_{16}^0 & = \kappa_1 \\ C_{17}^0 & = \kappa_2 \end{cases} \tag{3.103}$$

Considering all the components of fifth order coma in a misaligned system, C_{16} and C_{17} can be defined as:

$$C_{16} = \sum C_{16}^j = \kappa_0 y + \kappa_1$$
$$C_{17} = \sum C_{17}^j = \kappa_0 x + \kappa_2 \tag{3.104}$$

- Oblique spherical aberration

  The different components of oblique spherical aberration in equation 2.17 can be expressed in terms of the field components x and y and then the expressions of C_{12} and C_{13} can be derived.

$$
\begin{aligned}
W_{quadratic} &= \frac{1}{2} W_{242} \left(\vec{H}^2 \cdot \vec{\rho}^{\,2} \right) \rho^2 \\
&= \frac{1}{2} W_{242} \left(2xy\rho^4 \sin 2\phi + (y^2 - x^2)\rho^4 \cos 2\phi \right)
\end{aligned}
\tag{3.105}
$$

$$
\implies \quad
\begin{cases}
C_{12}^2 &= \eta_0 (y^2 - x^2) \\
C_{13}^2 &= \eta_0 2xy
\end{cases}
\tag{3.106}
$$

$$
\begin{aligned}
W_{linear} &= - \left(\left(\vec{H} \vec{A}_{242} \right) \cdot \vec{\rho}^{\,2} \right) \rho^2 \\
&= - \left(\left(A_{242x} y + A_{242y} x \right) \rho^4 \sin 2\phi + \left(- A_{242x} x + A_{242y} y \right) \rho^4 \cos 2\phi \right)
\end{aligned}
\tag{3.107}
$$

$$
\implies \quad
\begin{cases}
C_{12}^1 &= -\eta_1 x + \eta_2 y \\
C_{13}^1 &= \eta_1 y + \eta_2 x
\end{cases}
\tag{3.108}
$$

$$
\begin{aligned}
W_{constant} &= \frac{1}{2} \left(\vec{B}_{242}^2 \cdot \vec{\rho}^{\,2} \right) \rho^2 \\
&= \frac{1}{2} \left(2 B_{242x} B_{242y} \rho^4 \sin 2\phi + (B_{242y}^2 - B_{242x}^2) \rho^4 \cos 2\phi \right)
\end{aligned}
\tag{3.109}
$$

$$
\implies \quad
\begin{cases}
C_{12}^0 &= \eta_3 \\
C_{13}^0 &= \eta_4
\end{cases}
\tag{3.110}
$$

Considering all the components of oblique spherical aberration in a misaligned system, C_{12} and C_{13} can be defined as:

$$
\begin{aligned}
C_{12} &= \sum C_{12}^j = \eta_0 (y^2 - x^2) - \eta_1 x + \eta_2 y + \eta_3 \\
C_{13} &= \sum C_{13}^j = \eta_0 2xy + \eta_1 y + \eta_2 x + \eta_4
\end{aligned}
\tag{3.111}
$$

- Field curvature for oblique spherical aberration

  The different components of field curvature for oblique spherical aberration in equation 2.18 can be expressed in terms of the field components x and y which leads to the expressions of C_{11}.

$$W_{quadratic} = W_{240M}(x^2 + y^2)\rho^4 \tag{3.112}$$

$$\implies \quad C_{11}^2 = \delta_0(x^2 + y^2) \tag{3.113}$$

$$W_{linear} = -2(A_{240Mx}x + A_{240My}y)\rho^4 \tag{3.114}$$

$$\implies \quad C_{11}^1 = \delta_1 x + \delta_2 y \tag{3.115}$$

$$W_{constant} = B_{240M}\rho^4 \tag{3.116}$$

$$\implies \quad C_{11}^0 = \delta_3 \tag{3.117}$$

Considering all the components of field curvature for oblique spherical aberration in a misaligned system, C_{11} can be defined as:

$$C_{11} = \sum C_{11}^j = \delta_0(x^2 + y^2) + \delta_1 x + \delta_2 y + \delta_3 \tag{3.118}$$

- Fifth order distortion

  The different components of fifth order distortion in equation 2.19 can be expressed in terms of the field components x and y and then the expressions of C_2 and C_3 can be derived. To differentiate the components that have the same field dependence, the primes superscript will be used.

$$
\begin{aligned}
W_{fifth} &= W_{511}(x^2 + y^2)^2(x\rho\sin\phi + y\rho\cos\phi) \\
&= W_{511}\big((x^5 + xy^4 + 2x^3y^2)\rho\sin\phi + (x^4y + y^5 + 2x^2y^3)\rho\cos\phi\big)
\end{aligned}
\tag{3.119}
$$

$$\implies \begin{cases} C_2^5 &= \zeta_0(x^4y + y^5 + 2x^2y^3) \\ C_3^5 &= \zeta_0(x^5 + xy^4 + 2x^3y^2) \end{cases} \tag{3.120}$$

$$\begin{aligned} W_{quartic\#1} &= -4(x^2 + y^2)(xA_{511x} + yA_{511y})(x\rho\sin\phi + y\rho\cos\phi) \\ &= -4\Big(\big(A_{511x}(x^4 + x^2y^2) + A_{511y}(x^3y + xy^3)\big)\rho\sin\phi \\ &\quad + \big(A_{511x}(x^3y + xy^3) + A_{511y}(x^2y^2 + y^4)\big)\rho\cos\phi\Big) \end{aligned} \tag{3.121}$$

$$\implies \begin{cases} C_2^4 &= \zeta_1(x^3y + xy^3) + \zeta_2(x^2y^2 + y^4) \\ C_3^4 &= \zeta_1(x^4 + x^2y^2) + \zeta_2(x^3y + xy^3) \end{cases} \tag{3.122}$$

$$\begin{aligned} W_{cubic\#1} &= 6B_{511}(x^2 + y^2)\vec{H} \cdot (\rho\sin\phi\,\hat{i} + \rho\cos\phi\,\hat{j}) \\ &= 6B_{511}\big((x^3 + xy^2)\rho\sin\phi + (x^2y + y^3)\rho\cos\phi\big) \end{aligned} \tag{3.123}$$

$$\implies \begin{cases} C_2^3 &= \zeta_3(x^2y + y^3) \\ C_3^3 &= \zeta_3(x^3 + xy^2) \end{cases} \tag{3.124}$$

$$\begin{aligned} W_{cubic\#2} &= 2\big(4B_{511x}B_{511y}xy + (B_{511y}^2 - B_{511x}^2)(y^2 - x^2)\big)\big(x\rho\sin\phi + y\rho\cos\phi\big) \\ &= 2\Big(\big(4B_{511x}B_{511y}x^2y + (B_{511y}^2 - B_{511x}^2)(y^2x - x^3)\big)\rho\sin\phi \\ &\quad + \big((4B_{511x}B_{511y}xy^2 + (B_{511y}^2 - B_{511x}^2)(y^3 - x^2y)\big)\rho\cos\phi\Big) \end{aligned} \tag{3.125}$$

$$\implies \begin{cases} C_2^{3\prime} &= \zeta_4xy^2 + \zeta_5(y^3 - x^2y) \\ C_3^{3\prime} &= \zeta_4x^2y + \zeta_5(y^2x - x^3) \end{cases} \tag{3.126}$$

$$\begin{aligned} W_{quadratic\#1} &= -4(xC_{511x} + yC_{511y})(x\rho\sin\phi + y\rho\cos\phi) \\ &= -4\big((x^2C_{511x} + xyC_{511y})\rho\sin\phi + (xyC_{511x} + y^2C_{511y})\rho\cos\phi\big) \end{aligned} \tag{3.127}$$

$$\implies \begin{cases} C_2^2 &= \zeta_6xy + \zeta_7y^2 \\ C_3^2 &= \zeta_6x^2 + \zeta_7xy \end{cases} \tag{3.128}$$

$$W_{linear\#1} = 3D_{511}(x\rho\sin\phi + y\rho\cos\phi) \tag{3.129}$$

$$\implies \quad \begin{cases} C_2^1 & = \zeta_8 y \\ C_3^1 & = \zeta_8 x \end{cases} \tag{3.130}$$

$$W_{quartic\#2} = -(x^2 + y^2)^2(A_{511x}\rho\sin\phi + A_{511y}\rho\cos\phi)$$
$$= -\big(A_{511x}(x^4 + y^4 + 2x^2y^2)\rho\sin\phi + A_{511y}(x^4 + y^4 + 2x^2y^2)\rho\cos\phi\big) \tag{3.131}$$

$$\implies \quad \begin{cases} C_2^{4'} & = \zeta_9(x^4 + y^4 + 2x^2y^2) \\ C_3^{4'} & = \zeta_{10}(x^4 + y^4 + 2x^2y^2) \end{cases} \tag{3.132}$$

$$W_{cubic\#3} = 2(x^2 + y^2)(\vec{B}_{511}^2\vec{H}^*)\cdot(\rho\sin\phi\,\hat{i} + \rho\cos\phi\hat{j})$$
$$= 2\Big(\big(-(B_{511y}^2 - B_{511x}^2)(x^3 + xy^2) + 2B_{511x}B_{511y}(yx^2 + y^3)\big)\rho\sin\phi$$
$$+ \quad \big((B_{511y}^2 - B_{511x}^2)(x^2y + y^3) + 2B_{511x}B_{511y}(x^3 + xy^2)\big)\rho\cos\phi\Big) \tag{3.133}$$

$$\implies \quad \begin{cases} C_2^{3''} & = \zeta_{11}(x^2y + y^3) + \zeta_{12}(x^3 + xy^2) \\ C_3^{3''} & = -\zeta_{11}(x^3 + xy^2) + \zeta_{12}(yx^2 + y^3) \end{cases} \tag{3.134}$$

$$W_{quadratic\#2} = -4(x^2 + y^2)(C_{511x}\rho\sin\phi + C_{511y}\rho\cos\phi) \tag{3.135}$$

$$\implies \quad \begin{cases} C_2^{2'} & = \zeta_{13}(x^2 + y^2) \\ C_3^{2'} & = \zeta_{14}(x^2 + y^2) \end{cases} \tag{3.136}$$

$$W_{quadratic\#3} = -\big(\vec{H}^2(-C_{511x}\hat{i} + C_{511y}\hat{j})\big)\cdot\big(\vec{\rho}\big)$$
$$= -\Big(\big(-C_{511x}(y^2 - x^2) + 2C_{511y}xy\big)\rho\sin\phi \tag{3.137}$$
$$+ \big(2C_{511x}xy + C_{511y}(y^2 - x^2)\big)\rho\cos\phi\Big)$$

$$\implies \quad \begin{cases} C_2^{2''} & = 2\zeta_{15}xy + \zeta_{16}(y^2 - x^2) \\ C_3^{2''} & = -\zeta_{15}(y^2 - x^2) + 2\zeta_{16}xy \end{cases} \tag{3.138}$$

$$W_{quadratic\#4} = -(\vec{C}_{511}^3 \vec{H}^{2*}) \cdot \vec{\rho}$$

$$= \big(-2(C_{511y}^3 - 3C_{511x}^2 C_{511y})xy$$

$$+ (3C_{511y}^2 C_{511x} - C_{511x}^3)(y^2 - x^2)\big)\rho\sin\phi \qquad (3.139)$$

$$+ \big((C_{511y}^3 - 3C_{511x}^2 C_{511y})(y^2 - x^2)$$

$$+ 2(3C_{511y}^2 C_{511x} - C_{511x}^3)xy\big)\rho\cos\phi$$

$$\implies \begin{cases} C_2^{2'''} &= \zeta_{17}(y^2 - x^2) + 2\zeta_{18}xy \\ C_3^{2'''} &= -2\zeta_{17}xy + \zeta_{18}(y^2 - x^2) \end{cases} \qquad (3.140)$$

$$W_{linear\#2} = 2(\vec{D}_{511}^2 \vec{H}^*) \cdot (\rho\sin\phi\,\hat{i} + \rho\cos\phi\,\hat{j})$$

$$= 2\big((-(D_{511y}^2 - D_{511x}^2)x + 2D_{511x}D_{511y}y)\rho\sin\phi \qquad (3.141)$$

$$+ \quad ((D_{511y}^2 - D_{511x}^2)y + 2D_{511x}D_{511y}x)\rho\cos\phi\big)$$

$$\implies \begin{cases} C_2^{1'} &= \zeta_{19}y + \zeta_{20}x \\ C_3^{1'} &= -\zeta_{19}x + \zeta_{20}y \end{cases} \qquad (3.142)$$

$$W_{constant} = -(E_{511x}\rho\sin\phi + E_{511y}\rho\cos\phi) \qquad (3.143)$$

$$\implies \begin{cases} C_2^0 &= \zeta_{21} \\ C_3^0 &= \zeta_{22} \end{cases} \qquad (3.144)$$

Considering all the components of fifth order distortion in a misaligned system,

C_2 and C_3 can be defined as:

$$C_2 = \sum C_2^j = \zeta_0(x^4y + y^5 + 2x^2y^3) + \zeta_1(x^3y + xy^3) + \zeta_2(x^2y^2 + y^4)$$
$$+ \zeta_3(x^2y + y^3) + \zeta_4xy^2 + \zeta_5(y^3 - x^2y) + \zeta_6xy + \zeta_7y^2 + \zeta_8y$$
$$+ \zeta_9(x^4 + y^4 + 2x^2y^2) + \zeta_{11}(x^2y + y^3) + \zeta_{12}(x^3 + xy^2)$$
$$+ \zeta_{13}(x^2 + y^2) + 2\zeta_{15}xy + \zeta_{16}(y^2 - x^2) + \zeta_{17}(y^2 - x^2)$$
$$+ 2\zeta_{18}xy + \zeta_{19}y + \zeta_{20}x + \zeta_{21}$$

$$C_3 = \sum C_3^j = \zeta_0(x^5 + xy^4 + 2x^3y^2) + \zeta_1(x^4 + x^2y^2) + \zeta_2(x^3y + xy^3)$$
$$+ \zeta_3(x^3 + xy^2) + \zeta_4x^2y + \zeta_5(y^2x - x^3) + \zeta_6x^2 + \zeta_7xy + \zeta_8x$$
$$+ \zeta_{10}(x^4 + y^4 + 2x^2y^2) - \zeta_{11}(x^3 + xy^2) + \zeta_{12}(yx^2 + y^3)$$
$$+ \zeta_{14}(x^2 + y^2) - \zeta_{15}(y^2 - x^2) + 2\zeta_{16}xy - 2\zeta_{17}xy$$
$$+ \zeta_{18}(y^2 - x^2) - \zeta_{19}x + \zeta_{20}y + \zeta_{22}$$

$$(3.145)$$

3.4   Summary of the results

This section summarizes the results found in sections 3.2 and 3.3 for the field dependencies of the Zernike polynomials except that the equations shown here are expressed with the usual conventions used by optical software instead of the ones used in the dissertation of K.P. Thompson. Thus, a right-handed convention is considered here and that will lead to replace x by $-x$. But optical softwares usually "look" at the image plane from behind and therefore x is unchanged. Also, here ϕ is now defined from the $\vec{x}$ axis which will imply to replace $\cos\phi$ by $\sin\phi$ and vice versa. Thus to summarize, the only modification will be to exchange the expression of the cosine and the sine components for each aberration.

Again, the terms α_j, β_j, ξ_j, μ_j, χ_j, κ_j, γ_j, δ_j, ν_j, ζ_j, η_j ($\forall j$) are independent constants. The subscript $_0$ for these constants is used to express the aberrations of a centered system and the following numbers are used to express the aberrations that

can possibly arise in a misaligned system.

A two-element system is well described with third order aberrations. In this case the following equations are used:

Third order astigmatism: $\mathbf{W}_{222}$

$$C_5 = \alpha_0(y^2 - x^2) - \alpha_1 x + \alpha_2 y + \alpha_4$$

$$C_6 = 2\,\alpha_0 x\,y + \alpha_1 y + \alpha_2 x + \alpha_3$$

Third order coma: $\mathbf{W}_{131}$

$$C_7 = \beta_0 y + \beta_2$$

$$C_8 = \beta_0 x + \beta_1$$

Third order field curvature: $\mathbf{W}_{220M}$

$$C_4 = \gamma_0(x^2 + y^2) + \gamma_1 x + \gamma_2 y + \gamma_3$$

Third order distortion: $\mathbf{W}_{311}$

$$C_2 = \nu_0(x^3 + xy^2) + \nu_1 x^2 + \nu_2 xy + \nu_3 x + \nu_5(x^2 + y^2) - \nu_6 x + \nu_7 y + \nu_9$$

$$C_3 = \nu_0(x^2 y + y^3) + \nu_1 xy + \nu_2 y^2 + \nu_3 y + \nu_4(x^2 + y^2) + \nu_6 y + \nu_7 x + \nu_8$$

Fifth order aberrations give better results for systems with multiple degrees of freedom. Thus the following equations are used:

Field cubed coma: $\mathbf{W}_{331}$

$$C_7 = \xi_0(y^3 + x^2 y) + \xi_1\,xy + \xi_2\,y^2 + \xi_3\,y + \xi_5\,(x^2 + y^2) + \xi_6\,x - \xi_7\,y + \xi_9$$

$$C_8 = \xi_0(x^3 + xy^2) + \xi_1\,x^2 + \xi_2\,xy + \xi_3\,x + \xi_4\,(x^2 + y^2) + \xi_6\,y + \xi_7\,x + \xi_8$$

Elliptical coma (trefoil): $\mathbf{W}_{333}$

$$C_9 = \mu_0(y^3 - 3x^2 y) - 2\mu_1 xy + \mu_2(y^2 - x^2) + \mu_3 y - \mu_4 x + \mu_6$$

$$C_{10} = \mu_0(3y^2 x - x^3) + \mu_1(y^2 - x^2) + \mu_2 2xy + \mu_3 x + \mu_4 y + \mu_5$$

Quartic field curvature: W_{420}

$$
\begin{aligned}
C_4 =& \psi_0(x^4 + y^4 + 2x^2y^2) + \psi_1(x^3 + xy^2) + \psi_2(x^2y + y^3) \\
&+ \psi_3(x^2 + y^2) + \psi_4 xy + \psi_5(y^2 - x^2) + \psi_6 x + \psi_7 y + \psi_8
\end{aligned}
$$

Quartic astigmatism: W_{422}

$$
\begin{aligned}
C_5 =& \chi_0(y^4 - x^4) - \chi_1(x^3 + xy^2) + \chi_2(x^2y + y^3) + \chi_4(x^2 + y^2) \\
&- \chi_5(2x^2y) + \chi_6(y^3 - x^2y) + \chi_7 x + \chi_8 y + \chi_9(-x^2 + y^2) + \chi_{10}y - \chi_{11}x + \chi_{13}
\end{aligned}
$$

$$
\begin{aligned}
C_6 =& 2\chi_0(x^3y + xy^3) + \chi_1(x^2y + y^3) + \chi_2(x^3 + xy^2) + \chi_3(x^2 + y^2) \\
&+ \chi_5(xy^2 - x^3) + \chi_6(2xy^2) + \chi_7 y - \chi_8 x + \chi_9 2xy + \chi_{10}x + \chi_{11}y + \chi_{12}
\end{aligned}
$$

Fifth order coma: W_{151}

$$
C_{16} = \kappa_0 x + \kappa_2
$$

$$
C_{17} = \kappa_0 y + \kappa_1
$$

Oblique spherical aberration: W_{242}

$$
C_{12} = \eta_0 2xy + \eta_1 y + \eta_2 x + \eta_4
$$

$$
C_{13} = \eta_0(y^2 - x^2) - \eta_1 x + \eta_2 y + \eta_3
$$

Fifth order field curvature: W_{240M}

$$
C_{11} = \delta_0(x^2 + y^2) + \delta_1 x + \delta_2 y + \delta_3
$$

Fifth order distortion: W_{511}

$$
\begin{aligned}
C_2 =& \zeta_0(x^5 + xy^4 + 2x^3y^2) + \zeta_1(x^4 + x^2y^2) + \zeta_2(x^3y + xy^3) + \zeta_3(x^3 + xy^2) \\
&+ \zeta_4 x^2y + \zeta_5(y^2x - x^3) + \zeta_6 x^2 + \zeta_7 xy + \zeta_8 x + \zeta_{10}(x^4 + y^4 + 2x^2y^2) \\
&- \zeta_{11}(x^3 + xy^2) + \zeta_{12}(yx^2 + y^3) + \zeta_{14}(x^2 + y^2) - \zeta_{15}(y^2 - x^2) \\
&+ 2\zeta_{16}xy - 2\zeta_{17}xy + \zeta_{18}(y^2 - x^2) - \zeta_{19}x + \zeta_{20}y + \zeta_{22}
\end{aligned}
$$

$$
\begin{aligned}
C_3 =& \zeta_0(x^4y + y^5 + 2x^2y^3) + \zeta_1(x^3y + xy^3) + \zeta_2(x^2y^2 + y^4) + \zeta_3(x^2y + y^3) \\
&+ \zeta_4 xy^2 + \zeta_5(y^3 - x^2y) + \zeta_6 xy + \zeta_7 y^2 + \zeta_8 y + \zeta_9(x^4 + y^4 + 2x^2y^2) \\
&+ \zeta_{11}(x^2y + y^3) + \zeta_{12}(x^3 + xy^2) + \zeta_{13}(x^2 + y^2) + 2\zeta_{15}xy + \zeta_{16}(y^2 - x^2) \\
&+ \zeta_{17}(y^2 - x^2) + 2\zeta_{18}xy + \zeta_{19}y + \zeta_{20}x + \zeta_{21}
\end{aligned}
$$

These equations will be used in Chapter 4 and 5 to align two different telescopes. The first one is a two element telescope, so only the equations for third order aberrations will be used. And the second one is a four element telescope with a lot of degrees of freedom, thus the equations including the fifth order will be useful.

Chapter 4

APPLICATION ON THE 90" TELESCOPE

Now that the field dependencies of the Zernike polynomials in a misaligned system are known, it will be possible to develop a process to align an optical system.

To show the different steps, the process for the alignment will be shown on a real system. The optical system used in this chapter is a two-element telescope located on Kitt Peak.

In [6, 7, 8, 9, 10, 11, 12, 13, 14] it is claimed and proved that for a generic two-mirror telescope (Cassegrain, Ritchey Chretien...), a decenter of the secondary will introduce some constant coma over the whole field and a tilt about the "free coma point"[1] will generate mostly astigmatism linear with the field.

In this chapter, it will be shown how the results of the previous chapters apply for the particular case of the 90" telescope which is not a generic two-mirror telescope. It will be seen that the same type of relations found in [9, 10, 12] can be obtained. Since the process to align this type of telescope is not really complicated, it is a good way to verify the method by using the new set of Zernike. The method used in this chapter will follow the block diagram represented in figure 4.1

This system contains only two elements, so only the third order aberrations will be considered.

In a first section, the design of the telescope will be presented. Then, the different steps to get the relations between the Zernike coefficients and the perturbations will be studied. Finally, the results of the alignment process from the real system will be shown.

---

[1]This is the location for a two-mirror system where a rotation about it will not introduce coma, see [11] for more explanations.
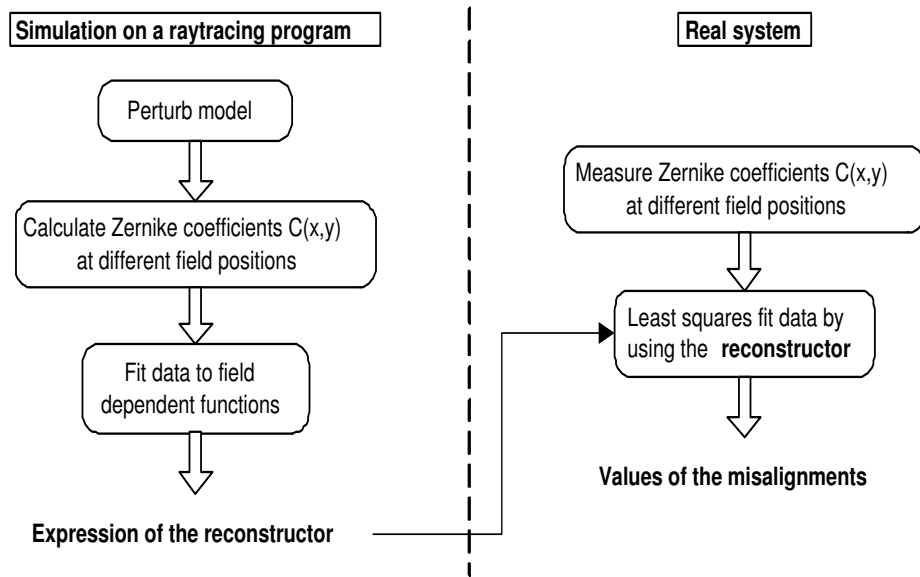
FIGURE 4.1. Block diagram of the method

4.1 Design of the 90"

The 90-inch telescope is a two element mirror-system with a primary mirror which has a diameter of 2286 mm and a radius of curvature of 12159 mm. The second element is a corrector; it is composed of four lenses and one filter made of Fused Silica. The group of lenses, the filter and the detector are already aligned in a box which can be moved relative to the primary. Thus the alignment of this telescope will consist of aligning this box relative to the primary mirror by using a wavefront measurement at the image plane. The actual system is equipped with a curvature sensor which fits the standard Zernike coefficients to the aberrated wavefront at the exit of the system. The design of the telescope is represented in figure 4.2 in three dimensions. Note that a hole is present in the primary because the original telescope was a Cassegrain-type design. And the box with the refractive elements was placed to replace the secondary mirror in order to have a wider field of view. The prescriptions of the design of the telescope can be found in Appendix B.
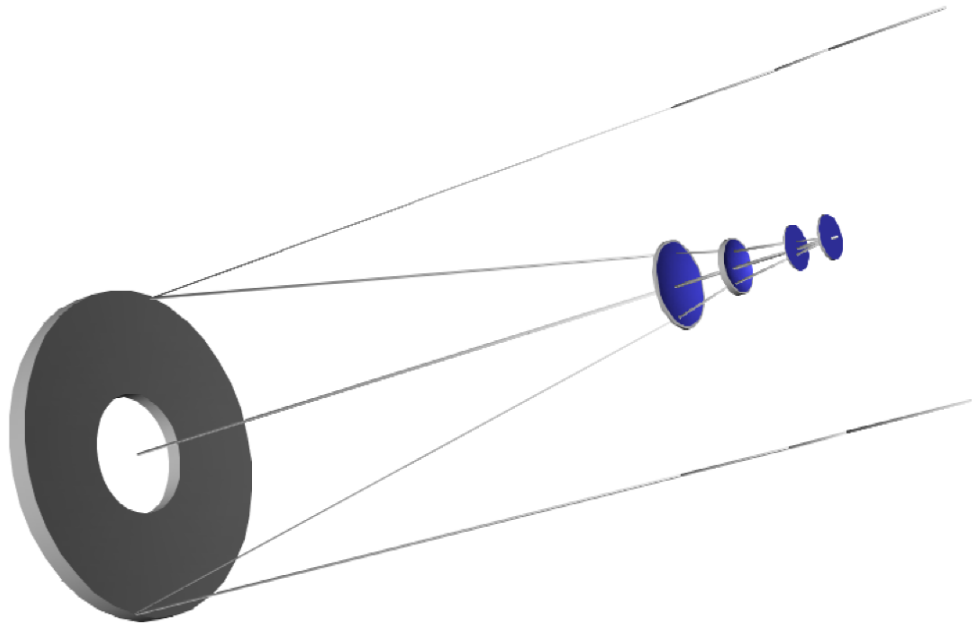
FIGURE 4.2. 3D view of the optical design

4.2 Procedure to find the alignment equations on Code V®

To align the telescope, first the alignment equations have to be found with an optical software. This corresponds to find the expression of the reconstructor in figure 4.1. The software chosen to show the results will be Code V®. But the same results were found using Zemax®. And what will be done in this section could also be done with a different software.

The alignment process with Code V® will be done in four steps:

1. Find the free coma point position

2. Perturb the corrector about the free coma point

3. Get the zernike coefficients at different field positions

4. Fit the zernike coefficients with a least-squares fit method to relate the perturbations, the zernike coefficients and the field coordinates

The general principle is to find the 4x4 matrix M which relates the Zernike coefficients to the perturbations as shown in equation 4.1. By knowing this matrix it will be possible to inverse it by a least-squares fit to get the perturbations when the Zernike coefficients are known which represents the last block in figure 4.1.

To find the matrix the Zernike values will be calculated for one perturbation at a time. Thus, each perturbation will allow to calculate one column of M. For instance, if the system is only tilted around $\vec{x}$, the first column of the matrix can be calculated.

$$\begin{bmatrix} C_5 \\ C_6 \\ C_7 \\ C_8 \end{bmatrix} = \begin{bmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ m_{21} & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \\ m_{41} & m_{42} & m_{43} & m_{44} \end{bmatrix} \cdot \begin{bmatrix} TiltX \\ TiltY \\ DecX \\ DecY \end{bmatrix} \tag{4.1}$$

Where m_{ij} $(1 \leq i \leq 4, \ 1 \leq j \leq 4)$ are the components of the matrix M.

This section consists in finding this Matrix M which represents the reconstructor in figure 4.1.

4.2.1   First step: Find the free coma point position

In a two-mirror system, it can be shown that there is a point where it is possible to rotate the secondary about without introducing any coma. It is called the "neutral point" or the "free coma point". In [11, 12, 13] the position of this point is derived by calculating the amount of coma introduced by a tilt and a decenter of the secondary about its vertex. It is shown that the coma introduced by a decenter can be cancelled by the coma introduced by a tilt. Then this combination of tilt and decenter can be reduced by only one tilt about a new axis of rotation which is located at the free coma point.

The alignment of the telescope will be done by choosing this point as the reference point for tilt and decenter. It is a convenient choice to decouple the effects of astigmatism and coma since a rotation about this point will introduce only astigmatism and

it will be shown later that a decenter of the secondary will introduce mostly constant coma.

Here the calculations to find the position of this free coma point will not be derived since the second element is not simply a mirror but it is made of four lenses. The calculations to find this point would have been long and complicated. Instead of doing all the calculations, Code V® will be used to realize this step.

This is done by introducing a coordinate break before the group of lenses and by setting the thickness of this coordinate break as a variable. Then, the right thickness can be found to get no coma on axis when a tilt is introduced at the coordinate break by using the optimization function in Code V®.

4.2.2 Second step: Perturbed the corrector about the free coma point

The second step in the process to find the equations which relate the Zernike coefficients to the perturbations is to perturb the secondary by a significant amount of tilt and decenter. One perturbation at a time will be done.

- Secondary tilted around the $\vec{x}$ axis

  When the secondary is tilted around $\vec{x}$ by .1 degree the spot diagram in the xy plane[2] in figure 4.3 is obtained[3].

  Figure 4.3 shows that linear astigmatism is dominant, it is almost the same pattern that was shown in figure 2.16 except that is not composed of straight segments since the centered system has aberrations.

  Thus the conclusion is that a tilt around $\vec{x}$ about the free coma point introduces mainly linear astigmatism.

---

[2]Plane perpendicular to the optical axis

[3]This corresponds to an alpha tilt of .1 degree in Code V®. This alpha should not be taken for the α_j defined previously for the astigmatism. The sign conventions used by Code V® for the angle can be found in Appendix C.

(a) Inside focus

(b) Outside focus
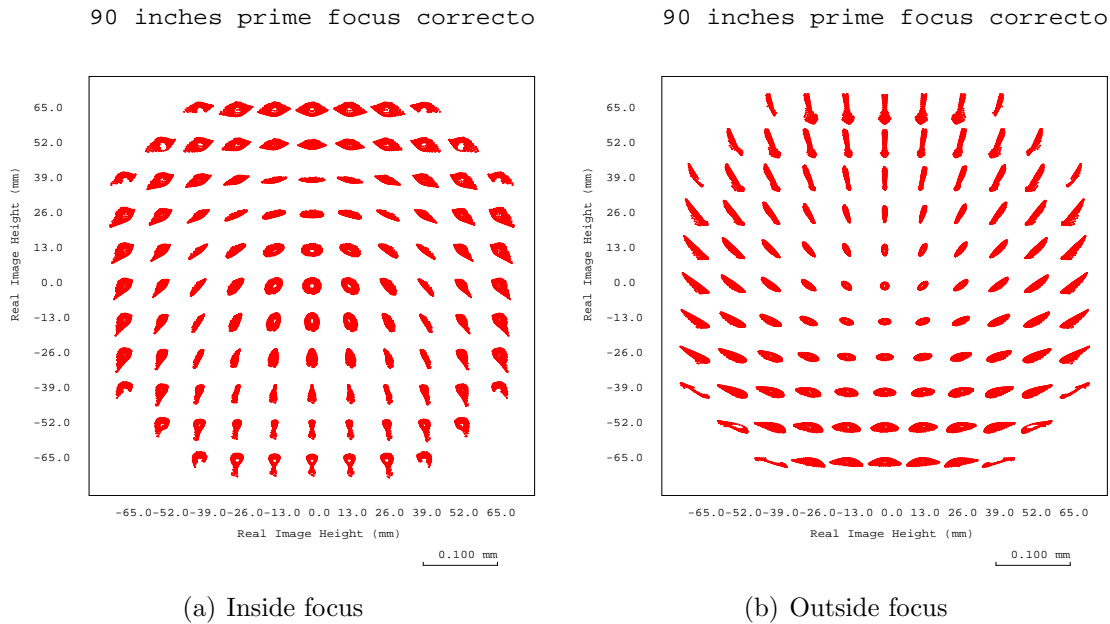
FIGURE 4.3. 2D spot diagram in the xy plane for alpha tilt=.1 degree

- Secondary tilted around the $\vec{y}$ axis

    When the secondary is tilted around $\vec{y}$ by .1 degree[4] the spot diagram of figure 4.4 is obtained.

    This perturbation introduces again mostly linear astigmatism as it can be seen in figure 4.4. It is the same spot diagram that was shown in figure 4.3 except that here the pattern is rotated by 90 degrees.

- Secondary decentered along the $\vec{x}$ axis

    When the secondary is decentered along $\vec{x}$ by 1 mm the spot diagram of figure 4.5 is observed.

    This perturbation introduces mostly constant coma along $\vec{x}$ as it can be seen in figure 4.5. It is not exactly constant coma due to the fact that the original centered design had a small amount of coma. So it is more linear coma with

---

[4]This corresponds to a beta tilt of .1 degree in Code V®. This beta should not be taken for the β_j defined previously for coma.

90 inches prime focus correcto   90 inches prime focus correcto

(a) Inside focus   (b) Outside focus
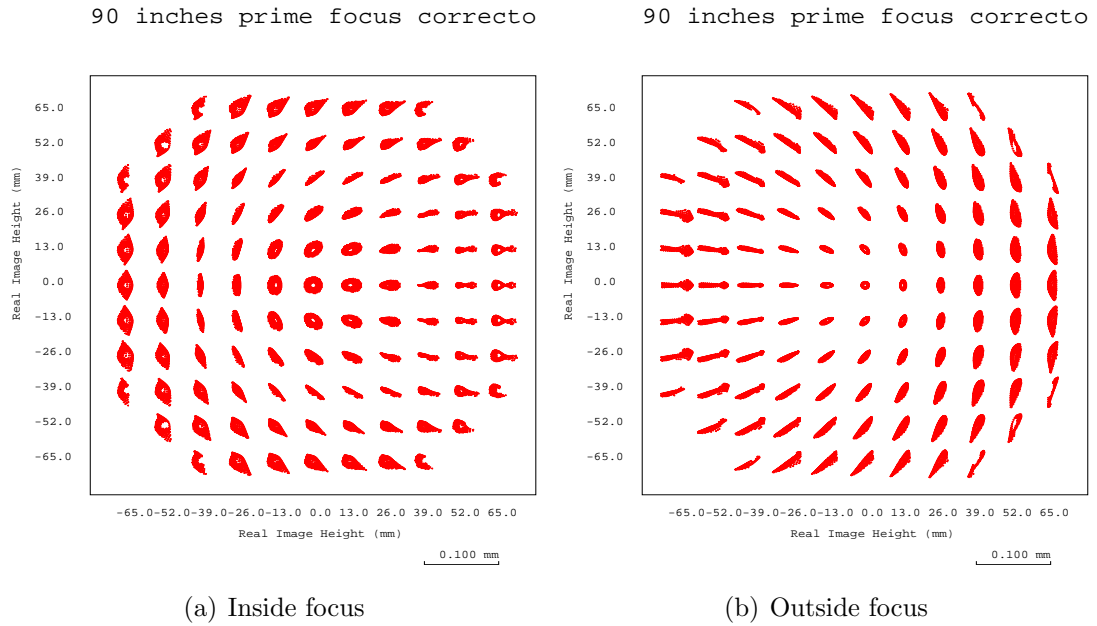
FIGURE 4.4. 2D spot diagram in the xy plane for beta tilt=.1 degree

its node shifted from the center of the Gaussian image plane. But since the constant part is much larger than the linear one, the spot diagram shows mostly a constant coma.

- Secondary decentered along the $\vec{y}$ axis

  When the secondary is decentered along $\vec{y}$ by 1 mm, the spot diagram of figure 4.6 is observed.

  This perturbation introduces mostly constant coma along $\vec{y}$ as it can be seen in figure 4.6. It is the same spot diagram that was shown on figure 4.5 for the decenter in the $\vec{x}$ direction except that here, the coma is oriented along $\vec{y}$.

4.2.3   Third step: Get the Zernike coefficients for different positions in the field

For each perturbation, the Zernike coefficients are calculated for different field positions with a Code V® macro which is given in appendix D. The equivalent macro

90 inches prime focus correcto
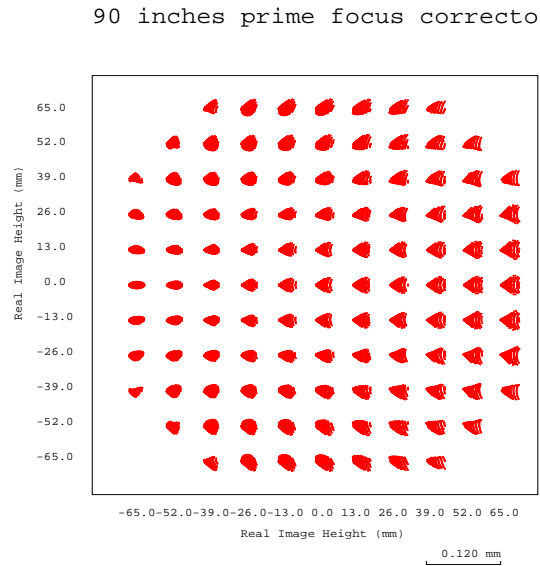
FIGURE 4.5. 2D spot diagram in the xy plane for a decenter along $\vec{x}$ of 1 mm

90 inches prime focus correcto
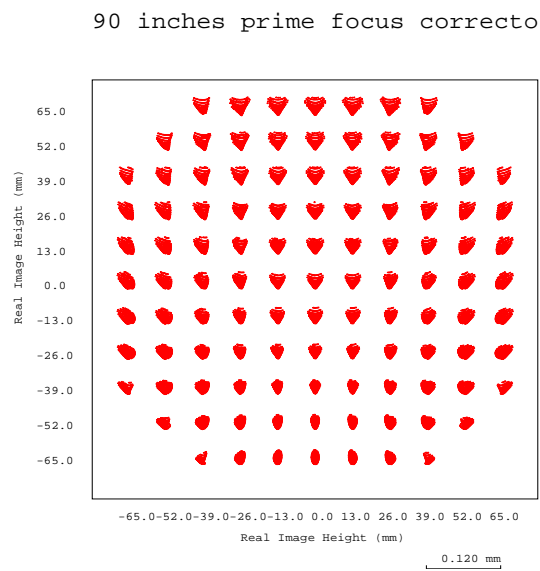
FIGURE 4.6. 2D spot diagram in the xy plane for a decenter along $\vec{y}$ of 1 mm
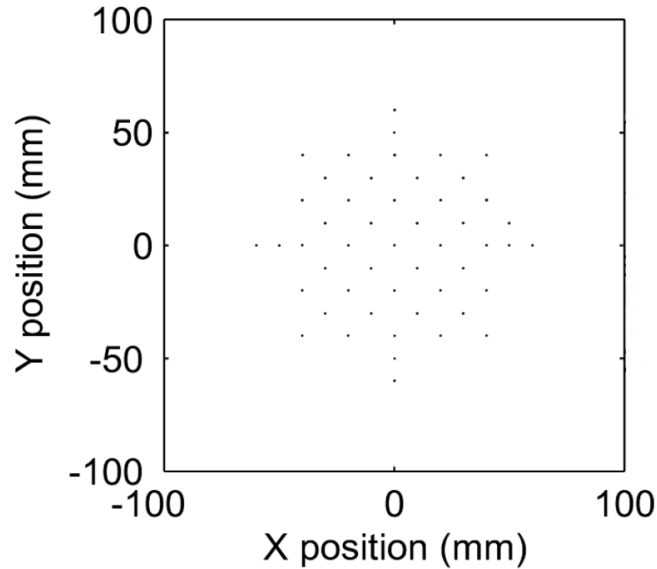
FIGURE 4.7. Positions of the field points in the image plane

for Zemax® can be found in appendix F (it is the macro for the LSST but the macro
is similar for the 90").

A grid of field points inside the field of view is created where the Zernike coefficients
are calculated. The 50 field points which are considered are represented in figure 4.7.

4.2.4   Fourth step: Least squares fit

To get the equations which relate the Zernike coefficients to the perturbations
and the field positions, a least-squares fit is used in a Matlab® program which can be
found in Appendix E. Basically, the Matlab® program fits the values of the Zernike
coefficients calculated in the previous step with the help of the equations found in
chapter 3 for each perturbation independently. In this process, the equations of
chapter 3 with the field dependencies are necessary.

To have a better visualization, some graphics were generated for each fit. For the
astigmatism for example, three graphics are generated for the constant, the linear and
the quadratic parts. Also, two other graphics are generated, one for the representa-

tion of the total amount of astigmatism and one for the residual astigmatism which represents the amount of astigmatism when the constant, the linear and the quadratic parts have been removed from the original data. Thus, the residual represents mostly the noise.

For the graphic representation of each aberration, the magnitude and the orientation of the Zernike coefficients are used. The expressions of the amplitude and the orientation are calculated by using the results of Appendix F.

For astigmatism, the magnitude and the orientation are represented by:

$$\begin{cases} M = 2\sqrt{6} \cdot \sqrt{(C_5)^2 + (C_6)^2} \\ A = \frac{1}{2} \tan^{-1} \frac{C_5}{C_6} \end{cases} \tag{4.2}$$

For coma, the amplitude and the orientation are represented by:

$$\begin{cases} M = 3\sqrt{8} \cdot \sqrt{(C_7)^2 + (C_8)^2} \\ A = \tan^{-1} \frac{C_7}{C_8} \end{cases} \tag{4.3}$$

When the system is aligned, the amount of astigmatism and coma is small as shown on figure 4.8 and 4.9.

The effects of each perturbation on the coma and the astigmatism are studied here.

- Tilt of .1 degree around $\vec{x}$

    A tilt of .1 degree around $\vec{x}$ produces mainly linear astigmatism as it was pointed out on the 2D spot diagram of figure 4.3.

    By fitting the Zernike coefficients calculated in the previous step with the matrix formulation of C_5 and C_6 in a perturbed system which is shown in equation 4.4, the values of α_0, α_1, α_2, α_3 and α_4 are calculated. Here, only the linear part is significant. The program returns: $\alpha_1 \approx -5.767$, where x and y are in mm and the Zernike coefficients in nm. Thus by considering that the secondary was tilted by .1 degree, the general equation is: $\alpha_1 = -57.67 \cdot TiltX$, where $TiltX$ represents the amount of tilt for the secondary around $\vec{x}$ in degrees.
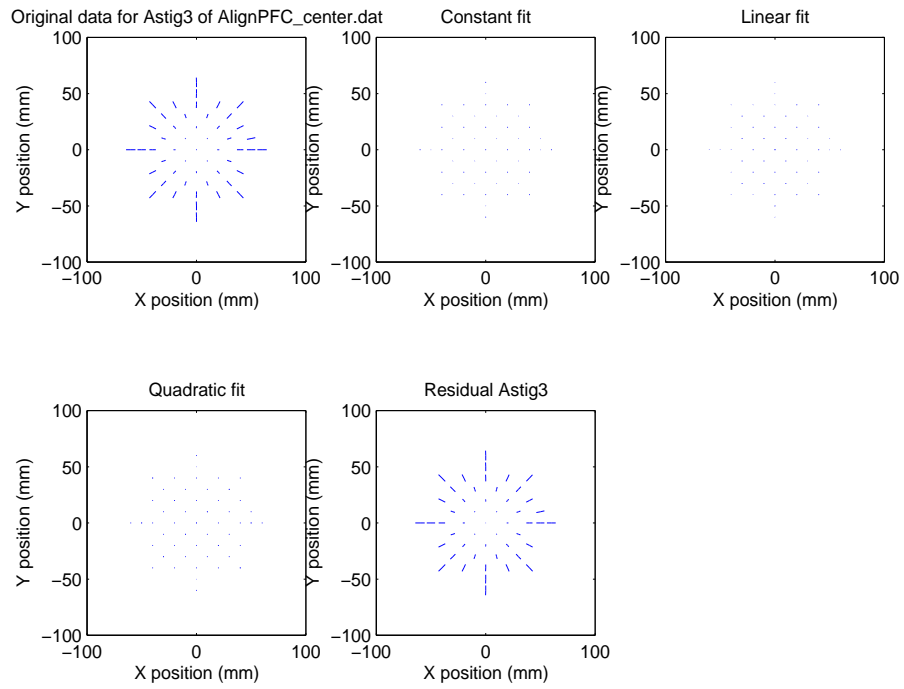
FIGURE 4.8. Astigmatism representation for the nominal centered system
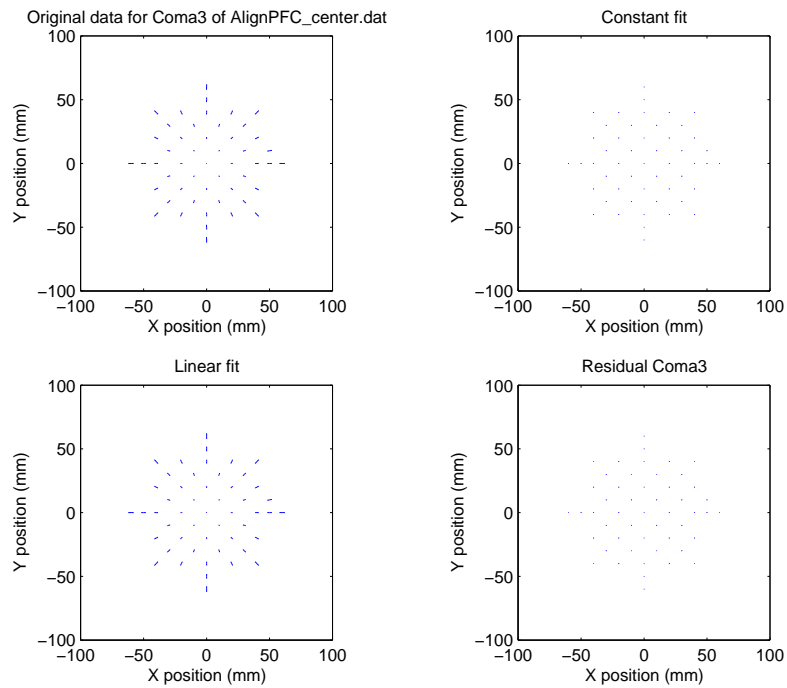
FIGURE 4.9. Coma representation for the nominal centered system
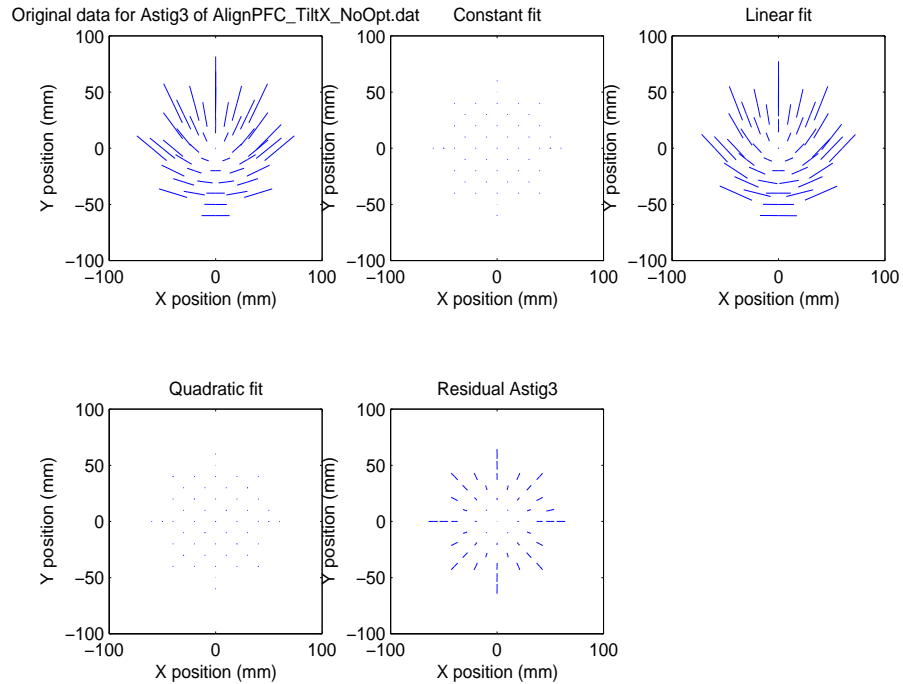
FIGURE 4.10. Astigmatism representation when secondary tilted .1 degree around $\vec{x}$

$$\begin{bmatrix} C_5 \\ C_6 \end{bmatrix} = \begin{bmatrix} (y^2 - x^2) & -x & y & 0 & 1 \\ 2xy & y & x & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} \alpha_0 \\ \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \end{bmatrix} \qquad (4.4)$$

The fact that linear astigmatism is the most important component can be seen in figure 4.10 and 4.11. The coma present is only the linear coma from the nominal centered system represented in figure 4.9 and the total astigmatism is almost similar to the linear part.

- Tilt of .1 degree around $\vec{y}$

    It was seen in the second step that a tilt around $\vec{y}$ will also produce only linear astigmatism but with a pattern tilted by 90 degrees compared to figure 4.10. This is confirmed in figure 4.12 and 4.13.

Here the program returns: $\alpha_2 \approx -5.767$, where x and y are in mm and the
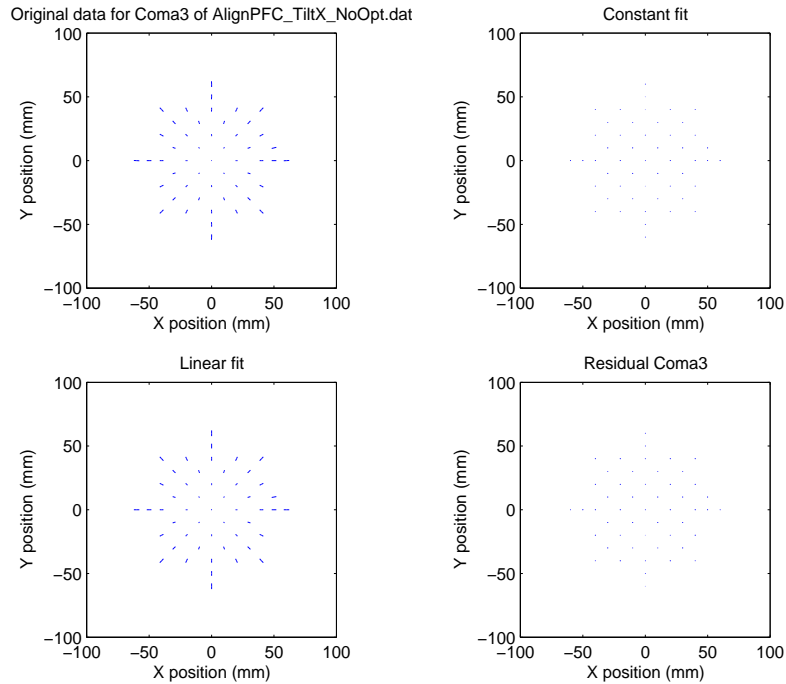
FIGURE 4.11. Coma representation when secondary tilted .1 degree around $\vec{x}$

Zernike coefficients in nm. Thus by considering that the secondary was tilted by .1 degree, the general equation is: $\alpha_2 = -57.67 \cdot TiltY$, where $TiltY$ represents the amount of tilt for the secondary around $\vec{y}$ in degrees. The other terms are negligible.

- Decenter of 1mm along $\vec{x}$

  It was seen previously that a decenter along $\vec{x}$ was producing constant coma along $\vec{x}$. This is confirmed with the graphics from Matlab® in figure 4.14 and 4.15.

By fitting the Zernike coefficients calculated in the previous step with the matrix formulation of C_7 and C_8 in a perturbed system which is shown in equation 4.5, the values of β_0, β_2 and β_2 are calculated. Here, only the constant part is significant. The program returns: $\beta_2 \approx -199.8$, where x and y are in mm and the Zernike coefficients in nm. Thus by considering that the secondary was
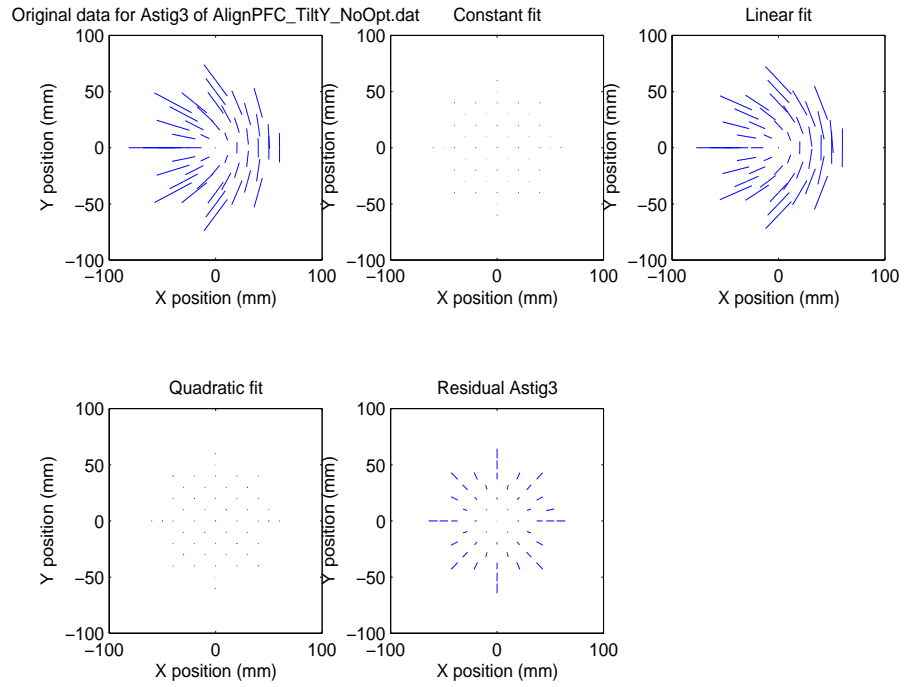
FIGURE 4.12. Astigmatism representation when secondary tilted .1 degree around $\vec{y}$
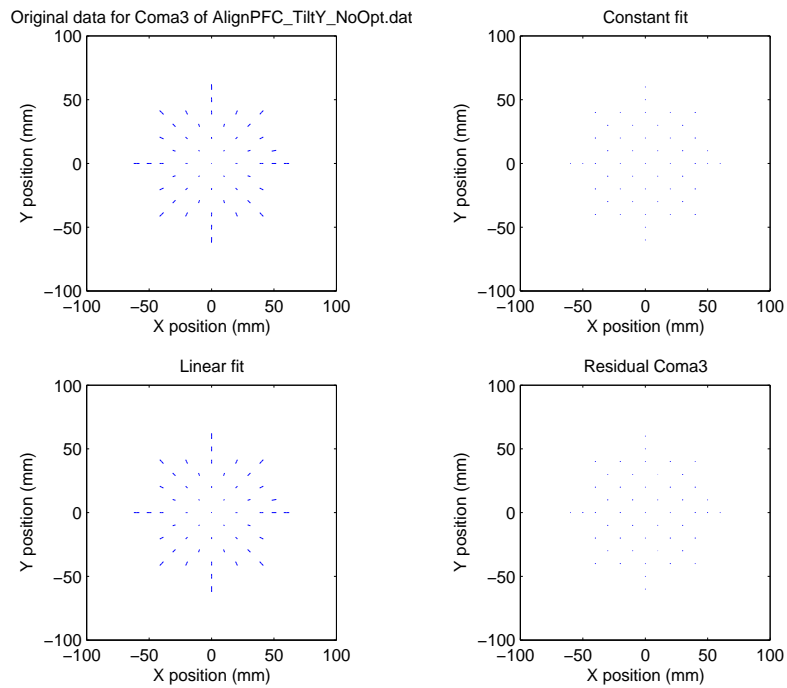
FIGURE 4.13. Coma representation when secondary tilted .1 degree around $\vec{y}$
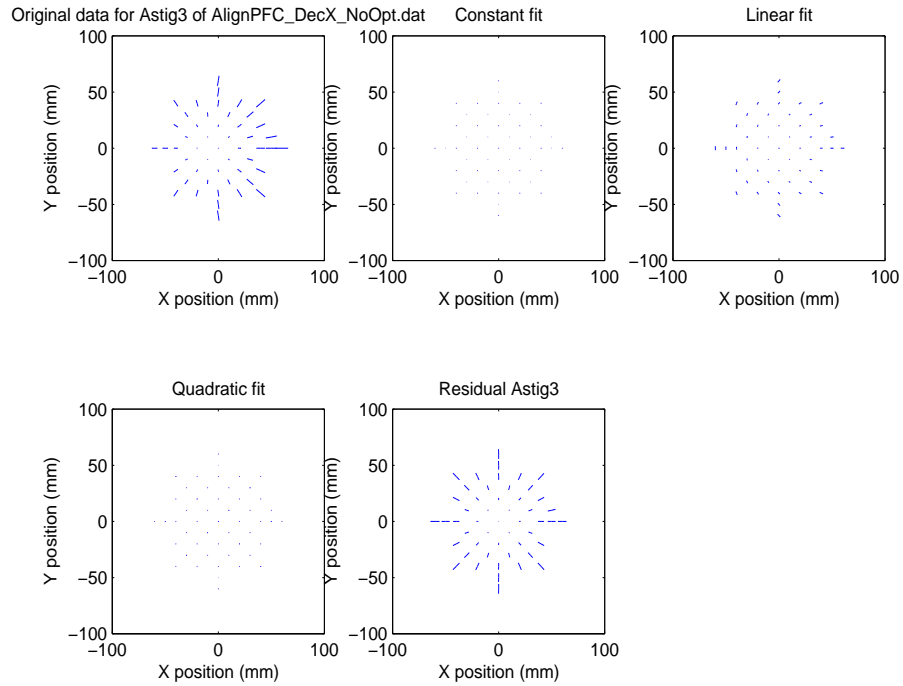
FIGURE 4.14. Astigmatism representation when secondary decentered 1mm along $\vec{x}$

decentered by 1mm, the general equation is: $\beta_2 = -199.8 \cdot DecX$, where $DecX$ represents the amount of decenter for the secondary in the $\vec{x}$ direction in mm.

$$\begin{bmatrix} C_7 \\ C_8 \end{bmatrix} = \begin{bmatrix} y & 0 & 1 \\ x & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \end{bmatrix} \tag{4.5}$$

- Decenter of 1mm along $\vec{y}$

    It was seen previously that a decenter along $\vec{y}$ will also produce only constant coma but orientated along $\vec{y}$ instead of $\vec{x}$. This is confirmed in figure 4.16 and 4.17.

Here the program returns: $\beta_1 \approx -199.8$, where x and y are in mm and the Zernike coefficients in nm. Thus by considering that the secondary was decentered by 1mm, the general equation is: $\beta_1 = -199.8 \cdot DecY$, where $DecY$ represents the amount of decenter for the secondary in the $\vec{y}$ direction in mm.
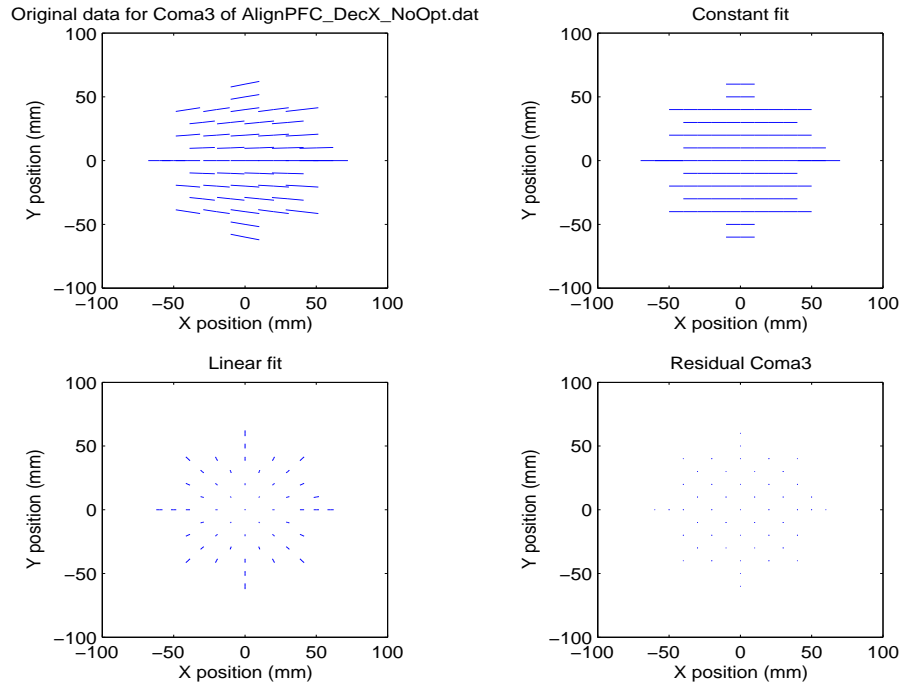
FIGURE 4.15. Coma representation when secondary decentered 1mm along $\vec{x}$

The other terms are negligible.

Finally, the Zernike coefficients can be expressed in terms of the perturbations and the field positions in a matrix form as shown in equation 4.6.

$$
\begin{bmatrix} C_5 \\ C_6 \\ C_7 \\ C_8 \end{bmatrix} = \begin{bmatrix} 57.67\,x & -57.67\,y & 0 & 0 \\ -57.67\,y & -57.67\,x & 0 & 0 \\ 0 & 0 & 0 & -199.78 \\ 0 & 0 & -199.78 & 0 \end{bmatrix} \cdot \begin{bmatrix} TiltX \\ TiltY \\ DecX \\ DecY \end{bmatrix} \tag{4.6}
$$

Where x and y are in mm, the Zernike coefficients in nm, the tilts in degrees and the decenters in mm.

Then, it is possible to invert the matrix which was just derived in equation 4.6 so as to retrieve the perturbations by having the values of the Zernike coefficients for one position in the field. Of course, in the real system one measure in the field will not be enough to get accurate values of the perturbations due to the noise. Thus, a
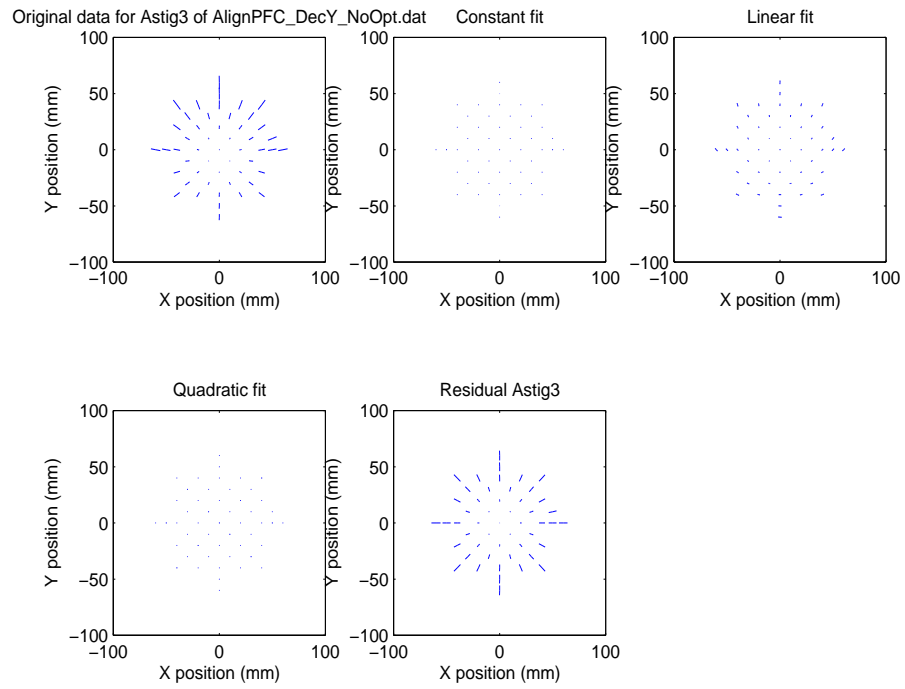
FIGURE 4.16. Astigmatism representation when secondary decentered 1mm along $\vec{y}$
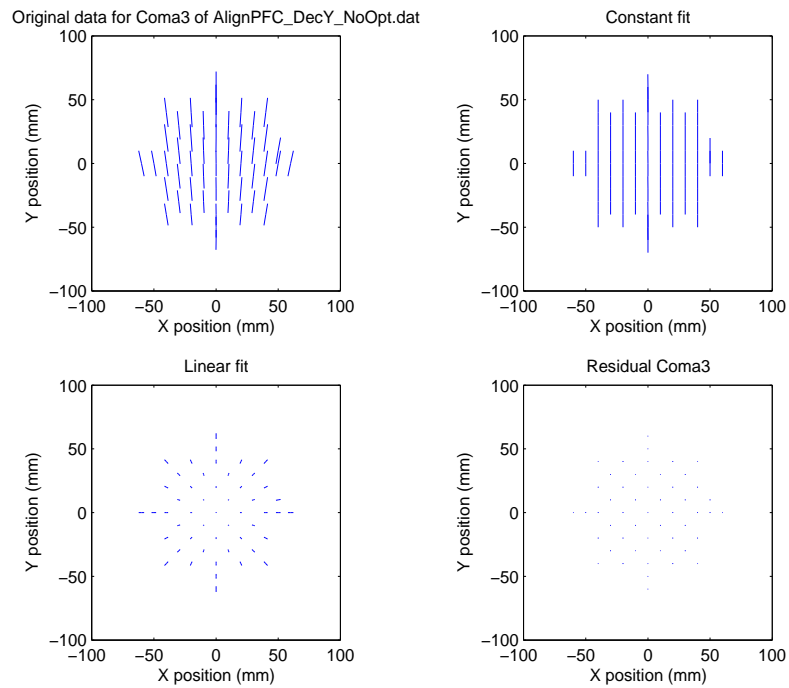
FIGURE 4.17. Coma representation when secondary decentered 1mm along $\vec{y}$

(a) Astigmatism representation   (b) Coma representation
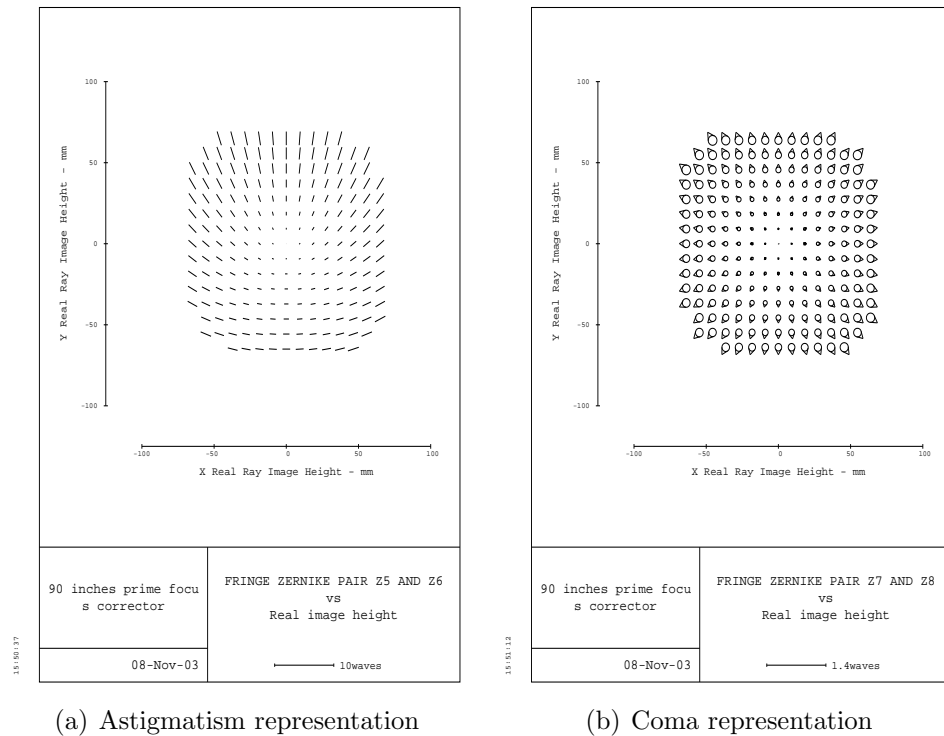
FIGURE 4.18. Aberration representations when the secondary is tilted by .1 degree around $\vec{x}$

least-squares fit with multiple measurements in the field will be used to minimize the effects of the noise.

The same kind of representation that was done with Matlab® for the total amount can be done in Code V® as shown in figure 4.18 and 4.19 where astigmatism and coma have been represented for a tilt of 0.1 degree around $\vec{x}$ and a decenter of 1mm along $\vec{x}$. It is a good way to get a fast representation of the aberrations at the image plane but there is no possibility to have a decomposition of the different field dependencies of each aberration. It is only a representation of the total amount of each aberration and there is no text output of the data that were fitted.
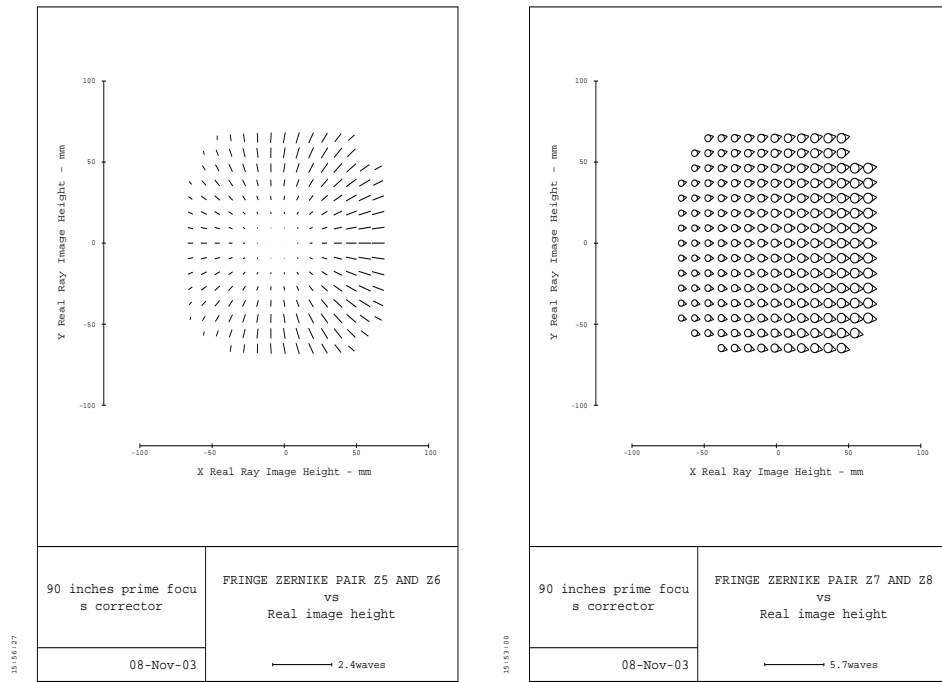
(a) Astigmatism representation

(b) Coma representation

FIGURE 4.19. Aberration representations when the secondary is decentered by 1mm along $\vec{x}$

4.3 A graphical method to verify the results from the previous procedure

The previous results can be verified by using a graphical analysis. This is done by following these steps:

1. Remove the aberrations of the nominal centered system

2. Find the "Free coma point"

3. Analyze each degree of freedom

4. Find the relation between the Zernike coefficients and the perturbations graphically

4.3.1 Aberrations removed from the nominal centered system

To see only the effects of misalignments in the system, it is possible to simplify the analysis by removing the aberrations present in the centered system. In the real system this can be done by subtracting the values of the Zernike coefficients calculated in Code V® from the wavefront measurement that will be made with the wavefront sensor of the real system.

In Code V®, instead of a subtraction, the aberrations can be removed by using aspherizations on the surfaces of the lenses. This will not greatly affect the results if the coefficients are not too large, it will only affect higher orders. This way, the direct effects of the misalignments of the corrector can be observed. This optimized system is a diffraction limited system as it can be seen on the spot diagram in figure 4.20 where all the rays are contained in the Airy disk represented by the black circle.

Since the way to find the "free coma point" was already discussed in the previous section, it will not be done here.
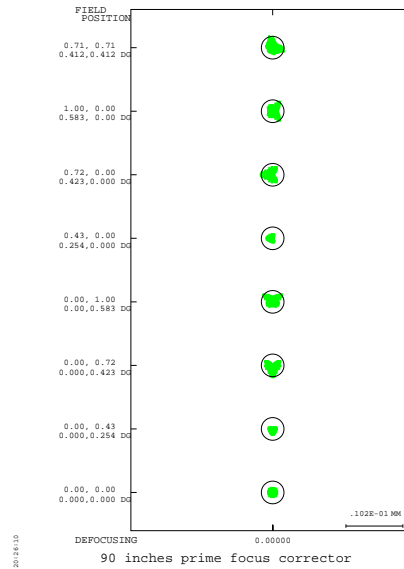
FIGURE 4.20. Spot diagram for different field positions for the optimized system

4.3.2 Analyzis of each degree of freedom

As it was done before, the system is perturbed for each degree of freedom. A tilt of the secondary around $\vec{x}$ or $\vec{y}$ will introduce only linear astigmatism as shown in figure 4.21 and 4.22. Due to the fact that the aberrations have been removed from the original design, only the linear astigmatism appears. This is confirmed by fitting the data into its different components as shown in figure 4.23. Only the representation of the astigmatism for the tilt around $\vec{x}$ is shown here since it is the same for the tilt around $\vec{y}$ except that there is a 90 degrees rotation.

By decentering the secondary along $\vec{x}$ and $\vec{y}$, constant coma orientated along $\vec{x}$ and $\vec{y}$ is introduced respectively as shown in figure 4.24.

4.3.3 Graphical analysis of the spot diagrams

Even if the relations found in chapter 3 are not known, the Zernike coefficients and the perturbations can still be found by using a graphical analysis.

The linear astigmatism of figure 4.21, where the secondary is tilted around $\vec{x}$, is
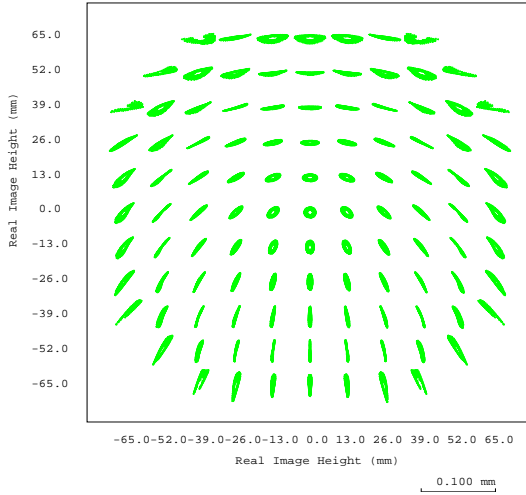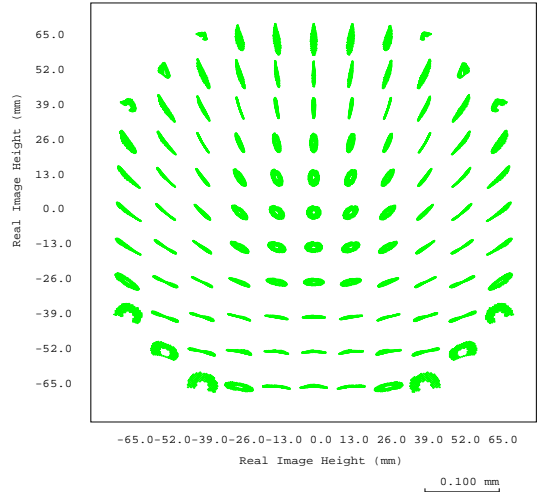
(a) Inside focus

(b) Outside focus

FIGURE 4.21. 2D spot diagram in the xy plane for alpha tilt=.1 degree for the optimized design
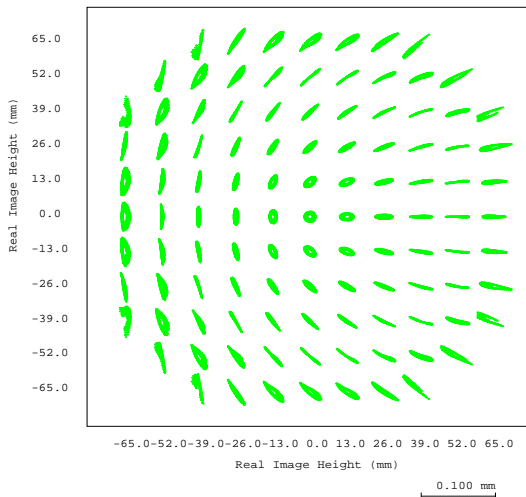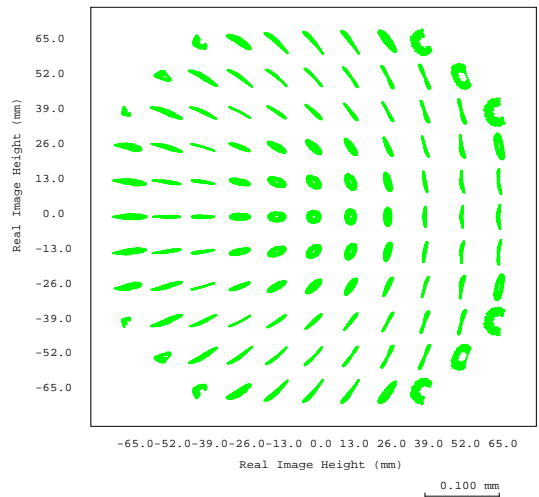
(a) Inside focus

(b) Outside focus

FIGURE 4.22. 2D spot diagram in the xy plane for beta tilt=.1 degree for the optimized design
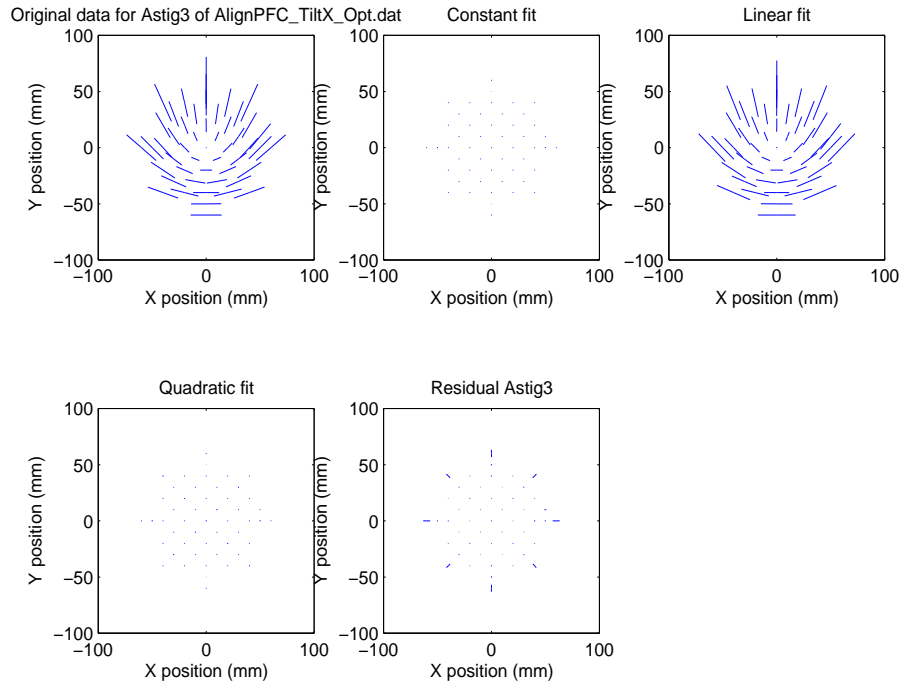
FIGURE 4.23. Astigmatism representation when the secondary is tilted by .1 degree around $\vec{x}$
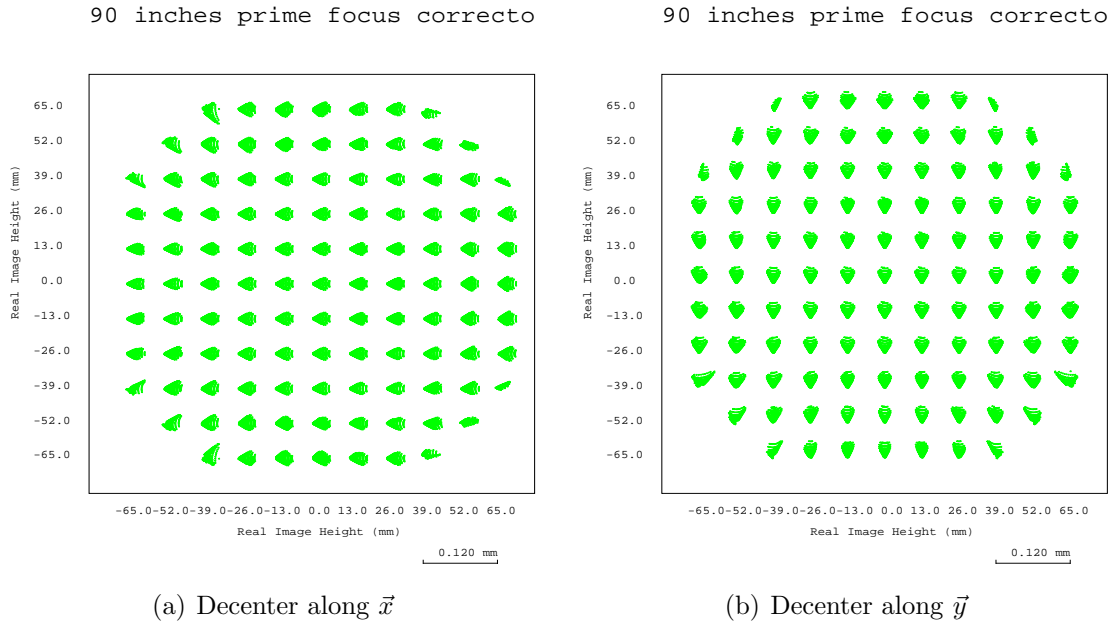
(a) Decenter along $\vec{x}$ (b) Decenter along $\vec{y}$

FIGURE 4.24. 2D spot diagram in the xy plane for a decenter of 1mm for the optimized design

(a) Case when y=0 ($\vec{x}$ axis) (b) Case when x=0 ($\vec{y}$ axis)
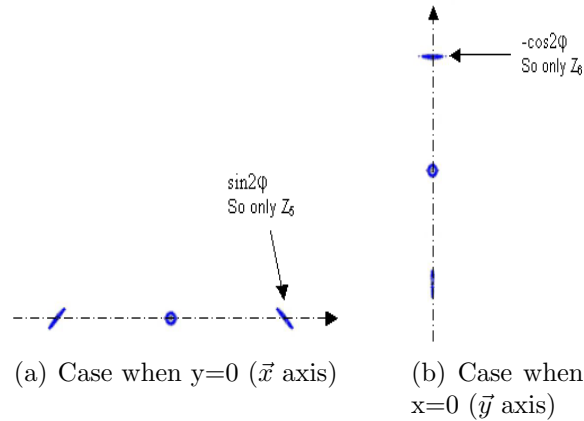
FIGURE 4.25. Graphical representation to get the expressions of C_5 and C_6

considered for the inside focus image. For y=0 ($\vec{x}$ axis) for the positive x values, only the $\sin 2\phi$ component is present which represents Z_5. Thus, for the case y=0, $C_5 = x \cdot TiltX \cdot Constant_1$. This is shown in figure 4.25.

For x=0 ($\vec{y}$ axis) for the positive values of y, there is only the $-\cos 2\phi$ component which represents $-Z_6$. Thus, for the case x=0, $C_6 = -y \cdot TiltX \cdot Constant_1$. This is shown in figure 4.25.

The same analysis is performed when the secondary is tilted around the $\vec{y}$ axis. The equations are then: $C_5 = -y \cdot TiltY \cdot Constant_2$ and $C_6 = -x \cdot TiltY \cdot Constant_2$.

Therefore, a linear combination of tilts around $\vec{x}$ and $\vec{y}$ will generate astigmatism which is represented by:

$$\begin{aligned} C_5 &= Constant_1 \cdot (-x \cdot TiltX + y \cdot TiltY) \\ C_6 &= Constant_2 \cdot (y \cdot TiltX + x \cdot TiltY) \end{aligned} \tag{4.7}$$

Note that the minus sign has been included in the constants. Thus, the equations of chapter 3 have been retrieved graphically. To get the exact coefficients, a least-squares fit can be used as it was done previously. The calculations with Matlab® give: $Constant_1 = Constant_2 = -58.3$.

For the coma, the analysis is even easier since the amplitude and the orientation are constant.

By including the decentrations, the final equation becomes:

$$\begin{bmatrix} C_5 \\ C_6 \\ C_7 \\ C_8 \end{bmatrix} = \begin{bmatrix} 58.3\,x & -58.3\,y & 0 & 0 \\ -58.3\,y & -58.3\,x & 0 & 0 \\ 0 & 0 & 0 & -199.88 \\ 0 & 0 & -199.88 & 0 \end{bmatrix} \cdot \begin{bmatrix} TiltX \\ TiltY \\ DecX \\ DecY \end{bmatrix} \quad (4.8)$$

Which is really close to the previous results. The small differences come from the introduction of the aspherizations on the lenses.

4.4 Results from the real system

During the first attempt to align the 90", the assumption was made that the system did not have a lot of initial aberrations referring to the nominal design in Code V®. Thus, only constant coma and linear astigmatism were expected. But the alignment based only on the linear astigmatism and the constant coma did not worked because the aberrations observed were not coming only from the misalignments.

Therefore, to understand the behavior of the aberrations, the data coming from the wavefront sensor of the telescope were fitted with all the components of the astigmatism and the coma and it was then possible to align the telescope as well as it can be for the tilts and almost for the decenters [5].

The results of the last run to align the telescope are shown in this section. They are important in the sense that they validate the equations and the method developed in this thesis.

4.4.1 Analysis of the astigmatism

The astigmatism representation of the system in figure 4.26 shows that the linear part is not significant, meaning that the secondary is aligned in terms of tilts.

[5]Not possible to fully aligned the telescope because a fixed amount of time was allowed for the alignment of this telescope

Also, it can be seen in figure 4.26 that an important quadratic astigmatism which was not expected is present. By comparing the total amount of astigmatism and this quadratic part, it can be seen that it is the main contribution. More quantitative results are shown in table 4.1 where the average and the maximum[6] values have been considered for each component.

Type of astigmatism	Average value		Maximum value	
	RMS wavefront error (nm)	RMS spot size diameters (")	RMS wavefront error (nm)	RMS spot size diameter (")
Constant	46	0.06	46	0.006
Linear	48	0.07	69	0.09
Quadratic	274	0.37	538	0.73
Quartic	2	0	7	0.01
Residual	39	0.05	227	0.31

TABLE 4.1. Results for the astigmatism for the data of the wavefront sensor from the 90" telescope

The constant part is orientated at 41 degrees and came probably from the primary mirror.

The residual linear part can be corrected by a tilt of 0.012 degrees around $\vec{x}$ and 0.002 degrees around $\vec{y}$. But the values are small enough not to have to tilt the secondary further.

Because the field dependence is mainly quadratic, this means that the astigmatism comes from the nominal centered system and not from the misalignments. There can be several reasons for the presence of this astigmatism. It can be due to the fact that the spacings between the elements and/or the radius of curvature of the primary and/or the conic constants are not as they were supposed to be. Thus, a further analysis needs to be done to see the influence of all these parameters and see if a solution could be found to correct for this aberration in a simple way. Note that an acceptable spot size is .7 arseconds, thus it will maybe not be necessary to correct for

---

[6]The maximum value is located at the edge of the field due to the field dependence of the astigmatism
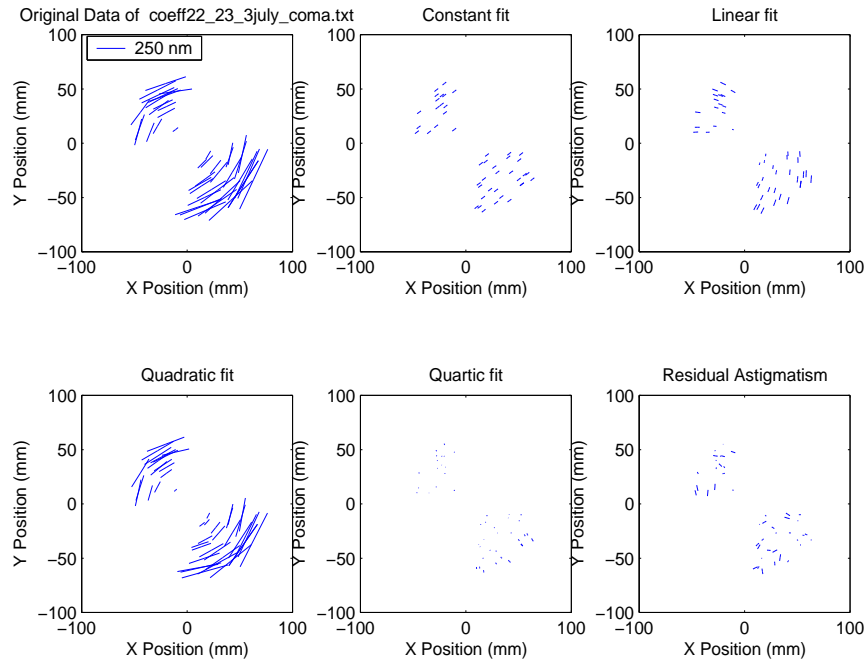
FIGURE 4.26. Astigmatism representation for the 90" telescope

the quadratic astigmatism.

4.4.2   Analysis of the coma

If the corrector is decentered, constant coma over the field is expected. To see if the data are consistent, each star is represented by a green square in figure 4.27 to show its value for the cosine and the sine coma. Since the coefficient for Z_8 (cosine coma) is related to a decenter in the $\vec{x}$ direction and the coefficient for Z_7 (sine coma) is related to a decenter in the $\vec{y}$ direction, the values of the decentrations are also represented on the axes . And finally, the average value with error bars for the standard deviations in both directions are represented.

Figure 4.27 shows that the measurements contain a large amount of noise; the average value is in the order of the value of the standard deviation.

As it was done for astigmatism, the coma is decomposed in its different components in figure 4.28 where the total amount of coma, the constant fit, the linear fit and the
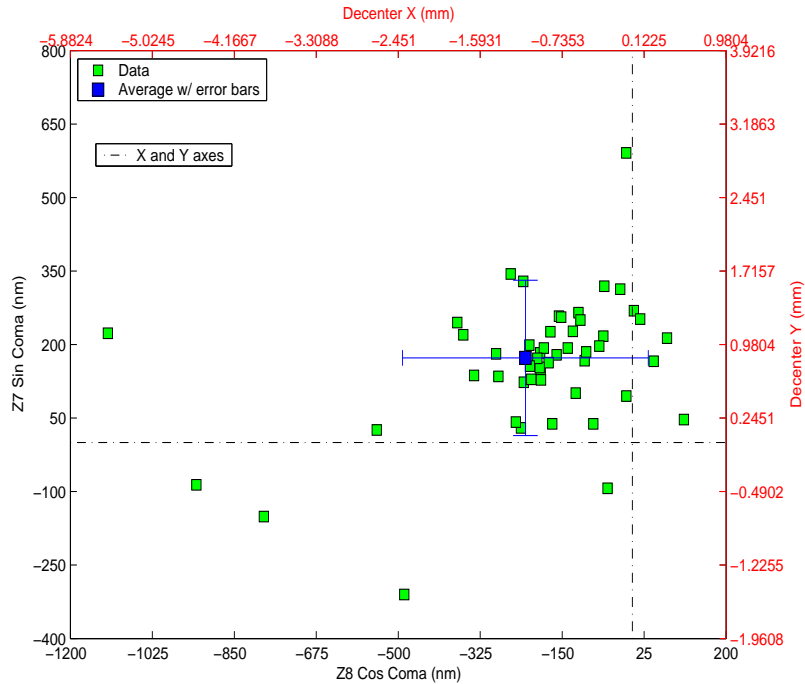
FIGURE 4.27. Plot of the coma values for the 90" telescope

residual coma are represented.

Figure 4.28 shows that the coma contains a constant part but there is also a lot of noise as it was said before. Indeed on the last graphic of the residual coma, it is not possible to discern any pattern, the directions and the amplitudes are erratic. Figure 4.28 also shows that linear coma is not significant.

Figures 4.27 and 4.28 show that there are a few points which deviate a lot from the average value. Therefore, they increase the value of the standard deviation. These points are located at the edge of the field and can be taken away from the data to get a better estimation of the error in the measurements. This was done in figure 4.29.

With the new set of data shown in figure 4.29, the new average values for C_7 and C_8 are:

$$\overline{C}_7 = 173.6nm$$
$$\overline{C}_8 = -159.3nm$$

(4.9)
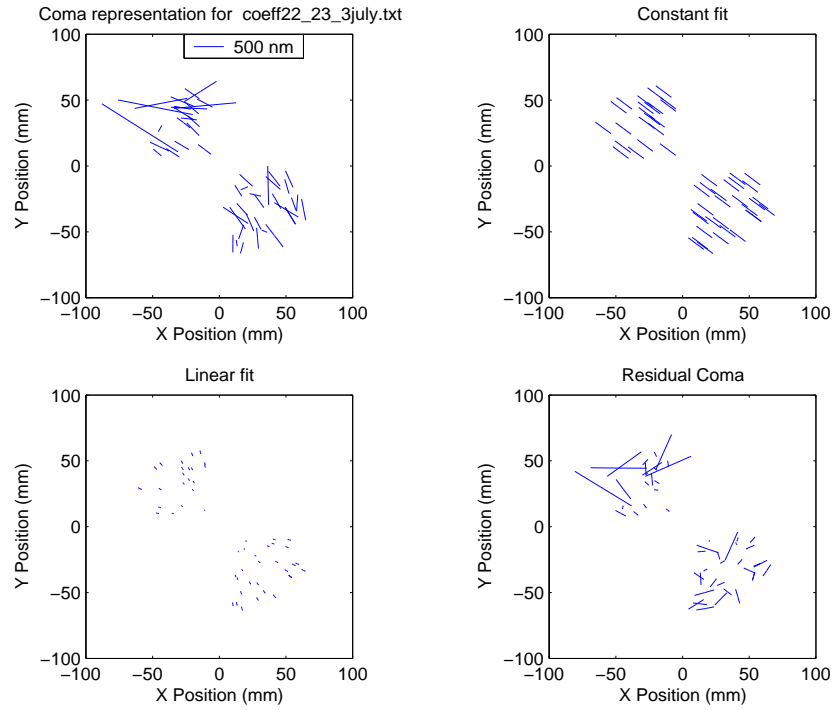
FIGURE 4.28. Decomposition of the coma for the real data from the wavefront sensor
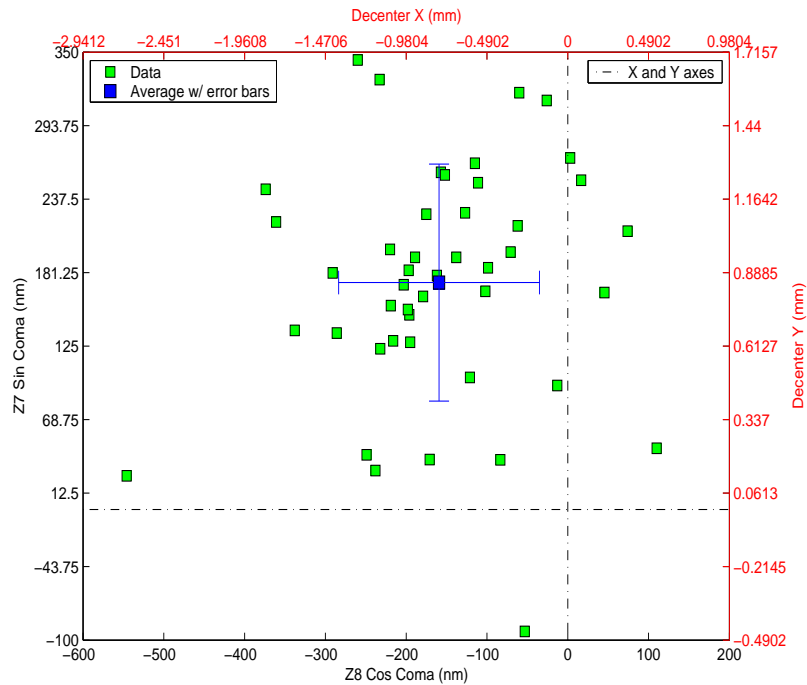
FIGURE 4.29. Plot of the coma values without the stars at the edge of the CCD

And the corresponding standard deviations for C_7 and C_8 are:

$$\sigma_{C_7} = 90.7nm$$
$$\sigma_{C_8} = 124.3nm$$

$$(4.10)$$

If gaussian distributions are assumed, the true value of C_7 is between $\overline{C}_7 - \frac{2\sigma_{C_7}}{\sqrt{N}}$ and $\overline{C}_7 + \frac{2\sigma_{C_7}}{\sqrt{N}}$ and C_8 is between $\overline{C}_8 - \frac{2\sigma_{C_8}}{\sqrt{N}}$ and $\overline{C}_8 + \frac{2\sigma_{C_8}}{\sqrt{N}}$[7] with a 95% confidence.

Thus:

$$C_7 = \quad 173.6 \pm 27nm$$
$$C_8 = -159.3 \pm 37nm$$

$$(4.11)$$

Equations 4.11 correspond to move the prime focus corrector:

- In the $\vec{x}$ direction by $0.78 \pm 0.18mm$

- In the $-\vec{y}$ direction by $0.85 \pm 0.13mm$

This section has shown that the equations developed in the previous section are useful in the process of aligning the telescope. But they need to be used with caution because as it was seen in this section, the system does not always behave as the model. Here the astigmatism was to be decomposed in terms of its field dependencies so as not to take into account the quadratic part in the alignment process.

The next chapter will use the same analysis which was performed for this system but for a more complicated system where the theory on fifth order aberrations is to be used.

---

[7]With N the number of measurements in the field so the number of stars

Chapter 5

APPLICATION TO THE LSST

In this chapter a more complicated design is considered. It will be seen that the method applied for the 90" telescope can be used in a similar way to the Large Synoptic Survey Telescope (LSST) which presents a difficult alignment problem due to the number of elements, the small f/# which is equal to 1.25, and the wide field of view.

Because of the complexity of the system, it will be necessary to consider the higher order aberrations which can not be neglected as for the 90" telescope. Thus, the expressions of the Zernike polynomials which include the fifth order will have to be used.

In a first section, the design of the telescope will be presented. Then a reverse optimization alignment method which does not use the equations developed earlier will be shown. Finally it will be underlined the efficiency of the technique used for the 90" telescope compared to a "brut force" method.

5.1   Design on the LSST

The optical design considered here and represented in figure 5.1 is the design realized by LLNL under contract to NOAO in support of LSST. The starting point of the design was a three-mirror telescope designed by Roger Angel et al. with a centrally obscured, 3.0 degree full field of view, 8.4m aperture telescope that operates at f/1.25. The instrument imaging assembly consists of two refractive correctors, a group of interchangeable spectral filters and a 55cm diameter array of CCD detectors. This instrument will be called later on: "the corrector". It is placed in the shadow of the secondary mirror obscuration. The central obscuration is about 50% by radius

11:38:27

Design of the LSST   Scale: 0.02   22-Oct-03

1470.59 MM
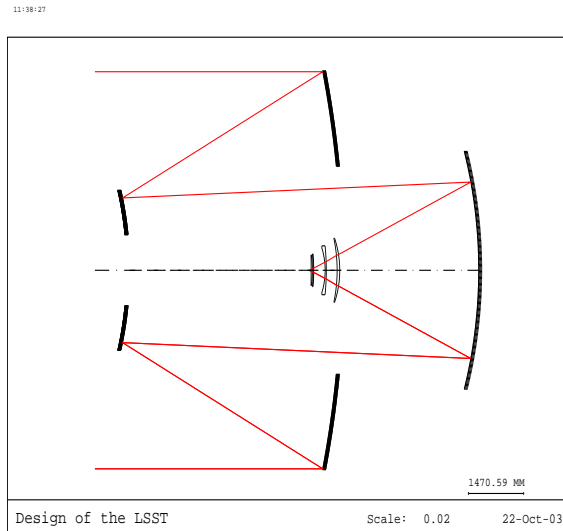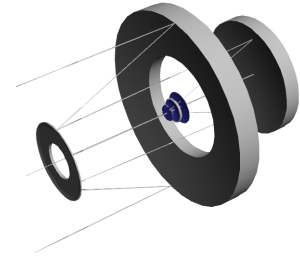
(a) 2D representation of the design   (b) 3D representation of the design

FIGURE 5.1. Design of the LSST

which represents an effective area that is equivalent to a 7m unobstructed telescope. For more details on the design of the system see [21].

5.2 Reverse optimization

To realize the alignment of the telescope it is not necessary to have all the tools which were developed in the previous chapters and it is always possible to use a "brut force" which uses only numerical calculations. Of course this kind of approach is not the best way to tackle the problem since no insights are gained but it allows to get to a solution without a time consuming analysis.

This section will present how to realize this "brut force" analysis on a smaller scaled version of the LSST by running Monte Carlo simulations.

5.2.1 Degrees of freedom considered for the analysis

The scaled version considered here is similar to the design presented in the previous section. Thus the results shown here can be applied directly to the LSST.

In this design, the primary mirror is considered as the reference. So, it will be fixed and the other elements will be perturbed relative to it.

It is possible to consider a large number of degrees of freedom, but here only 19 will be considered.

For the bending modes, only the surface of the primary is considered. Since the design here is a smaller version of the LSST, it will be necessary to account only for the astigmatism (Z_5 and Z_6) and the trefoil (Z_{10} and Z_{11}) which corresponds to 4 degrees of freedom. For the LSST it is necessary to consider 12 bending modes, Z_4-Z_{15}, on the primary and also 6 modes, Z_4-Z_9 on the secondary.

The other 15 degrees of freedom will be represented by the 3 decenters along each axis and the tilts around the $\vec{x}$ and $\vec{y}$ axes for the other three elements. Because the system is axially symmetric, a rotation around $\vec{z}$ has no effect.

5.2.2 Scheme of the analysis

The general procedure used here can be summarized into 5 steps:

1. Perturbation of each degree of freedom

2. Calculation of the wavefront of the perturbed system given by an optical software

3. Simulation of the wavefront measurement by adding noise to the wavefront data

4. Estimation of the perturbations with optical software by using a Merit function

5. Correction of the perturbations with the previous estimation

During the first step, each degree of freedom is perturbed randomly in a reasonable range in Zemax®. Then, the wavefront is calculated and a real wavefront measurement is simulated by adding noise to the calculated wavefront. Then a merit function is used with the unperturbed model of the telescope to get a combination of

perturbations that will produced an identical wavefront so as to get an estimation of the misalignments. And finally, the estimated perturbations are used to correct for the misalignments. Since the problem is underconstrained, multiple combinations of perturbations will produced the same wavefront. Therefore, the system will not converge with one iteration considering also that some noise was added. Thus the process will be repeated 5 times to see when the system converges to an aligned situation.

5.2.3 Starting point

As it was said before, the first step consists in randomly perturbing the system for each degree of freedom. The random value RV is generated from a uniform distribution with a value PV that is set for each degree as shown in table 5.1. RV is defined as:

$$RV = rand(PV) - \frac{PV}{2} \tag{5.1}$$

Where $x \mapsto rand(x)$ represents a function which generates a random floating point number uniformly distributed between 0 and x.

Element	Degree of freedom	PV
M1	Z_5 and Z_6	$.56\mu$m
	Z_{10} and Z_{11}	$.47\mu$m
M2, M3, corrector	DecX and DecY	400μm
	TiltX and TiltY	.01degrees
Spacings	Distance M1-M2	148μm
	Distance M2-M3	936μm
	Distance last lens-focal plane	4mm

TABLE 5.1. PV values for each degree of freedom

The contribution of all the perturbations produces a certain RMS spot radius at the image plane. The distribution of the inital RMS spot radius for all the models are represented in the histogram of figure 5.2.
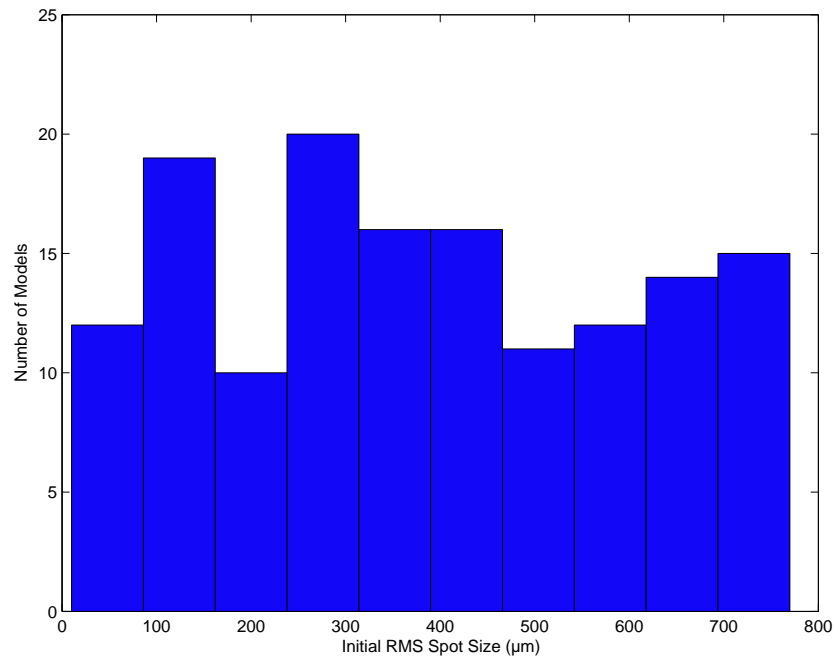
FIGURE 5.2. Distribution of the initial RMS spot radius for all the models

5.2.4 Results of the analysis

The scheme presented previously has been iterated five times and at the end for the five iterations the RMS spot radius relative to the centroid was calculated for each model. The different iterations are represented in figure 5.3 where the RMS spot radius of the corrected system is a function of the initial value of the model. It shows the improvement of the spot size at the image plane compared to the original perturbed system. Note that for the unperturbed model, the spot size is equal to $1.8\mu m$.

Figure 5.3 shows that with the first iteration a lot of the models converge to a small RMS spot radius close to the target value of the unperturbed system. Thus, this analysis has shown that it was possible to align the telescope with a good accuracy even if the original spot radius was large.

The same analysis on the original design of the LSST has been done by C. Claver
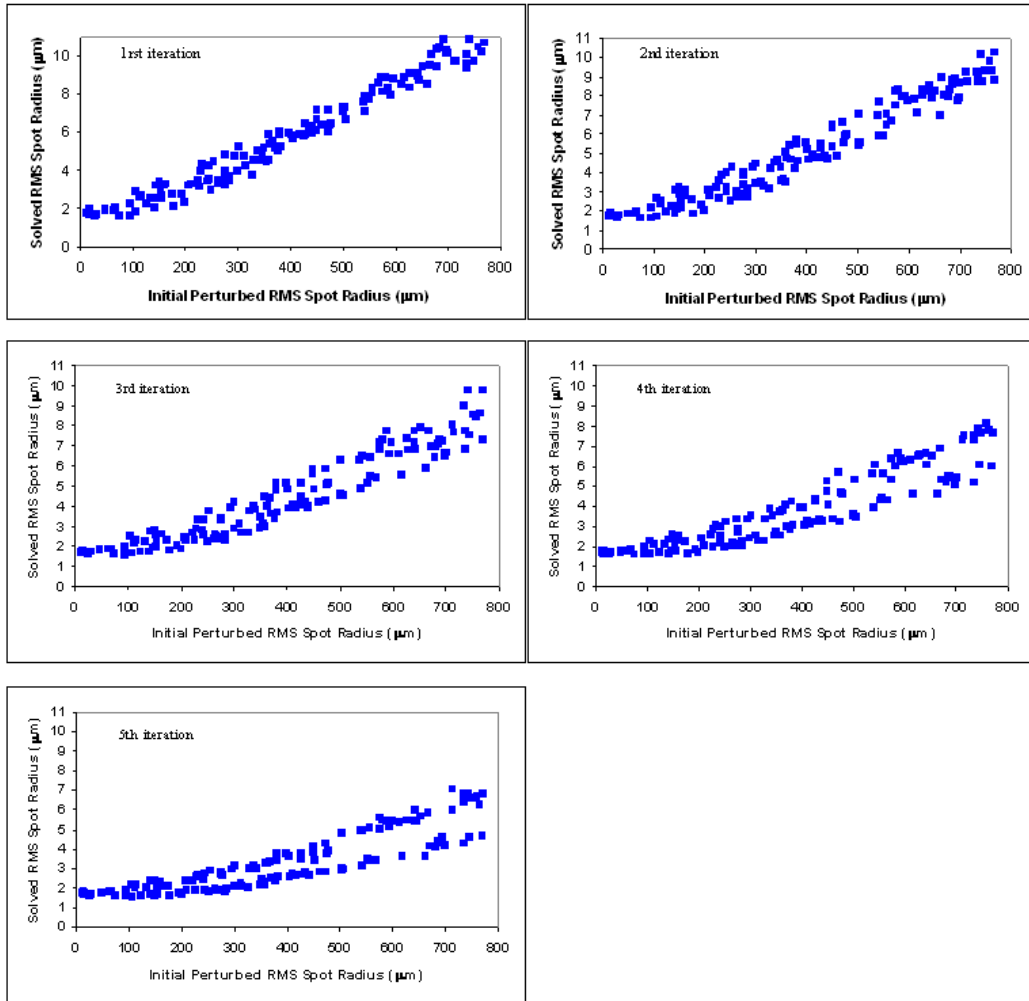
Figure 5.3. Iterations of the process

and J.H. Burge [22]. Due to the different scale, only half of the models were converging during the first iteration to a reasonable spot size. But after 5 iterations about 91% of the models were meeting the requirements for the image spot size as shown in figure 5.4 which was taken from [22].

But this analysis does not account for the motions of the mirrors. Some of the solutions use large tilts and decenters for the elements which may not be acceptable for the real system from a mechanical point of view. Also, for the LSST the alignment process can require five iterations and sometimes never meet the requirements if the system is in a local minimum. Therefore the technique used for the 90" by fitting the wavefront with the equations found in chapter 3 give better insight on the behavior of the system and will lead to a better procedure. It is worth pointing out that it is possible to align the telescope using only a numerical method but it is more efficient to find a structured procedure.

5.3 Relations between the perturbations and the Zernike coefficients

The previous section has shown the limits of a numerical based method and the importance to have some insight in the behavior of the system. Thus this section will follow the same procedure that was used with the 90" in order to have a better understanding of the behavior of the system.

The goal to perform a good alignment is to find the matrix A defined in equation 5.2 . Thus, by having a wavefront measurement for the real system, it will be possible to determine the perturbations of the system by inverting the matrix. This matrix represents the reconstructor of figure 4.1.
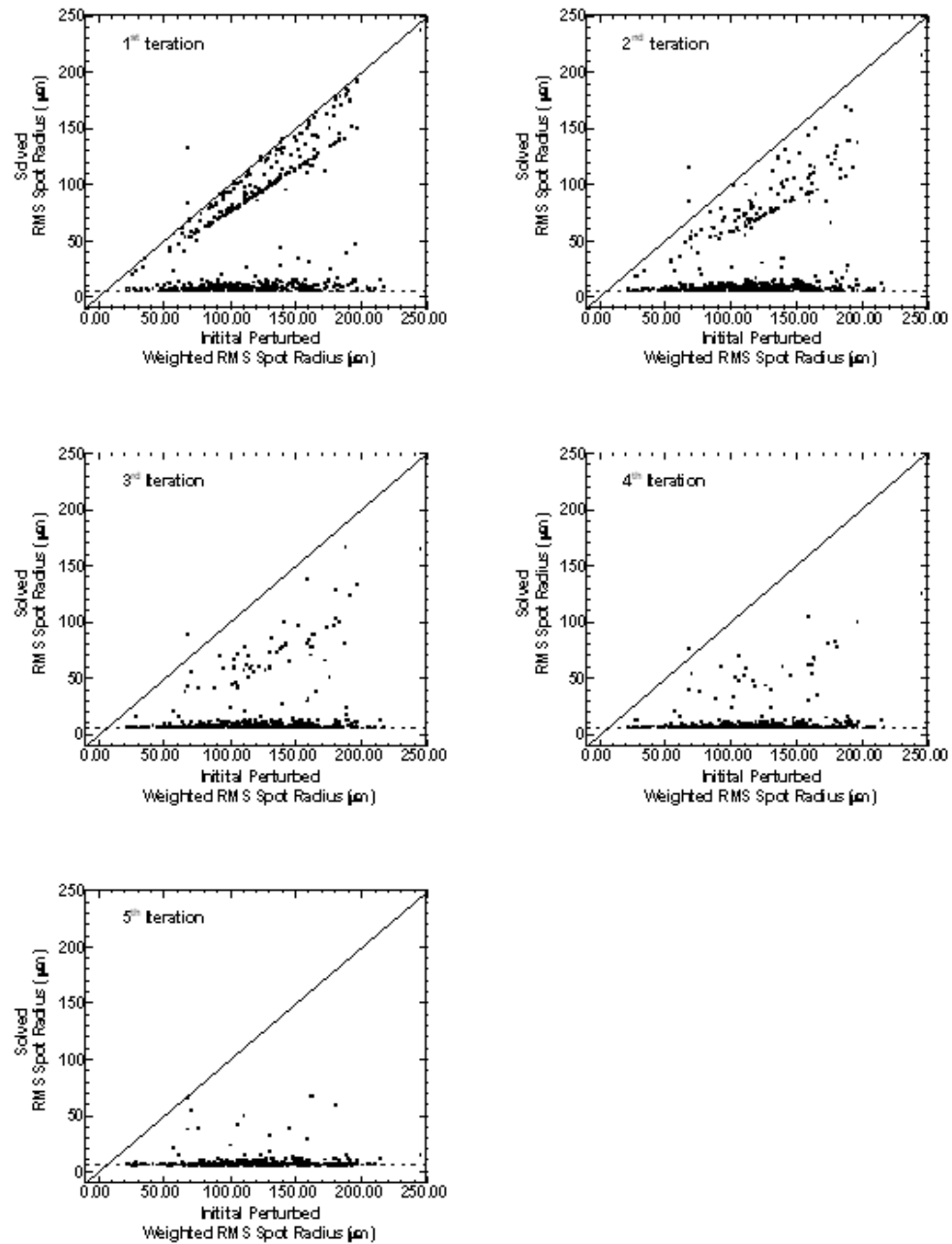
FIGURE 5.4. The merit function for the solved models versus initial merit function of the perturbed model. In each panel from top-left to bottom-right the solutions from successive iterations are shown. Points above the solid diagonal have degraded, those below have improved. The dashed horizontal line is the unperturbed merit function target.

$$
\begin{bmatrix} C_5 \\ C_6 \\ C_7 \\ C_8 \\ C_9 \\ C_{10} \\ C_{12} \\ C_{13} \\ C_{16} \\ C_{17} \end{bmatrix} = \begin{bmatrix} a_{1,1} & a_{1,2} & a_{1,3} & \cdots & a_{1,10} & a_{1,11} & a_{1,12} \\ a_{2,1} & a_{2,2} & a_{2,3} & \cdots & a_{2,10} & a_{2,11} & a_{2,12} \\ a_{3,1} & a_{3,2} & a_{3,3} & \cdots & a_{3,10} & a_{3,11} & a_{3,12} \\ a_{4,1} & a_{4,2} & a_{4,3} & \cdots & a_{4,10} & a_{4,11} & a_{4,12} \\ a_{5,1} & a_{5,2} & a_{5,3} & \cdots & a_{5,10} & a_{5,11} & a_{5,12} \\ a_{6,1} & a_{6,2} & a_{6,3} & \cdots & a_{6,10} & a_{6,11} & a_{6,12} \\ a_{7,1} & a_{7,2} & a_{7,3} & \cdots & a_{7,10} & a_{7,11} & a_{7,12} \\ a_{8,1} & a_{8,2} & a_{8,3} & \cdots & a_{8,10} & a_{8,11} & a_{8,12} \\ a_{9,1} & a_{9,2} & a_{9,3} & \cdots & a_{9,10} & a_{9,11} & a_{9,12} \\ a_{10,1} & a_{10,2} & a_{10,3} & \cdots & a_{10,10} & a_{10,11} & a_{10,12} \end{bmatrix} \cdot \begin{bmatrix} t_x^{M2} \\ t_y^{M2} \\ d_x^{M2} \\ d_y^{M2} \\ t_x^{M3} \\ t_y^{M3} \\ d_x^{M3} \\ d_y^{M3} \\ t_x^{Corr.} \\ t_y^{Corr.} \\ d_x^{Corr.} \\ d_y^{Corr.} \end{bmatrix} \tag{5.2}
$$

Where a_{ij} ($1 \leq i \leq 10$, $1 \leq j \leq 12$) are the components of the matrix A, d and t represent respectively the decenter and the tilt, $M2$, $M3$ and $Corr.$ represent respectively the secondary, the tertiary and the corrector. Note that here, only 12 degrees of freedom were considered and it will be necessary later on to account for the spacing errors and the bending modes of the primary and the secondary which represent the more complex part in the alignment process[1]. Also, note that it was assumed in equation 5.2 that the Zernike coefficients are linear with the perturbations.

Recalling the equations through fifth order of chapter 3 and summarized in section

---

[1]This part will not be realized in this thesis.

3.4, the components of the matrix A are ($\forall \ 1 \leq i \leq 12$):

$$a_{1,i} = \chi_0(y^4 - x^4) - \chi_1(x^3 + xy^2) + \chi_2(x^2y + y^3) + \chi_4(x^2 + y^2) - \chi_5(2x^2y)$$
$$+ \chi_6(y^3 - x^2y) + \chi_7 x + \chi_8 y + \chi_9(-x^2 + y^2) + \chi_{10}y - \chi_{11}x + \chi_{13}$$

$$a_{2,i} = 2\chi_0(x^3y + xy^3) + \chi_1(x^2y + y^3) + \chi_2(x^3 + xy^2) + \chi_3(x^2 + y^2)$$
$$+ \chi_5(xy^2 - x^3) + \chi_6(2xy^2) + \chi_7 y - \chi_8 x + \chi_9 2xy + \chi_{10}x + \chi_{11}y + \chi_{12}$$

$$a_{3,i} = \xi_0(y^3 + x^2y) + \xi_1 xy + \xi_2 y^2 + \xi_3 y + \xi_5 (x^2 + y^2) + \xi_6 x - \xi_7 y + \xi_9$$

$$a_{4,i} = \xi_0(x^3 + xy^2) + \xi_1 x^2 + \xi_2 xy + \xi_3 x + \xi_4 (x^2 + y^2) + \xi_6 y + \xi_7 x + \xi_8$$

$$a_{5,i} = \mu_0(y^3 - 3x^2y) - 2\mu_1 xy + \mu_2(y^2 - x^2) + \mu_3 y - \mu_4 x + \mu_6$$

$$a_{6,i} = \mu_0(3y^2x - x^3) + \mu_1(y^2 - x^2) + \mu_2 2xy + \mu_3 x + \mu_4 y + \mu_5$$

$$a_{7,i} = \eta_0 2xy + \eta_1 y + \eta_2 x + \eta_4$$

$$a_{8,i} = \eta_0(y^2 - x^2) - \eta_1 x + \eta_2 y + \eta_3$$

$$a_{9,i} = \kappa_0 x + \kappa_2$$

$$a_{10,i} = \kappa_0 y + \kappa_1$$

(5.3)

To create the matrix A, each degree of freedom has to be perturbed independently and the corresponding Zernike coefficients has to be calculated. Here, the fifth order aberrations have been considered with the third order terms since the system is a fast wide field angle telescope with multiple elements.

Basically, the same procedure that was followed for the 90" will be used except that each element will be perturbed about its vertex instead of the no coma point. The reason for that is that there may not exist a single point where all the elements can be perturbed about and introducing only one aberration.

The scheme will contain three major steps and will be done into Zemax® this time but the same analysis has been done in Code V® (the macro in Zemax® for the LSST can be found in Appendix G):

1. Perturb each element with two tilts around $\vec{x}$ and $\vec{y}$ and two decenters along $\vec{x}$ and $\vec{y}$
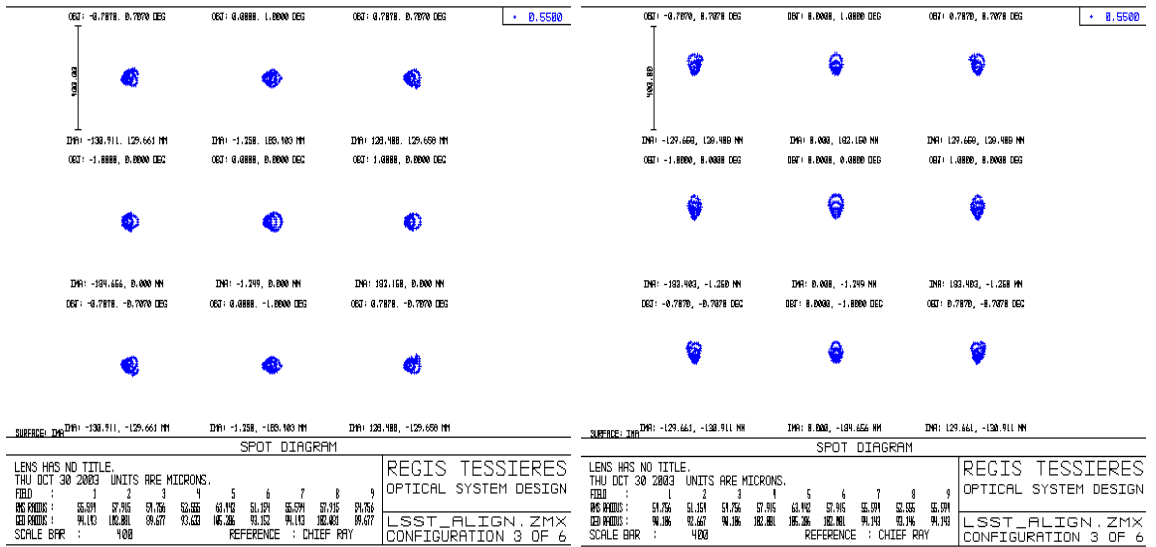
(a) Decenter along the $\vec{x}$ direction

(b) Decenter along the $\vec{y}$ direction

FIGURE 5.5. Spot diagrams when the secondary is decentered by 1mm

2. Get the Zernike coefficients at different field positions

3. Fit the Zernike coefficients with a least-squares fit to find the reconstructor A

5.3.1 First step: Perturbation of each degree of freedom

To find the values of the reconstructor, each degree of freedom is perturbed independently. As said before only the tilts and decenters of the mirrors and the corrector are considered. For the decenters a value of 1mm is chosen and for the tilts, .1 degree will be considered.

- Secondary mirror

  Decentering the secondary by 1mm along $\vec{x}$ or $\vec{y}$ produces mostly constant coma as shown in figure 5.5. The same cubic shape of the OPD plot in Zemax® for all field points confirms this assessment.

  Now if the mirror is tilted by .1 degree about its vertex around $\vec{x}$ and $\vec{y}$, the spot diagram of figure 5.6 will be observed. Again constant coma along $\vec{x}$ and $\vec{y}$ is

(a) Tilt around $\vec{x}$

(b) Tilt around $\vec{y}$
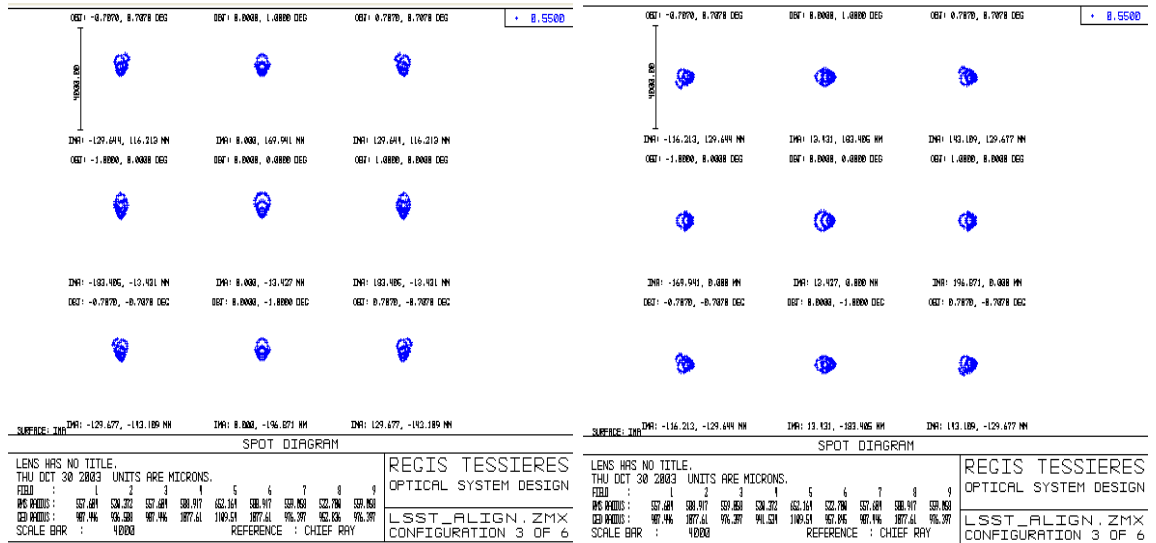
FIGURE 5.6. Spot diagrams when the secondary is tilted by .1 degree

observed but here there is a factor 10 of difference in the spot radius compared to the decenters and the patterns are rotated by 90°.

- Tertiary mirror

  The same kind of spot diagrams for the decenters and the tilts are observed for the tertiary (compared to the secondary). It is the same conclusion, coma is the main contribution to the aberrated image.

- Corrector

  For the corrector the effects of the perturbations are different. The decenters will produce mainly a combination of astigmatism and coma as shown on the spot diagram of figure 5.7. And the tilts will produce mainly field curvature and astigmatism as shown on the spot diagram of figure 5.8.

The observations made here were really general. The effects of the higher order were hidden by the large amount of the third order aberrations. And their contributions will be underlined in the third step by fitting the Zernike coefficients with the
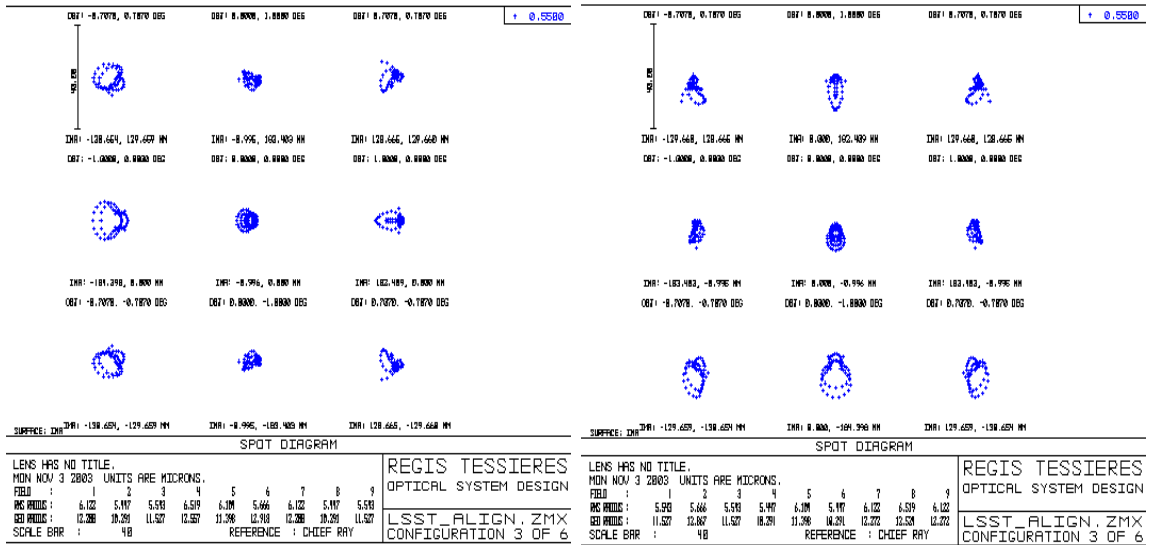
(a) Decenter along the $\vec{x}$ direction

(b) Decenter along the $\vec{y}$ direction

FIGURE 5.7. Spot diagrams when the corrector is decentered by 1mm
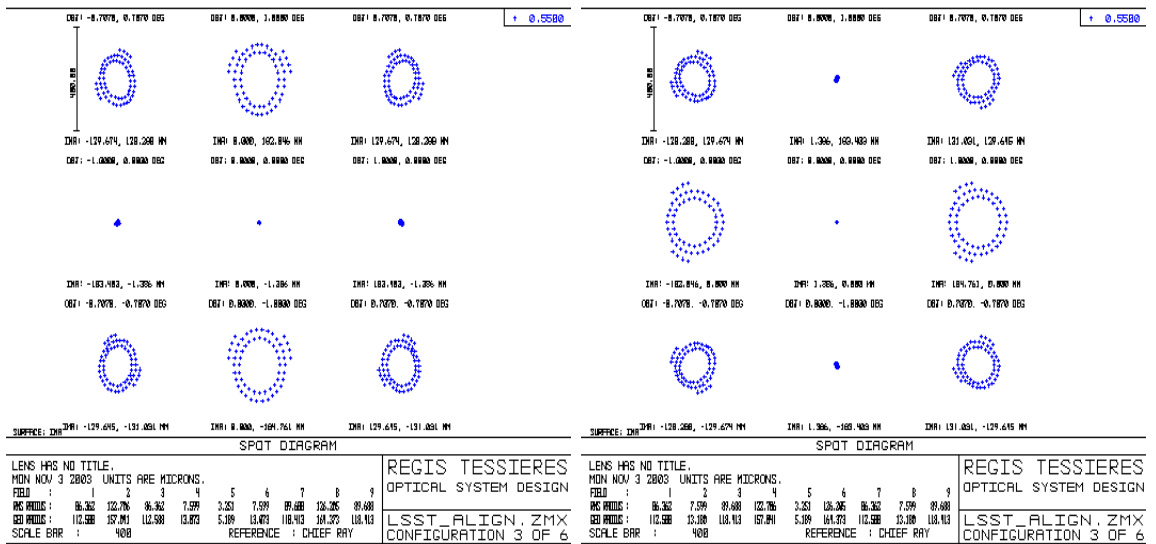
(a) Tilt around $\vec{x}$

(b) Tilt around $\vec{y}$

FIGURE 5.8. Spot diagrams when the corrector is tilted by .1 degree
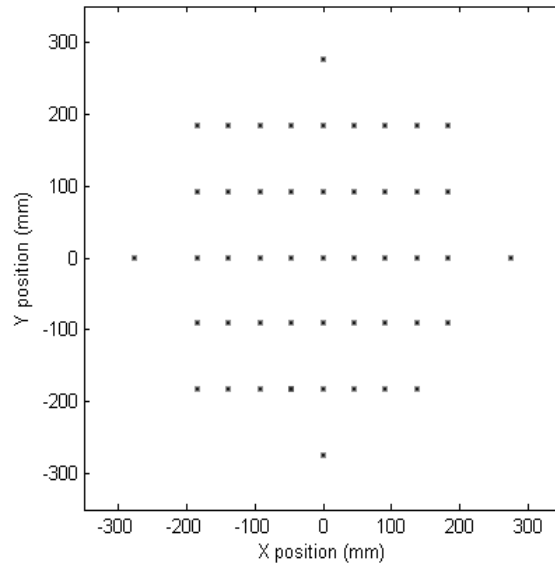
FIGURE 5.9. Positions of the field points in the image plane for the LSST

equations found in chapter 3.

5.3.2 Second step: Calculations of the Zernike coefficients for different field positions

The focal length of the telescope is 10496.4mm and its field of view is equal to $\pm 1.5°$. Thus, the equivalent size of the image plane is defined by a circle with a radius of 274.9mm.

Therefore, to account for a different variety of rays contained in this field of view, the grid of field points represented in figure 5.9 is considered. The Zernike coefficients are calculated at these field positions[2] so as to be fitted in the third step with their field dependencies.

---

[2] 48 field points are considered here

5.3.3 Third step: Least-squares fit to get the reconstructor

To find the coefficients $a_{i,j}$ defined in equations 5.3 of the reconstructor, each degree of freedom is perturbed independently so as to find one column at a time by doing a least-squares fit with the Zernike coefficients calculated in the previous step for the different field positions. To calculate the coefficients, a Matlab® program, which can be found in Appendix H, was used.

Each element is tilted around $\vec{x}$ and $\vec{y}$ by .1 degree and decentered along $\vec{x}$ and $\vec{y}$ by 1mm.

- Secondary mirror perturbed

  When the secondary mirror is tilted around $\vec{x}$ the following coefficients can be calculated:

$$
\begin{aligned}
a_{1,1} &= 416.4 \cdot x \\
a_{2,1} &= 2069 - 416.4 \cdot y \\
a_{3,1} &= -177507 + 0.165 \cdot y - 0.0119 \cdot (x^2 + y^2) + 0.466 \cdot y \\
a_{4,1} &= -0.165 \cdot x + 0.466 \cdot x \\
a_{5,1} &= -0.181 \cdot y + 0.0054 \cdot (y^2 - x^2) \\
a_{6,1} &= -0.181 \cdot x + 0.0108 \cdot xy \\
a_{7,1} &= 31.3 + 2.1 \cdot y \\
a_{8,1} &= -2.1 \cdot x \\
a_{9,1} &= 0 \\
a_{10,1} &= 2364.25
\end{aligned}
\qquad (5.4)
$$

The second column is given by tilting the secondary about $\vec{y}$:

$$a_{1,2} = -416.4 \cdot y$$

$$a_{2,2} = -2069 - 416.4 \cdot x$$

$$a_{3,2} = -0.165 \cdot y + 0.466 \cdot y$$

$$a_{4,2} = 177507 + 0.165 \cdot x + 0.0119 \cdot (x^2 + y^2) + 0.466 \cdot x$$

$$a_{5,2} = 0.181 \cdot y + 0.0108 \cdot xy$$

$$a_{6,2} = 0.181 \cdot x - 0.0054 \cdot (y^2 - x^2)$$

$$a_{7,2} = -31.3 + 2.1 \cdot x$$

$$a_{8,2} = 2.1 \cdot y$$

$$a_{9,2} = -2364.25$$

$$a_{10,2} = 0$$

(5.5)

Then the third and fourth columns of A represented by equations 5.6 and 5.7 are calculated by decentering the secondary mirror along $\vec{x}$ and $\vec{y}$ respectively.

$$a_{1,3} = 3.793 \cdot y$$

$$a_{2,3} = 3.793 \cdot x$$

$$a_{3,3} = 0$$

$$a_{4,3} = -1755.24$$

$$a_{5,3} = 0$$

$$a_{6,3} = 0$$

$$a_{7,3} = 0.054 \cdot x$$

$$a_{8,3} = 0.054 \cdot y$$

$$a_{9,3} = 60.64$$

$$a_{10,3} = 0$$

(5.6)

$$a_{1,4} = 3.793 \cdot x$$

$$a_{2,4} = -3.793 \cdot y$$

$$a_{3,4} = -1755.24$$

$$a_{4,4} = 0$$

$$a_{5,4} = 0$$

$$a_{6,4} = 0 \tag{5.7}$$

$$a_{7,4} = -0.054 \cdot y$$

$$a_{8,4} = 0.054 \cdot x$$

$$a_{9,4} = 0$$

$$a_{10,4} = 60.64$$

These coefficients correspond to the field dependencies of the Zernike coefficients when the secondary is perturbed. Some examples of the graphical representation of the fitted coefficients can be found at the end of this section.

- Tertiary mirror perturbed

  As it was done for the secondary mirror, the fifth and the sixth columns are calculated by tilting the tertiary mirror around $\vec{x}$ and $\vec{y}$ respectively and the seventh and eighth columns are found by decentering the tertiary along $\vec{x}$ and $\vec{y}$ respectively. Their values are given in equations 5.8 through 5.11.

$$a_{1,5} = -130.6 \cdot x - 0.000083 \cdot (x^3 + xy^2)$$

$$a_{2,5} = -5164 + 130.6 \cdot y + 0.0036 \cdot (x^2 + y^2) + 0.000083 \cdot (x^2y + y^3)$$

$$a_{3,5} = -186150 - 0.454 \cdot y + 0.0277 \cdot (x^2 + y^2) - 1.12 \cdot y + 0.0464 \cdot y^2$$

$$a_{4,5} = 0.454 \cdot x - 1.12 \cdot x + 0.0464 \cdot xy$$

$$a_{5,5} = 0.545 \cdot y - 0.0065 \cdot (y^2 - x^2)$$

$$a_{6,5} = 0.545 \cdot x - 0.013 \cdot xy$$

$$a_{7,5} = -229 + 0.62 \cdot y$$

$$a_{8,5} = -0.62 \cdot x$$

$$a_{9,5} = 0$$

$$a_{10,5} = -4315$$

$$(5.8)$$

$$a_{1,6} = 130.6 \cdot y + 0.000083 \cdot (y^3 + x^2y)$$

$$a_{2,6} = 5164 + 130.6 \cdot x - 0.0036 \cdot (x^2 + y^2) + 0.000083 \cdot (xy^2 + x^3)$$

$$a_{3,6} = 0.454 \cdot y - 1.12 \cdot y - 0.0464 \cdot xy$$

$$a_{4,6} = 186150 - 0.454 \cdot x - 0.0277 \cdot (x^2 + y^2) - 1.12 \cdot x - 0.0464 \cdot x^2$$

$$a_{5,6} = -0.545 \cdot y - 0.013 \cdot xy$$

$$a_{6,6} = -0.545 \cdot x + 0.0065 \cdot (y^2 - x^2)$$

$$a_{7,6} = 229 + 0.62 \cdot x$$

$$a_{8,6} = 0.62 \cdot y$$

$$a_{9,6} = 4315$$

$$a_{10,6} = 0$$

$$(5.9)$$

$$a_{1,7} = -0.8997 \cdot y$$

$$a_{2,7} = -0.8997 \cdot x$$

$$a_{3,7} = 0.00053 \cdot xy$$

$$a_{4,7} = -1237.7 + 0.0002936 \cdot (x^2 + y^2) + 0.00053 \cdot x^2$$

$$a_{5,7} = -0.000298 \cdot xy$$

$$a_{6,7} = 0.000149 \cdot (y^2 - x^2)$$

$$a_{7,7} = 0.025 \cdot x$$

$$a_{8,7} = 0.025 \cdot y$$

$$a_{9,7} = -25.88$$

$$a_{10,7} = 0$$

$$(5.10)$$

$$a_{1,8} = -0.8997 \cdot x$$

$$a_{2,8} = 0.8997 \cdot y$$

$$a_{3,8} = -1237.7 + 0.0002936 \cdot (x^2 + y^2) + 0.00053 \cdot y^2$$

$$a_{4,8} = 0.00053 \cdot xy$$

$$a_{5,8} = -0.000149 \cdot (y^2 - x^2)$$

$$a_{6,8} = -0.000298 \cdot xy$$

$$a_{7,8} = -0.025 \cdot y$$

$$a_{8,8} = 0.025 \cdot x$$

$$a_{9,8} = 0$$

$$a_{10,8} = -25.88$$

$$(5.11)$$

The coefficients calculated in this subsection correspond to the field dependencies of the Zernike coefficients when the tertiary is perturbed.

- Corrector perturbed

  Finally, to find the values for the last four columns of A, the corrector is tilted and decentered like it was done for the mirrors. The expressions of the last columns are given in equations 5.12 through 5.15.

$$a_{1,9} = -38.35 \cdot x + 0.000084 \cdot (x^3 + xy^2)$$

$$a_{2,9} = -38 + 38.35 \cdot y - 0.000084 \cdot (x^2y + y^3)$$

$$a_{3,9} = 869 + 0.006797 \cdot (x^2 + y^2) - 0.02109 \cdot y^2$$

$$a_{4,9} = -0.02109 \cdot xy$$

$$a_{5,9} = -0.00609 \cdot (y^2 - x^2)$$

$$a_{6,9} = -0.01218 \cdot xy \qquad (5.12)$$

$$a_{7,9} = 1.27 \cdot y$$

$$a_{8,9} = -1.27 \cdot x$$

$$a_{9,9} = 0$$

$$a_{10,9} = 0$$

$$a_{1,10} = +38.35 \cdot y - 0.000084 \cdot (x^2y + y^3)$$

$$a_{2,10} = 38 + 38.35 \cdot x - 0.000084 \cdot (x^3 + xy^2)$$

$$a_{3,10} = 0.02109 \cdot xy$$

$$a_{4,10} = -869 - 0.006797 \cdot (x^2 + y^2) + 0.02109 \cdot x^2$$

$$a_{5,10} = -0.01218 \cdot xy$$

$$a_{6,10} = 0.00609 \cdot (y^2 - x^2) \qquad (5.13)$$

$$a_{7,10} = 1.27 \cdot x$$

$$a_{8,10} = 1.27 \cdot y$$

$$a_{9,10} = 0$$

$$a_{10,10} = 0$$

$$a_{1,11} = -0.705 \cdot y$$

$$a_{2,11} = -0.705 \cdot x$$

$$a_{3,11} = -0.000635 \cdot xy$$

$$a_{4,11} = 160.56 - 0.0002345 \cdot (x^2 + y^2) - 0.000635 \cdot x^2$$

$$a_{5,11} = -0.00034 \cdot xy$$

$$a_{6,11} = 0.00017 \cdot (y^2 - x^2)$$

$$a_{7,11} = -0.0824 \cdot y$$

$$a_{8,11} = 0.0824 \cdot x$$

$$a_{9,11} = 0$$

$$a_{10,11} = 0$$

$$(5.14)$$

$$a_{1,12} = -0.705 \cdot x$$

$$a_{2,12} = 0.705 \cdot y$$

$$a_{3,12} = 160.56 - 0.0002345 \cdot (x^2 + y^2) - 0.000635 \cdot y^2$$

$$a_{4,12} = -0.000635 \cdot xy$$

$$a_{5,12} = 0.00017 \cdot (y^2 - x^2)$$

$$a_{6,12} = 0.00034 \cdot xy$$

$$a_{7,12} = 0.0824 \cdot x$$

$$a_{8,12} = 0.0824 \cdot y$$

$$a_{9,12} = 0$$

$$a_{10,12} = 0$$

$$(5.15)$$

The coefficients calculated and shown here for the four last columns of the reconstructor correspond to the field dependencies of the Zernike coefficients when the corrector is perturbed.

The coefficients $a_{i,j}$ introduced earlier correspond to the field dependencies of the different aberrations. Some examples of their representations are shown in the following pages where the original aberrations of the centered system have been removed by subtraction. They have been represented with their magnitude and orientation by using the results of Appendix F as it was done for the 90" telescope. Except for the third order astigmatism and coma where the representation in polar coordinates is used instead of the cartesian representation with the sine and cosine. It is different for these two cases because there is some coupling between the fifth order and the third order. The fifth order aberrations modify the expressions of the third order. Since the important idea was to get a visual representation of the amount of each field component, the simple polar representation was used. The Matlab® program used for the graphical representation is the same that was used to have the numerical values of the Zernike coefficients and can be found in Appendix H.

Since there is no point to show the representations for all the coefficients seen previously, only some significant ones are presented here in figures 5.10 through 5.14.
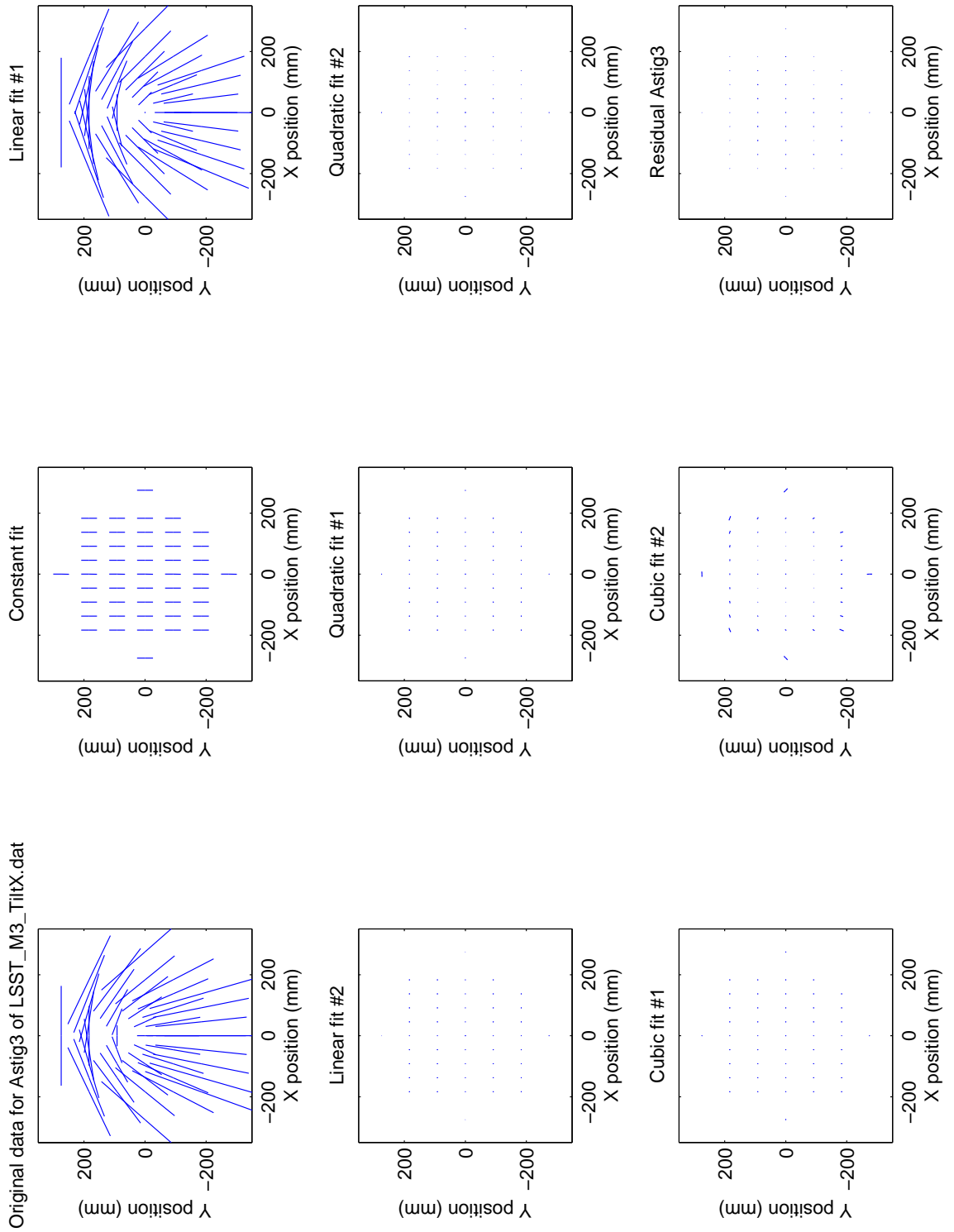
FIGURE 5.10. Representation of the third order astigmatism when M3 is tilted by 0.1 degree around $\vec{x}$
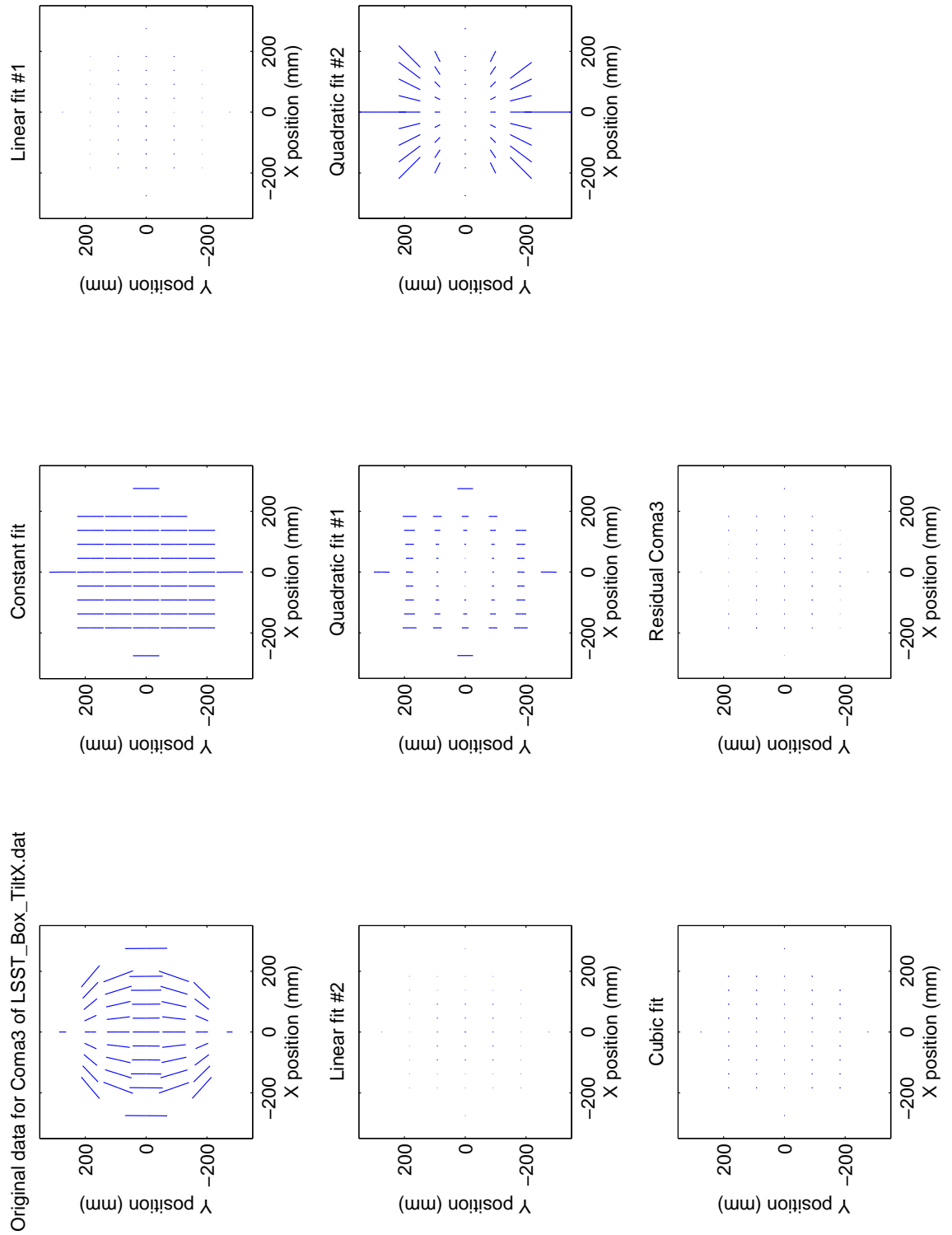
FIGURE 5.11. Representation of the third order coma when the corrector is tilted by 0.1 degree around $\vec{x}$
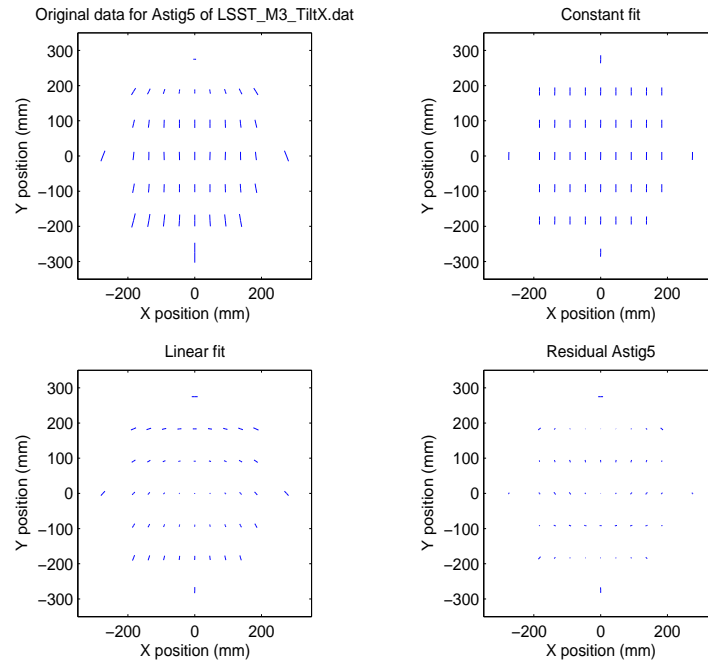
FIGURE 5.12. Representation of the fifth order astigmatism when M3 is tilted by 0.1 degree around $\vec{x}$
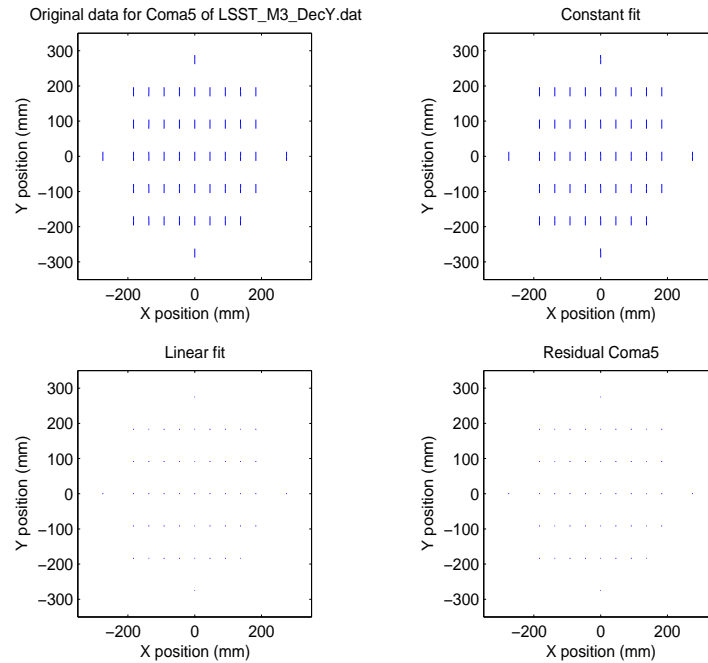
FIGURE 5.13. Representation of the fifth order coma when M3 is decentered by 1mm along $\vec{y}$
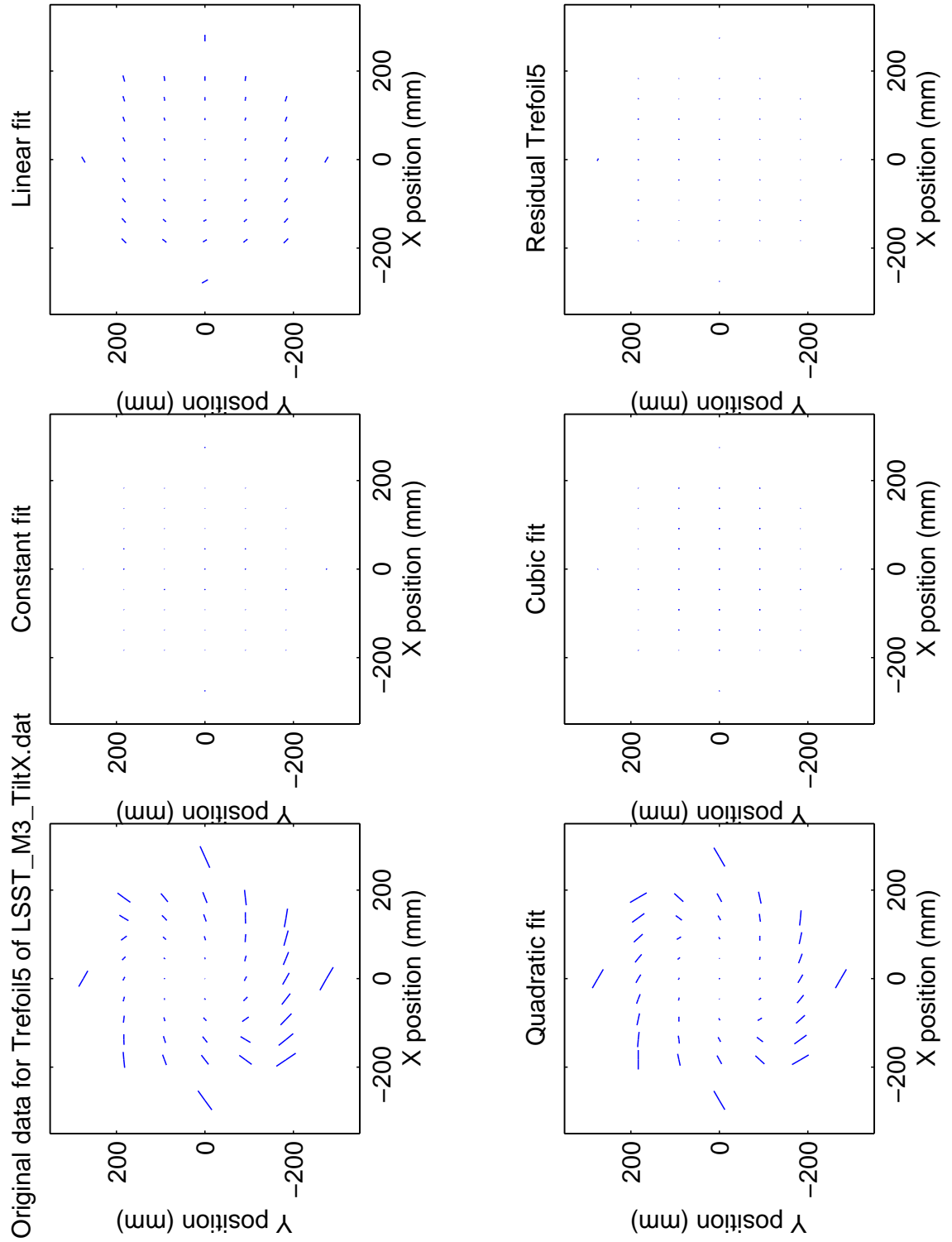
FIGURE 5.14. Representation of the trefoil when M3 is tilted by 0.1 degree around $\vec{x}$

5.4   Simulation of the real system

To test the alignment process with the reconstructor calculated in the previous section 5.3, wavefront measurements are simulated as if they were coming from the real system. To realize this task, Zemax® is used.

The design in Zemax® is randomly perturbed for the 12 degrees of freedom considered in the previous section to simulate the perturbed system. Then, the Zernike coefficients and the reconstructor are calculated at different field positions (48 points are considered) which were represented earlier in figure 5.9. After that, the perturbations can be calculated with the values of the Zernike coefficients by using a least squares fit because the expressions of the matrix A (the reconstructor) is known. Then to test if the calculated perturbations are right, the design is corrected with these predicted misalignments and the RMS spot size at the image plane is compared with its original value before the correction. The procedure can be summarized with the following four steps:

1. Randomly perturb the model

2. Calculate the Zernike coefficients for 48 field positions

3. Calculate the perturbations with a least squares fit

4. Evaluate the image quality with the "corrected" system

It was not done here but to simulate a real wavefront measurement from the system, some noise needs to be added to the values of the Zernike coefficients as it was done for the "reverse optimization" technique.

The first, second and fourth steps have been done with Zemax®'s macros and they can be found in Appendix I. The third step has been done with a Matlab® program and can be found in Appendix J.

5.4.1 Original models considered for the alignment procedure

To evaluate the performance of the perturbed models which will be considered for the simulation, a merit function, which contains the RMS spot size for different field points, is created. The field points used to create the merit function are shown in table 5.2 where Hx represents the field position in the $\vec{x}$ direction and Hy the field position in the $\vec{y}$ direction.

Point number	Hx	Hy
1	0	0
2	0	0.2
3	0	0.4
4	0	0.6
5	0	0.8
6	0	1
7	0.2	0
8	0.4	0
9	0.6	0
10	0.8	0
11	1	0

TABLE 5.2. Field positions considered for the merit function

For the simulation, 100 models are generated and their merit function values in μm are represented in the histogram of figure 5.15.

5.4.2 Results after the correction

After the perturbations have been calculated with a least squares fit by using the values of the Zernike coefficients, they are entered in the perturbed models and the merit function is calculated so as to see if the misalignments have been well predicted. The results are shown in figure 5.16 were the initial merit function is represented as a function of the solved merit function. The solved merit function is the merit function that is calculated for the perturbed model after it has been corrected with the predicted perturbations. If a model is under the solid line of figure 5.16 the
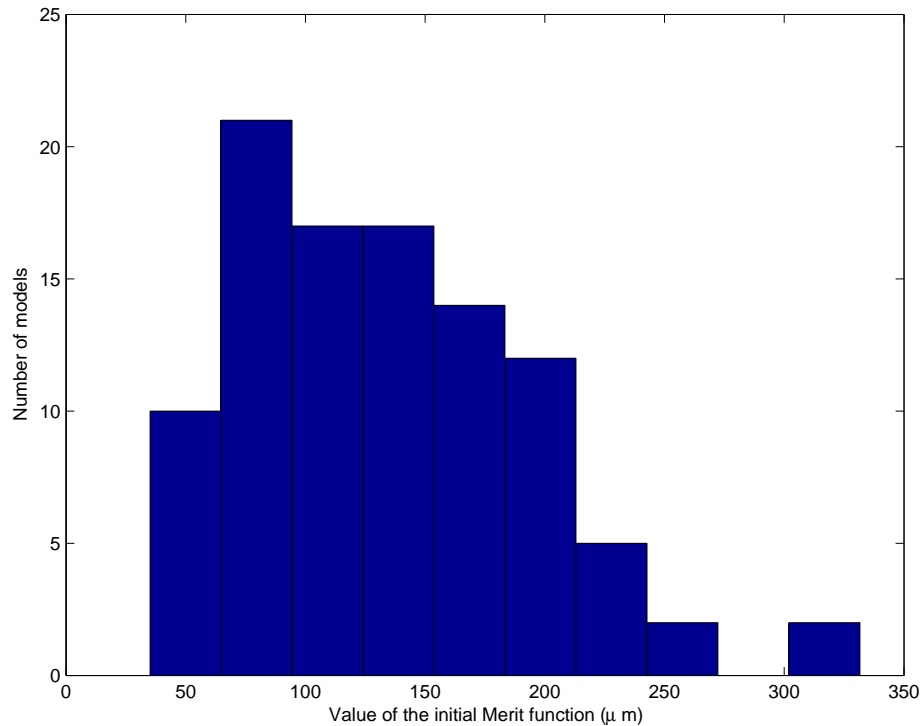
FIGURE 5.15. Histogram representation of the values of the initial merit functions for the 100 perturbed models.

system has improved and if it is above it has degraded. The dashed horizontal line corresponds to the unperturbed merit function target.

Figure 5.16 shows with one implementation of the procedure that the models converge to the target value. But they do not converge enough. Thus, a second iteration of the process was realized and represented in figure 5.17. On this last figure, it is shown that the results have improved but not enough, they still do not converge exactly to the target value. By looking at the remaining aberrations in the "corrected" models, the field curvature was dominant. Therefore the fact that the values do not converge exactly it is due to the field curvature that has not been consider in the expression of the reconstructor. Thus for the alignment process it will be necessary to consider Z_4 by using its expression in a perturbed system which was developed in Chapter 3.
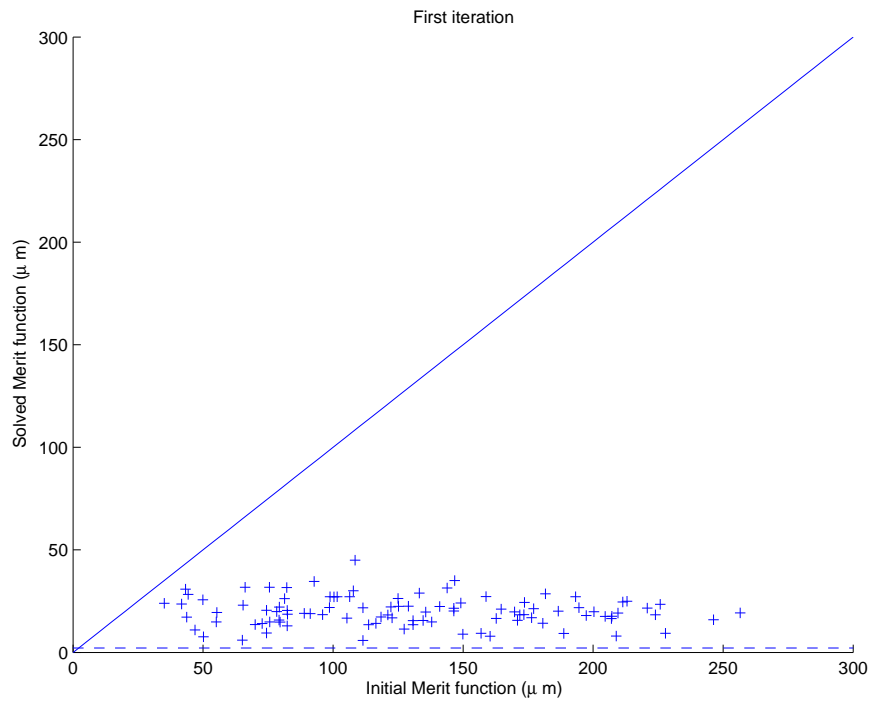
FIGURE 5.16. Results of the alignment process for the LSST for the first iteration
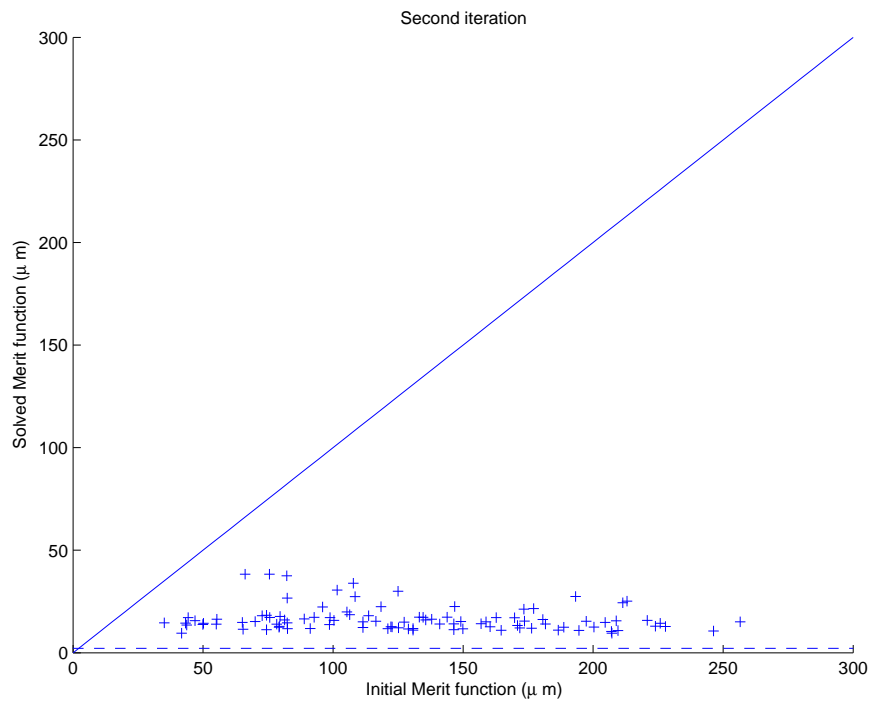
FIGURE 5.17. Results of the alignment process for the LSST for the second iteration

Chapter 6

CONCLUSION

This thesis has shown that it was possible to develop the expressions of field dependencies for the Zernike polynomials to fully describe misaligned system. And that it was possible to use them as a tool to retrieve the perturbations present in a given system.

Also, it was shown that this procedure to analyze a misaligned system gives a good insight and ease the procedure to align a system. It allows to detect immediately possible sources of error during an alignment procedure compared to a "brut force" method where only numerical calculations are considered. Moreover, as it was shown with the 90", it allows to minimize the effects of the noise and to make an alignment analysis even if a centered system contains original aberrations.

Some future work need to be done to fully describe the alignment process of the LSST by including the field curvature. Also, the errors in the spacings have not been included in the process. That should be straightforward to investigate by considering the expressions of C_4 (Zernike coefficient for the defocus). Then, it could be interesting to analyze the bending modes of the mirrors by developing a new set of equations that were not developed in this thesis. Furthermore, the effects of the different degrees of freedom can be analyzed more clearly by doing a single value decomposition of the reconstructor. Then to really test the procedure an analysis which consider the noise in the measurements should be performed. But this last part should not really affect the results since the method applied in this thesis already accounts for the noise.

To conclude, the equations developed in this thesis in terms of field dependencies have shown their convenience in an alignment process to get insight. But they can also be applied in a tolerance analysis or during the design of unobstructed systems.

Appendix A

DERIVATIONS OF THE WAVEFRONT EXPANSION AT THE MEDIAL FOCUS

The wavefront expansion at the sagittal focal surface when only field curvature and astigmatism are considered is:

$$W = W_{220}H^2\rho^2 + W_{222}H^2\rho^2\cos^2\phi \tag{A.1}$$

The medial surface is obtained from the coefficient $W_{220_M} = W_{220} + \frac{1}{2}W_{222}$. Thus to express the wavefront relative to the medial focal plane, the trigonometric relation $\cos^2\phi = \frac{1}{2} + \frac{1}{2}\cos 2\phi$ is used in equation A.1:

$$
\begin{aligned}
W &= W_{220}H^2\rho^2 + W_{222}H^2\rho^2\left(\frac{1}{2} + \frac{1}{2}\cos 2\phi\right) \\
&= W_{220_M}H^2\rho^2 + \frac{1}{2}W_{222}H^2\rho^2\cos 2\phi
\end{aligned}
\tag{A.2}
$$

In the vector notation, the wavefront is expressed as:

$$W = W_{220_M}\left(\vec{H}\cdot\vec{H}\right)\left(\vec{\rho}\cdot\vec{\rho}\right) + \frac{1}{2}W_{222}\left(\vec{H}^2\cdot\vec{\rho}^2\right) \tag{A.3}$$

Appendix B

DESIGN OF THE 90" TELESCOPE

SURFACE DATA SUMMARY:

Surf	Type	Comment	Radius	Thickness	Glass	Diameter	Conic
OBJ	STANDARD		Infinity	Infinity		0	0
1	STANDARD		Infinity	0	MIRROR	0	0
2	STANDARD		Infinity	-400		2461.42	0
3	STANDARD		Infinity	-800		2450.003	0
4	STANDARD		Infinity	-5000		2427.171	0
STO	STANDARD	PRIMARY	12159	4954.482	MIRROR	2286	-1.0646
6	STANDARD	DUMMY 1	Infinity	0		571.8458	0
7	COORDBRK		-	0		-	-
8	STANDARD	DUMMY 2	Infinity	0		571.8458	0
9	STANDARD	LENS 1	621.886	43.67	SILICA	520	0
10	STANDARD		813.09	429.9		500	0
11	STANDARD		Infinity	0		354.9686	0
12	STANDARD	LENS 2	1128.38	13.1	SILICA	320	0
13	STANDARD		312.53	441.63		310	-0.249087
14	STANDARD		Infinity	0		293.3061	0
15	STANDARD	LENS 3	478.59	30.21	SILICA	260	0
16	STANDARD		Infinity	197.1726		260	0
17	STANDARD		Infinity	-0.1316008		213.4136	0
18	STANDARD	FILTER	Infinity	8	SILICA	213.4642	0
19	STANDARD		Infinity	19		211.3761	0
20	STANDARD	LENS 4	2370.5	22.5	SILICA	252	0
21	STANDARD		Infinity	5		252	0
IMA	STANDARD		Infinity			195.2212	0

Appendix C

Code V® conventions for tilts

Positive alpha (α)
Tilt in Y-Z plane
(Rotate about X axis)
(most common)

Positive beta (β)
Tilt in X-Z plane
(Rotate about Y axis)

Positive gamma (γ)
Tilt in X-Y plane
(Rotate about Z axis)

Notes:

- Tilts are in degrees

- X, Y and Z decenters are in lens units

- Decentrations are performed first, then ADE, then BDE, then CDE, then refract/reflect

- For REV surface first refract/reflect, then tilt -CDE, then -BDE, then -ADE, then decenter with reversed signs (-XDE, -YDE, -ZDE)

FIGURE C.1. Tilt definitions for tilt(ADE), tilt(BDE) and tilt(CDE).

Appendix D

MACRO IN CODE V® FOR THE ALIGNMENT OF THE 90"

```
!Purpose of the macro:
!
!The general goal of this macro is to perturb the corrector of the 90" telescope
!for all its degrees of freedom and calculate the Zernike coefficients for
!50 different field positions when the system is perturbed.
!The values of the Zernike coefficients are then stored
!in 4 different ASCII files (extension .dat) so as to be
!used later on with Matlab. The 4 files correspond to the 2 tilts and the 2
!decenters. Note the use of the GOTO statements. It is never suitable to use them
!in a program but they were used here as a subroutine. After the execution of the
!part of the macro that goes with the GOTO, the macro comes back to the point
!where the GOTO was called.
!
! Regis Tessieres - September 24th 2003
!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!

!Definitions of the field points!
!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!

^FIELD==0 !Iniatialization of the variable ^FIELD which will be used as a counter
!The first set of 25 field points are entered (Code V can only defines 25
!points at a time)
XRI 0    0    -40  -20  0    20   40   -30  -10  10   30   -40  -20  0    20
    40   50   -30  -1010 30   -60  -50  -40  -20
YRI 60   50   40   40   40   40   40   30   30   30   30   20   20   20   20   20
    10   10   10   10   10   0    0    0    0
^FIELD==1
!The variable ^Field is increment to keep track that the first set of field
!points have been defined
GOT MAIN
!After the definition of the first points, the macro goes to the main
!part of the program

LBL FIELD_2 !After the main part of the program has been executed
            !the macro comes back to this point
^FIELD==2   !The variable ^Field is increment to keep track
            !that the second set of field points have been defined
!The last set of 25 field points are entered
XRI 0   20   40   50   60   -30  -10  10   30   -40  -20  0    20   40   -30
    -10  10   30   -40  -20  0    20   40   0    0
YRI 0   0    0    0    0    -10  -10  -10  -10  -20  -20  -20  -20  -20  -30
    -30  -30  -30  -40  -40  -40  -40  -40  -50  -60
GOT MAIN    !After the definition of the last set of points,
```

```
                !the macro goes to the main part of the program again

!Main part of the macro: the correcto is perturbed with tilts and decenters!
!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!


LBL MAIN
^STEP==0 !Initialization of the counter ^STEP
ADE s7 0;BDE s7 0;XDE s7 0;YDE s7 0
!The decenters and tilts are initialized to 0


ADE s7 .1 !Alpha tilt of .1 degree at the no coma point
^STEP==1
GOT ANALYSIS !Go to the subroutine analysis which allows
             !to calculate the Zernike coefficients  for coma
             !and astigmatism and store them into an ASCII file
LBL STEP_1   !After the Zernike coefficients have been calculated
             !and stored the macro comes back here
ADE s7 0     !Put back the alpha tilt to zero


BDE s7 .1 !Beta tilt of .1 degree
^STEP==2
GOT ANALYSIS !Go to the subroutine analysis which allows
             !to calculate the Zernike coefficients  for coma and astigmatism
             ! and store them into an ASCII file
LBL STEP_2   !After the Zernike coefficients have been calculated and
             !stored the macro comes back here
BDE s7 0     !Put back the beta tilt to zero


XDE s7 1  !Decenter of 1 mm in the x direction
^STEP==3
GOT ANALYSIS !Go to the subroutine analysis which allows
             !to calculate the Zernike coefficients  for coma
             !and astigmatism and store them into an ASCII file
LBL STEP_3   !After the Zernike coefficients have been calculated
             !and stored the macro comes back here
XDE s7 0     !Put back the decenter in the x direction to zero


YDE s7 1  !Decenter of 1mm in the y direction
^STEP==4
GOT ANALYSIS !Go to the subroutine analysis which allows
             !to calculate the Zernike coefficients  for coma and astigmatism
             !and store them into an ASCII file
LBL STEP_4   !After the Zernike coefficients have been calculated
             !and stored the macro comes back here
YDE s7 0     !Put back the decenter in the y direction to zero


!Little loop to know which field set have been defined so as to know
!of the macro needs to redo the analysis for
!the second set or of it is the end of the program
```

```
IF ^FIELD=1
    GOT FIELD_2
    ELSE
        GOT END
END IF

!Subroutine ANALYSIS!
!!!!!!!!!!!!!!!!!!!!!!

!This subroutine allows to calculate and store the zernike
!coefficients for coma and astigmatism
LBL ANALYSIS

FOR  ^field_num  1  25  1
!Calculation of the Zernike coefficients
^Z_5==ZFRCOEF(1, ^field_num, 1, 5, 121, 37, 'ENP')
^Z_6==ZFRCOEF(1, ^field_num, 1, 6, 121, 37, 'ENP')
^Z_7==ZFRCOEF(1, ^field_num, 1, 7, 121, 37, 'ENP')
^Z_8==ZFRCOEF(1, ^field_num, 1, 8, 121, 37, 'ENP')

!Loop to store the data into ASCII files
IF ^STEP=1
    !The data for the alpha tilt
    OPE APP U^unit_align AlignPFC_TiltX_Opt
    WRI U^unit_align   (XRI F^field_num)   (YRI F^field_num)  ^Z_5*550/sqrt(6)
    ^Z_6*550/sqrt(6)   ^Z_7*550/sqrt(8)      ^Z_8*550/sqrt(8)
    CLO U^unit_align

    ELSE IF ^STEP=2
        !The data for the beta tilt
        OPE APP U^unit_align AlignPFC_TiltY_Opt
        WRI U^unit_align (XRI F^field_num) (YRI F^field_num)   ^Z_5*550/sqrt(6)
        ^Z_6*550/sqrt(6)      ^Z_7*550/sqrt(8)       ^Z_8*550/sqrt(8)
        CLO U^unit_align

    ELSE IF ^STEP=3
        !The data for the decenter along the x direction
        OPE APP U^unit_align AlignPFC_DecX_Opt
        WRI U^unit_align (XRI F^field_num) (YRI F^field_num)  ^Z_5*550/sqrt(6)
        ^Z_6*550/sqrt(6)       ^Z_7*550/sqrt(8)    ^Z_8*550/sqrt(8)
        CLO U^unit_align


    ELSE IF ^STEP=4
        !The data for the decenter along the y direction
        OPE APP U^unit_align AlignPFC_DecY_Opt
        WRI U^unit_align (XRI F^field_num)   (YRI F^field_num)   ^Z_5*550/sqrt(6)
        ^Z_6*550/sqrt(6)        ^Z_7*550/sqrt(8)       ^Z_8*550/sqrt(8)
        CLO U^unit_align
```

```
    ELSE


END IF
END  FOR

!Loop to know which perturbation is studied
IF ^STEP=1
    GOT STEP_1
    ELSE IF ^STEP=2
        GOT STEP_2
    ELSE IF ^STEP=3
        GOT STEP_3
    ELSE IF ^STEP=4
        GOT STEP_4
    ELSE

END IF

LBL END !End of the program
```

Appendix E

Matlab® program for the 90" telescope alignment

```
% pfc: Program to fit the data from a ray tracing program or a wavefront
% sensor by using a least squares fit with the equations of the Zernike
% coefficients in misaligned systems.
%
%Output: Coefficients of each field dependence for coma and astigmatism and
%graphical representation of each field dependence.
%
% Regis Tessieres - September 24th 2003

format long %Double precision
clear all

%Load file selected by the user
[filename, pathname] = uigetfile( ... {'*.txt;*.dat;*.xls','ASCII
Files (*.txt;*.dat;*.xls)';
    '*.txt',  'Text files (*.txt)'; ...
    '*.dat','Data files (*.dat)'; ...
    '*.xls','Excel files (*.xls)'; ...
    '*.*',  'All Files (*.*)'}, ...
    'Select an ASCII file');

%Store the values of the file into a matrix D
D= importdata(filename,'\t');
%Get the number of points
nbpoints=size(D); nbpoints=nbpoints(1);

%read the data in the file%
%%%%%%%%%%%%%%%%%%%%%%%%%%%

%initialization of the variables
Z56=[]; Z5=[]; Z6=[]; Z78=[]; Z7=[]; Z8=[];

for i=1:nbpoints

    %Store the values of astigmatism
    z5(i)=D(i,3);
    z6(i)=D(i,4);
    %Calculation of the amplitude and the orientation for astigmatism
    amplZ56(i)=sqrt(z5(i).^2+z6(i).^2);
    phiZ56(i)=1./2.*atan2(z6(i),z5(i));
    Z5=[Z5;z5(i)];
```

```
    Z6=[Z6;z6(i)];
    Z56=[Z56;z5(i);z6(i)];

    %Store the values of coma
    z7(i)=D(i,5);
    z8(i)=D(i,6);
    %Calculation of the amplitude and the orientation for coma
    amplZ78(i)=sqrt(z7(i).^2+z8(i).^2);
    phiZ78(i)=atan2(z8(i),z7(i));
    Z7=[Z7;z7(i)];
    Z8=[Z8;z8(i)];
    Z78=[Z78;z7(i);z8(i)];

    x(i)=D(i,1);
    y(i)=D(i,2);
end
%Conversion to vector
x=x'; y=y'; amplZ56=amplZ56'; phiZ56=phiZ56'; amplZ78=amplZ78';
phiZ78=phiZ78';

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Creation of the matrix and Least-squares fit (LSF) for the zernike coefficients%

%Matrix for the LSF for Astig3 (third order astigmatism)
MAstig3=[]; %initialization of the matrix for Astig3 (z5 and z6)

for i=1:nbpoints
    M1=[0 1 x(i) y(i) 2.*x(i).*y(i)];
    M2=[1 0 y(i) -x(i) ((y(i)).^2-(x(i)).^2)];

    A=[M1;M2];
    MAstig3=[MAstig3;A];
end

%least-squares fit for z5 and z6 (Output of the program)
RAstig3=MAstig3\Z56


%Matrix for the LSF for Coma3 (third order coma)%
MComa3=[]; %initialization of the matrix for Coma3 (z7 and z8)
for i=1:nbpoints

    M1=[1 0 x(i)];
    M2=[0 1 y(i)];

    A=[M1;M2];
    MComa3=[MComa3;A];

end
```

```
%least-squares fit for z7 and z8 (Output of the program)
RComa3=MComa3\Z78

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Graphic representation of the 5 aberrations with their fit for astigmatism

%Astig3

%Constant part
B=ones(nbpoints); B=B(:,1); Z5cst=RAstig3(2).*B;
Z6cst=RAstig3(1).*B;
%Amplitude of the constant part
amplcst=((Z5cst).^2+(Z6cst).^2).^(1./2);
%Orientation of the constant part
phicst=1./2.*atan2(Z6cst,Z5cst);

%Linear part 1
Z5lin1=RAstig3(3).*x+RAstig3(4).*y;
Z6lin1=RAstig3(3).*y-RAstig3(4).*x;
%Amplitude of the linear part
ampllin1=((Z6lin1).^2+(Z5lin1).^2).^(1./2);
%Orientation of the linear part
philin1=1./2.*atan2(Z6lin1,Z5lin1);

%Quadratic part 1
Z5quad1=RAstig3(5).*2.*x.*y; Z6quad1=RAstig3(5).*(y.^2-x.^2);
%Amplitude of the quadratic part
amplquad1=((Z6quad1).^2+(Z5quad1).^2).^(1./2);
%Orientation of the quadratic part
phiquad1=1./2.*atan2(Z6quad1,Z5quad1);

%Residual part
Z5res=Z5-Z5cst-Z5lin1-Z5quad1; Z6res=Z6-Z6cst-Z6lin1-Z6quad1;
%Amplitude of the residual astigmatism
amplres=((Z6res).^2+(Z5res).^2).^(1./2);
%Orientation of the residual astigmatism
phires=1./2.*atan2(Z6res,Z5res);

%Creation of the plot with the 5 graphs
figure subplot(2,3,1)
quiv(x,y,1./10.*amplZ56.*cos(phiZ56),1./10.*amplZ56.*sin(phiZ56),0);
axis square title(['Original data for Astig3 of ',filename]);
xlabel('X position (mm)'); ylabel('Y position (mm)'); axis([-100
100 -100 100]);

subplot(2,3,2)
quiv(x,y,1./10.*amplcst.*cos(phicst),1./10.*amplcst.*sin(phicst),0);
```

```
axis square title('Constant fit'); xlabel('X position (mm)');
ylabel('Y position (mm)'); axis([-100 100 -100 100]);

subplot(2,3,3)
quiv(x,y,1./10.*ampllin1.*cos(philin1),1./10.*ampllin1.*sin(philin1),0);
axis square title('Linear fit'); xlabel('X position (mm)');
ylabel('Y position (mm)'); axis([-100 100 -100 100]);

subplot(2,3,4)
quiv(x,y,1./10.*amplquad1.*cos(phiquad1),1./10.*amplquad1.*sin(phiquad1),0);
axis square title('Quadratic fit'); xlabel('X position (mm)');
ylabel('Y position (mm)'); axis([-100 100 -100 100]);

subplot(2,3,5)
quiv(x,y,1./10.*amplres.*cos(phires),1./10.*amplres.*sin(phires),0);
axis square title('Residual Astig3'); xlabel('X position (mm)');
ylabel('Y position (mm)'); axis([-100 100 -100 100]);

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Coma3

%Constant part
Z7cst=RComa3(1).*B; Z8cst=RComa3(2).*B;

%Linear part 1
Z7lin1=RComa3(3).*x; Z8lin1=RComa3(3).*y;

%Residual part
Z7res=Z7-Z7cst-Z7lin1; Z8res=Z8-Z8cst-Z8lin1;

%Creation of the plot with the 4 graphs
figure subplot(2,2,1) quiv(x,y,1./10.*Z7,1./10.*Z8,0); axis square
title(['Original data for Coma3 of ',filename]); xlabel('X
position (mm)'); ylabel('Y position (mm)'); axis([-100 100 -100
100]);

subplot(2,2,2) quiv(x,y,1./10.*Z7cst,1./10.*Z8cst,0); axis square
title('Constant fit'); xlabel('X position (mm)'); ylabel('Y
position (mm)'); axis([-100 100 -100 100]);

subplot(2,2,3) quiv(x,y,1./10.*Z7lin1,1./10.*Z8lin1,0); axis
square title('Linear fit'); xlabel('X position (mm)'); ylabel('Y
position (mm)'); axis([-100 100 -100 100]);

subplot(2,2,4) quiv(x,y,1./10.*Z7res,1./10.*Z8res,0); axis square
title('Residual Coma3'); xlabel('X position (mm)'); ylabel('Y
position (mm)'); axis([-100 100 -100 100]);
```

Appendix F

CALCULATIONS OF THE AMPLITUDE AND THE ORIENTATION WITH THE ZERNIKE COEFFICIENTS

F.1   Third order aberrations

Since only third order coma and astigmatism are analyzed for the alignment of the 90" telescope, the calculations to find the amplitude and the orientation will be done only for these two aberrations.

F.1.1   Third order astigmatism

The expression of the wavefront error in terms of Zernike considering only third order astigmatism is:

$$
\begin{aligned}
\Delta W &= C_5 \cdot \sqrt{6}\rho^2 \sin 2\phi + C_6 \cdot \sqrt{6}\rho^2 \cos 2\phi \\
&= \sqrt{6}\rho^2 \cdot \left(C_5 \sin 2\phi + C_6 \cos 2\phi\right)
\end{aligned}
\tag{F.1}
$$

By using the relation $a \cos x + b \sin x = \sqrt{a^2 + b^2} \cdot \cos\left(x - \tan^{-1}(\frac{b}{a})\right)$[1] equation F.1 can be rewritten as:

$$
\Delta W = \rho^2 \sqrt{6}\sqrt{(C_5)^2 + (C_6)^2} \cdot \cos\left(2\left(\phi - \frac{1}{2}\tan^{-1}\frac{C_5}{C_6}\right)\right)
\tag{F.2}
$$

Then by using the trigonometric relation $\cos 2\phi = 2\cos^2 \phi - 1$, equation F.2 becomes:

$$
\Delta W = \rho^2 \sqrt{6}\sqrt{(C_5)^2 + (C_6)^2} \cdot 2\cos^2\left(\phi - \frac{1}{2}\tan^{-1}\frac{C_5}{C_6}\right) - \rho^2 \sqrt{6}\sqrt{(C_5)^2 + (C_6)^2}
\tag{F.3}
$$

The second term in equation F.3 is for defocus, thus only the first is considered for astigmatism. From equation F.3 the magnitude and the orientation for third order astigmatism can be derived:

$$
\begin{cases}
M = 2\sqrt{6} \cdot \sqrt{(C_5)^2 + (C_6)^2} \\
A = \frac{1}{2}\tan^{-1}\frac{C_5}{C_6}
\end{cases}
\tag{F.4}
$$

---

[1]This equation will also be used in the expressions of the next aberrations for simplification

F.1.2 Third order coma

The expression of the wavefront error in terms of Zernike considering only third order coma is:

$$
\begin{aligned}
\Delta W &= C_7 \cdot \sqrt{8}(3\rho^3 - 2\rho)\sin\phi + C_8 \cdot \sqrt{8}(3\rho^3 - 2\rho)\cos\phi \\
&= \sqrt{8}(3\rho^3 - 2\rho)(C_7\sin\phi + C_8\cos\phi) \\
&= \sqrt{8}(3\rho^3 - 2\rho)\sqrt{(C_7)^2 + (C_8)^2}\cos\left(\phi - \tan^{-1}\frac{C_7}{C_8}\right) \\
&= \sqrt{8} \cdot 3\rho^3\sqrt{(C_7)^2 + (C_8)^2}\cos\left(\phi - \tan^{-1}\frac{C_7}{C_8}\right) \\
&\quad - \sqrt{8} \cdot 2\rho\sqrt{(C_7)^2 + (C_8)^2}\cos\left(\phi - \tan^{-1}\frac{C_7}{C_8}\right)
\end{aligned}
\tag{F.5}
$$

The last term of equation F.5 is tilt. Thus only the first term is considered to get the magnitude and the orientation of third order coma:

$$
\begin{cases}
M = 3\sqrt{8} \cdot \sqrt{(C_7)^2 + (C_8)^2} \\
A = \tan^{-1}\frac{C_7}{C_8}
\end{cases}
\tag{F.6}
$$

F.2 Fifth order aberrations

When the fifth order is considered, it is more complicated in the sense that fifth order aberrations will also create third order aberrations as it will be seen in the calculations.

Here only third order astigmatism and coma, trefoil and fifth order astigmatism and coma will be considered since they are the only aberrations considered for the alignment of the LSST.

F.2.1 Third order astigmatism

By considering the contributions of third order and fifth order astigmatism, the wavefront error can be written as:

$$
\begin{aligned}
\Delta W &= C_5 \cdot \sqrt{6}\rho^2 \sin 2\phi + C_6 \cdot \sqrt{6}\rho^2 \cos 2\phi - 3\sqrt{10} \cdot C_{12}\rho^2 \cos 2\phi - 3\sqrt{10} \cdot C_{13}\rho^2 \sin 2\phi \\
&= \rho^2 \left[\left(\sqrt{6}\,C_5 - 3\sqrt{10}\,C_{13} \right) \sin 2\phi + \left(\sqrt{6}\,C_6 - 3\sqrt{10}\,C_{12} \right) \cos 2\phi \right] \\
&= \rho^2 \sqrt{ \left(\sqrt{6}\,C_5 - 3\sqrt{10}\,C_{13} \right)^2 + \left(\sqrt{6}\,C_6 - 3\sqrt{10}\,C_{12} \right)^2 } \\
&\quad \cdot \cos \left(2\phi - \tan^{-1} \frac{\sqrt{6}\,C_5 - 3\sqrt{10}\,C_{13}}{\sqrt{6}\,C_6 - 3\sqrt{10}\,C_{12}} \right)
\end{aligned}
\tag{F.7}
$$

By considering only the components for third order astigmatism:

$$
\begin{aligned}
\Delta W =& 2\rho^2 \sqrt{ \left(\sqrt{6}\,C_5 - 3\sqrt{10}\,C_{13} \right)^2 + \left(\sqrt{6}\,C_6 - 3\sqrt{10}\,C_{12} \right)^2 } \\
& \cdot \cos^2 \left(\phi - \frac{1}{2}\tan^{-1} \frac{\sqrt{6}\,C_5 - 3\sqrt{10}\,C_{13}}{\sqrt{6}\,C_6 - 3\sqrt{10}\,C_{12}} \right)
\end{aligned}
\tag{F.8}
$$

Thus, the magnitude and the orientation of third order astigmatism are:

$$
\begin{cases}
M = 2\sqrt{ \left(\sqrt{6}\,C_5 - 3\sqrt{10}\,C_{13} \right)^2 + \left(\sqrt{6}\,C_6 - 3\sqrt{10}\,C_{12} \right)^2 } \\
A = \frac{1}{2}\tan^{-1} \frac{\sqrt{6}\,C_5 - 3\sqrt{10}\,C_{13}}{\sqrt{6}\,C_6 - 3\sqrt{10}\,C_{12}}
\end{cases}
\tag{F.9}
$$

F.2.2 Third order coma

By considering the contributions of third order and fifth order coma, the wavefront error can be written as:

$$
\begin{aligned}
\Delta W = & \, C_7\,\sqrt{8}(3\rho^3 - 2\rho)\sin\phi + C_8\,\sqrt{8}(3\rho^3 - 2\rho)\cos\phi \\
& - 12\sqrt{12}\,C_{16}\rho^3 \cos\phi - 12\sqrt{12}\,C_{17}\rho^3 \sin\phi
\end{aligned}
\tag{F.10}
$$

By considering only the components for third order coma, the wavefront error can be rewritten as:

$$
\begin{aligned}
\Delta W &= \rho^3 \left[\left(3\sqrt{8}\,C_7 - 12\sqrt{12}\,C_{17} \right) \sin\phi + \left(3\sqrt{8}\,C_8 - 12\sqrt{12}\,C_{16} \right) \cos\phi \right] \\
&= \rho^3 \sqrt{ \left(3\sqrt{8}\,C_7 - 12\sqrt{12}\,C_{17} \right)^2 + \left(3\sqrt{8}\,C_8 - 12\sqrt{12}\,C_{16} \right)^2 } \\
&\quad \cdot \cos \left(\phi - \tan^{-1} \frac{3\sqrt{8}\,C_7 - 12\sqrt{12}\,C_{17}}{3\sqrt{8}\,C_8 - 12\sqrt{12}\,C_{16}} \right)
\end{aligned}
\tag{F.11}
$$

Thus the magnitude and the orientation of third order coma are:

$$\begin{cases} M = \sqrt{\left(3\sqrt{8}\,C_7 - 12\sqrt{12}\,C_{17}\right)^2 + \left(3\sqrt{8}\,C_8 - 12\sqrt{12}\,C_{16}\right)^2} \\ A = \tan^{-1}\frac{3\sqrt{8}\,C_7 - 12\sqrt{12}\,C_{17}}{3\sqrt{8}\,C_8 - 12\sqrt{12}\,C_{16}} \end{cases} \tag{F.12}$$

F.2.3 Trefoil

The expression of the wavefront error in terms of Zernike considering only trefoil is:

$$\begin{aligned} \Delta W &= \rho^3\,\sqrt{8}\big(C_9 \sin 3\phi + C_{10} \cos 3\phi\big) \\ &= \rho^3\,\sqrt{8} \cdot \left(\sqrt{(C_9)^2 + (C_{10})^2} \cdot \cos\left[3\left(\phi - \frac{1}{3}\tan^{-1}\frac{C_9}{C_{10}}\right)\right] \right) \end{aligned} \tag{F.13}$$

By using the trigonometric relation $\cos 3x = 4\cos^3 x - 3\cos x$ and considering only the components for trefoil, the wavefront error can be expressed as:

$$\Delta W = \rho^3\,8\sqrt{2}\,\sqrt{(C_9)^2 + (C_{10})^2} \cdot \cos^3\left(\phi - \frac{1}{3}\tan^{-1}\frac{C_9}{C_{10}}\right) \tag{F.14}$$

Thus the magnitude and the orientation of trefoil are:

$$\begin{cases} M = 8\sqrt{2}\,\sqrt{(C_9)^2 + (C_{10})^2} \\ A = \frac{1}{3}\tan^{-1}\frac{C_9}{C_{10}} \end{cases} \tag{F.15}$$

F.2.4 Fifth order astigmatism

The expression of the wavefront error in terms of Zernike considering only fifth order astigmatism is:

$$\begin{aligned} \Delta W &= 4\sqrt{10}\,\rho^4\,\cos 2\phi\,C_{12} + 4\sqrt{10}\,\rho^4\,\sin 2\phi\,C_{13} \\ &= 4\sqrt{10}\rho^4 \cdot \big(C_{12}\cos 2\phi + C_{13}\sin 2\phi\big) \\ &= 4\sqrt{10}\rho^4\,\sqrt{(C_{12})^2 + (C_{13})^2} \cdot \cos\left[2\left(\phi - \frac{1}{2}\tan^{-1}\frac{C_{13}}{C_{12}}\right)\right] \end{aligned} \tag{F.16}$$

By using the trigonometric relation $\cos 2\phi = 2\cos^2\phi - 1$ and considering only fifth order astigmatism, the wavefront error can be expressed as:

$$\Delta W = 8\sqrt{10}\rho^4\,\sqrt{(C_{12})^2 + (C_{13})^2} \cdot \cos^2\left(\phi - \frac{1}{2}\tan^{-1}\frac{C_{13}}{C_{12}}\right) \tag{F.17}$$

Thus the magnitude and the orientation of fifth order astigmatism are:

$$\begin{cases} M = 8\sqrt{10}\sqrt{(C_{12})^2 + (C_{13})^2} \\ A = \frac{1}{2}\tan^{-1}\frac{C_{13}}{C_{12}} \end{cases} \tag{F.18}$$

F.2.5 Fifth order coma

The expression of the wavefront error in terms of Zernike considering only fifth order coma is:

$$\begin{aligned} \Delta W &= 10\sqrt{12}\rho^5 \cdot (C_{16}\cos\phi + C_{17}\sin\phi) \\ &= 10\sqrt{12}\rho^5 \sqrt{(C_{16})^2 + (C_{17})^2} \cdot \cos\left(\phi - \tan^{-1}\frac{C_{17}}{C_{16}}\right) \end{aligned} \tag{F.19}$$

According to equation F.19, the magnitude and orientation of fifth order coma are:

$$\begin{cases} M = 10\sqrt{12}\sqrt{(C_{16})^2 + (C_{17})^2} \\ A = \tan^{-1}\frac{C_{17}}{C_{16}} \end{cases} \tag{F.20}$$

Appendix G

Macro in Zemax® for the LSST

```
!This macro defines 48 points, calculate the Zernike coefficients for these points
!and store the data
!into an ASCII file defined by the user.
!
! Regis Tessieres - September 24th 2003

path$ = "c:\LSST\analysis\"
!Define the path where the file is going to be stored

REWIND
INPUT "Enter the name of the output file:" , filename$
!Pop up window to ask for the file name
outfile$ = path$ + filename$ + ".dat"
PRINT outfile$
pi=4*ATAN(1)
!Definition of pi to use it later on


!First set of field points (Only 12 field points can be defined at a time)
NUMFIELD 12
!fix the number of field and define each field

FTYP=0
FLDX 1 = 0
FLDY 1 = 1.5
FWGT 1 = 1
FLDX 2 = -1
FLDY 2 = 1
FWGT 2 = 1
FLDX 3 = -0.75
FLDY 3 = 1
FWGT 3 = 1
FLDX 4 = -0.5
FLDY 4 = 1
FWGT 4 = 1
FLDX 5 = -0.25
FLDY 5 = 1
FWGT 5 = 1
FLDX 6 = 0
FLDY 6 = 1
FWGT 6 = 1
FLDX 7 = 0.25
FLDY 7 = 1
```

```
FWGT 7 = 1
FLDX 8 = 0.5
FLDY 8 = 1
FWGT 8 = 1
FLDX 9 = 0.75
FLDY 9 = 1
FWGT 9 = 1
FLDX 10 = 1
FLDY 10 = 1
FWGT 10 = 1
FLDX 11 = -1
FLDY 11 = 0.5
FWGT 11 = 1
FLDX 12 = -0.75
FLDY 12 = 0.5
FWGT 12 = 1


!Optimize the system to have the best focus
OPTIMIZE 5
!Update the windows
UPDATE ALL
GOSUB ZERN
!Go to the subroutine Zern to calulate and store the zernike coefficients

FTYP=0
FLDX 1 = -0.5
FLDY 1 = 0.5
FWGT 1 = 1
FLDX 2 = -0.25
FLDY 2 = 0.5
FWGT 2 = 1
FLDX 3 = 0
FLDY 3 = 0.5
FWGT 3 = 1
FLDX 4 = 0.25
FLDY 4 = 0.5
FWGT 4 = 1
FLDX 5 = 0.5
FLDY 5 = 0.5
FWGT 5 = 1
FLDX 6 = 0.75
FLDY 6 = 0.5
FWGT 6 = 1
FLDX 7 = 1
FLDY 7 = 0.5
FWGT 7 = 1
FLDX 8 = -1.5
FLDY 8 = 0
FWGT 8 = 1
```

```
FLDX 9 = -1
FLDY 9 = 0
FWGT 9 = 1
FLDX 10 = -0.75
FLDY 10 = 0
FWGT 10 = 1
FLDX 11 = -0.5
FLDY 11 = 0
FWGT 11 = 1
FLDX 12 = -0.25
FLDY 12 = 0
FWGT 12 = 1


!Update the windows
UPDATE ALL
GOSUB ZERN
!Go to the subroutine Zern to calulate and store the zernike coefficients

FTYP=0
FLDX 1 = 0
FLDY 1 = 0
FWGT 1 = 1
FLDX 2 = 0.25
FLDY 2 = 0
FWGT 2 = 1
FLDX 3 = 0.5
FLDY 3 = 0
FWGT 3 = 1
FLDX 4 = 0.75
FLDY 4 = 0
FWGT 4 = 1
FLDX 5 = 1
FLDY 5 = 0
FWGT 5 = 1
FLDX 6 = 0
FLDY 6 = -1.5
FWGT 6 = 1
FLDX 7 = 1.5
FLDY 7 = 0
FWGT 7 = 1
FLDX 8 = -1
FLDY 8 = -0.5
FWGT 8 = 1
FLDX 9 = -0.75
FLDY 9 = -0.5
FWGT 9 = 1
FLDX 10 = -0.5
FLDY 10 = -0.5
FWGT 10 = 1
```

```
FLDX 11 = -0.25
FLDY 11 = -0.5
FWGT 11 = 1
FLDX 12 = 0
FLDY 12 = -0.5
FWGT 12 = 1

!Update the windows
UPDATE ALL
GOSUB ZERN
!Go to the subroutine Zern to calulate and store the zernike coefficients

FTYP=0
FLDX 1 = 0.25
FLDY 1 = -0.5
FWGT 1 = 1
FLDX 2 = 0.5
FLDY 2 = -0.5
FWGT 2 = 1
FLDX 3 = 0.75
FLDY 3 = -0.5
FWGT 3 = 1
FLDX 4 = 1
FLDY 4 = -0.5
FWGT 4 = 1
FLDX 5 = -1
FLDY 5 = -1
FWGT 5 = 1
FLDX 6 = -0.75
FLDY 6 = -1
FWGT 6 = 1
FLDX 7 = -0.5
FLDY 7 = -1
FWGT 7 = 1
FLDX 8 = -0.25
FLDY 8 = -1
FWGT 8 = 1
FLDX 9 = 0
FLDY 9 = -1
FWGT 9 = 1
FLDX 10 = 0.25
FLDY 10 = -1
FWGT 10 = 1
FLDX 11 = 0.5
FLDY 11 = -1
FWGT 11 = 1
FLDX 12 = 0.75
FLDY 12 = -1
FWGT 12 = 1
```

```
!Update the windows
UPDATE ALL
GOSUB ZERN
!Go to the subroutine Zern to calulate and store the zernike coefficients

END

!Subroutine to get the Zernike polynomial and store them into a file

SUB ZERN

OUTPUT outfile$ APPEND

FOR j=1,12,1
    GETZERNIKE 17,1,j,3,1,0
    PRINT TANG(FLDX(j)*pi/180)*10496.5,"  ",TANG(FLDY(j)*pi/180)*10496.5,
    "  ",VEC1(12)*550/sqrt(3)
NEXT

OUTPUT screen
RETURN
```

Appendix H

Matlab® program for the LSST alignment

```
% lsstC: Program to fit the data from a raytracing program or a wavefront
% sensor by using a least squares fit with the equations of the Zernike
% coefficients in misaligned systems.
%
%Output: Coefficients of each field dependence for each aberration and
%graphical representation of each field dependence.
%
% Regis Tessieres - September 24th 2003

format long
clear all

%Load file selected by the user
[filename, pathname] = uigetfile( ...
{'*.txt;*.dat;*.xls','ASCII Files (*.txt;*.dat;*.xls)';
    '*.txt',  'Text files (*.txt)'; ...
    '*.dat','Data files (*.dat)'; ...
    '*.xls','Excel files (*.xls)'; ...
    '*.*',  'All Files (*.*)'}, ...
    'Select an ASCII file');

%Store the values of the file into a matrix D when the system is perturbed
D= importdata(filename,'\t');
nbpoints=size(D);
nbpoints=nbpoints(1);

%Load file of the centered system
[filename0, pathname0] = uigetfile( ...
{'*.txt;*.dat;*.xls','ASCII Files (*.txt;*.dat;*.xls)';
    '*.txt',  'Text files (*.txt)'; ...
    '*.dat','Data files (*.dat)'; ...
    '*.xls','Excel files (*.xls)'; ...
    '*.*',  'All Files (*.*)'}, ...
    'Select the file for centered sys');
%The values of the Zernike
%coefficients of the centered system willl be subtracted from the Zernike
%coefficients when the system is perturbed.
D0= importdata(filename0,'\t'); %Store the values of the centered system.

%read the data in the file%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%initialization
```

```
Z56=[];
Z5=[];
Z6=[];
Z78=[];
Z7=[];
Z8=[];
Z910=[];
Z9=[];
Z10=[];
Z1213=[];
Z12=[];
Z13=[];
Z1617=[];
Z16=[];
Z17=[];

for i=1:nbpoints

    %Store the values for third order astigmatism and calculate its
    %amplitude and orientation
    z5(i)=D(i,3)-D0(i,3);
    z6(i)=D(i,4)-D0(i,4);
    amplZ56(i)=sqrt(z5(i).^2+z6(i).^2);
    phiZ56(i)=1./2.*atan2(z6(i),z5(i));
    Z5=[Z5;z5(i)];
    Z6=[Z6;z6(i)];
    Z56=[Z56;z5(i);z6(i)];

    %Store the values for third order coma and calculate its
    %amplitude and orientation
    z7(i)=D(i,5)-D0(i,5);
    z8(i)=D(i,6)-D0(i,6);
    amplZ78(i)=sqrt(z7(i).^2+z8(i).^2);
    phiZ78(i)=atan2(z8(i),z7(i));
    Z7=[Z7;z7(i)];
    Z8=[Z8;z8(i)];
    Z78=[Z78;z7(i);z8(i)];

    %Store the values for trefoil and calculate its
    %amplitude and orientation
    z9(i)=D(i,7)-D0(i,7);
    z10(i)=D(i,8)-D0(i,8);
    Z9=[Z9;z9(i)];
    Z10=[Z10;z10(i)];
    amplZ910(i)=sqrt(z9(i).^2+z10(i).^2);
    phiZ910(i)=1./3.*atan2(z10(i),z9(i));
    Z910=[Z910;z9(i);z10(i)];

    %Store the values for fifth order astigmatism and calculate its
```

```
    %amplitude and orientation
    z12(i)=D(i,10)-D0(i,10);
    z13(i)=D(i,9)-D0(i,9);
    Z12=[Z12;z12(i)];
    Z13=[Z13;z13(i)];
    amplZ1213(i)=sqrt(z12(i).^2+z13(i).^2);
    phiZ1213(i)=1./2.*atan2(z12(i),z13(i));
    Z1213=[Z1213;z12(i);z13(i)];

    %Store the values for fifth order coma and calculate its
    %amplitude and orientation
    z16(i)=D(i,12)-D0(i,12);
    z17(i)=D(i,11)-D0(i,11);
    Z16=[Z16;z16(i)];
    Z17=[Z17;z17(i)];
    amplZ1617(i)=sqrt(z16(i).^2+z17(i).^2);
    phiZ1617(i)=atan2(z16(i),z17(i));
    Z1617=[Z1617;z16(i);z17(i)];

    %Store the field positions
    x(i)=D(i,1);
    y(i)=D(i,2);
end
%Conversion to vector
x=x';
y=y';
amplZ56=amplZ56';
phiZ56=phiZ56';
amplZ78=amplZ78';
phiZ78=phiZ78';
amplZ910=amplZ910';
phiZ910=phiZ910';
amplZ1213=amplZ1213';
phiZ1213=phiZ1213';
amplZ1617=amplZ1617';
phiZ1617=phiZ1617';


%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Creation of the matrix and Least-squares fit (LSF) for the zernike coefficients%

%Matrix for the LSF for Astig3 (third order astigmatism)
MAstig3=[]; %initialization of the matrix for Astig3 (z5 and z6)

for i=1:nbpoints
    M1=[0 1 y(i) x(i) 2.*x(i).*y(i) -x(i) y(i) 2.*x(i).*(y(i)).^2 ...
            x(i).*(y(i)).^2-(x(i)).^3 0 ((x(i)).^2+(y(i)).^2) ...
        ((x(i)).^3+x(i).*(y(i)).^2) (x(i)).^2.*y(i)+(y(i)).^3 ...
        2.*((x(i)).^3.*y(i)+x(i).*(y(i)).^3)];
```

```
    M2=[1 0 -x(i) y(i) ((y(i)).^2-(x(i)).^2) y(i) x(i) ...
            ((y(i)).^3-(x(i)).^2.*y(i)) -2.*(x(i)).^2.*y(i) ...
        ((x(i)).^2+(y(i)).^2) 0 (x(i)).^2.*y(i)+(y(i)).^3 ...
        -((x(i)).^3+x(i).*(y(i)).^2) ((y(i)).^4-(x(i)).^4)];

    A=[M1;M2];
    MAstig3=[MAstig3;A];
end

%least-squares fit for z5 and z6 (Output of the program)
RAstig3=MAstig3\Z56

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Matrix for the LSF for Coma3 (third order coma)%

MComa3=[]; %initialization of the matrix for Coma3 (z7 and z8)
for i=1:nbpoints
    M1=[0 1 x(i) y(i) 0 ((x(i)).^2+(y(i)).^2) x(i) x(i).*y(i) ...
            (x(i)).^2 ((x(i)).^3+x(i).*(y(i)).^2)];
    M2=[1 0 -y(i) x(i) ((x(i)).^2+(y(i)).^2) 0 y(i) (y(i)).^2 ...
            (x(i).*y(i)) ((y(i)).^3+y(i).*(x(i)).^2)];

    A=[M1;M2];
    MComa3=[MComa3;A];
end

%least-squares fit for z7 and z8 (Output of the program)
RComa3=MComa3\Z78

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Matrix for the LSF for Trefoil5%

MTrefoil=[]; %initialization of the matrix for Trefoil5 (z9 and z10)
for i=1:nbpoints

    M1=[0 1 y(i) x(i) 2.*x(i).*y(i) ((y(i)).^2-(x(i)).^2) ...
            (3.*(y(i)).^2.*x(i)-(x(i)).^3)];
    M2=[1 0 -x(i) y(i) ((y(i)).^2-(x(i)).^2) -2.*x(i).*y(i) ...
            (-3.*(x(i)).^2.*y(i)+(y(i)).^3)];

    A=[M1;M2];
    MTrefoil=[MTrefoil;A];

end

%least-squares fit for z9 and z10 (Output of the program)
RTrefoil=MTrefoil\Z910

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

```
%Matrix for the LSF for Astig5 (fifth order astigmatism)%

MAstig5=[]; %initialization of the matrix for Astig5 (z12 and z13)
for i=1:nbpoints

    M1=[0 1 y(i) -x(i) ((y(i)).^2-(x(i)).^2)];
    M2=[1 0  x(i) y(i) 2.*x(i).*y(i)];

    A=[M1;M2];
    MAstig5=[MAstig5;A];

end

%least-squares fit for z12 and z13 (Output of the program)
RAstig5=MAstig5\Z1213

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Matrix for the LSF for Coma5 (fifth order coma)%

MComa5=[]; %initialization of the matrix for Coma5 (z16 and z17)
for i=1:nbpoints

    M1=[0 1 y(i)];
    M2=[1 0 x(i)];

    A=[M1;M2];
    MComa5=[MComa5;A];

end

%least-squares fit for z16 and z17 (Output of the program)
RComa5=MComa5\Z1617


%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Graphic representation of the 5 aberrations with their fit

%Astig3%
%%%%%%%

%Constant part
B=ones(nbpoints);
B=B(:,1);
Z5cst=RAstig3(2).*B;
Z6cst=RAstig3(1).*B;
amplcst=((Z5cst).^2+(Z6cst).^2).^(1./2);
phicst=1./2.*atan2(Z6cst,Z5cst);
```

```
%Linear part 1
Z5lin1=RAstig3(3).*y+RAstig3(4).*x;
Z6lin1=-RAstig3(3).*x+RAstig3(4).*y;
ampllin1=((Z5lin1).^2+(Z6lin1).^2).^(1./2);
philin1=1./2.*atan2(Z6lin1,Z5lin1);

%Linear part 2
Z5lin2=-RAstig3(6).*x+RAstig3(7).*y;
Z6lin2=RAstig3(6).*y+RAstig3(7).*x;
ampllin2=((Z5lin2).^2+(Z6lin2).^2).^(1./2);
philin2=1./2.*atan2(Z6lin2,Z5lin2);

%Quadratic part 1
Z5quad1=RAstig3(5).*2.*x.*y;
Z6quad1=RAstig3(5).*(y.^2-x.^2);
amplquad1=((Z5quad1).^2+(Z6quad1).^2).^(1./2);
phiquad1=1./2.*atan2(Z6quad1,Z5quad1);

%Quadratic part 2
Z5quad2=RAstig3(11).*(x.^2+y.^2);
Z6quad2=RAstig3(10).*(x.^2+y.^2);
amplquad2=((Z5quad2).^2+(Z6quad2).^2).^(1./2);
phiquad2=1./2.*atan2(Z6quad2,Z5quad2);

%Cubic part 1
Z5cub1=RAstig3(8).*2.*x.*(y.^2)+RAstig3(9).*(x.*(y.^2)-x.^3);
Z6cub1=RAstig3(8).*(y.^3-(x.^2).*y)-RAstig3(9).*(2.*(x.^2).*y);
amplcub1=((Z5cub1).^2+(Z6cub1).^2).^(1./2);
phicub1=1./2.*atan2(Z6cub1,Z5cub1);

%Cubic part 2
Z5cub2=RAstig3(12).*(x.^3+x.*(y.^2))+RAstig3(13).*((x.^2).*y+y.^3);
Z6cub2=RAstig3(12).*((x.^2).*y+y.^3)-RAstig3(13).*(x.^3+x.*(y.^2));
amplcub2=((Z5cub2).^2+(Z6cub2).^2).^(1./2);
phicub2=1./2.*atan2(Z6cub2,Z5cub2);

%Quartic part (Astig of an aligned system)
Z5quar=RAstig3(14).*(2.*((x.^3).*y+x.*(y.^3)));
Z6quar=RAstig3(14).*((y.^4-x.^4));
amplquar=((Z5quar).^2+(Z6quar).^2).^(1./2);
phiquar=1./2.*atan2(Z6quar,Z5quar);


%Residual part
Z5res=Z5-Z5cst-Z5lin1-Z5lin2-Z5quad1-Z5quad2-Z5cub1-Z5cub2-Z5quar;
Z6res=Z6-Z6cst-Z6lin1-Z6lin2-Z6quad1-Z6quad2-Z6cub1-Z6cub2-Z6quar;
amplres=((Z5res).^2+(Z6res).^2).^(1./2);
phires=1./2.*atan2(Z6res,Z5res);
```

```
%Creation of the plot with the 9 graphs for Astig3%

figure('Position',[1 29 1024 670],...
  'PaperOrientation','Landscape',...
  'PaperPosition',[0.25 0.25 11.193 7.7677])
subplot('Position',[.08 .69 .20 .20])%(3,3,1)
quiv(x,y,1./10.*amplZ56.*cos(phiZ56),1./10.*amplZ56.*sin(phiZ56),0);
axis square
title(['Original data for Astig3 of ',filename]);
xlabel('X position (mm)');
ylabel('Y position (mm)');
axis([-350 350 -350 350]);

subplot('Position',[.38 .69 .20 .20])%(3,3,2)
quiv(x,y,1./10.*amplcst.*cos(phicst),1./10.*amplcst.*sin(phicst),0);
axis square
title('Constant fit');
xlabel('X position (mm)');
ylabel('Y position (mm)');
axis([-350 350 -350 350]);

subplot('Position',[.68 .69 .20 .20])%(3,3,3)
quiv(x,y,1./10.*ampllin1.*cos(philin1),1./10.*ampllin1.*sin(philin1),0);
axis square
title('Linear fit #1');
xlabel('X position (mm)');
ylabel('Y position (mm)');
axis([-350 350 -350 350]);

subplot('Position',[.08 .39 .20 .20])%(3,3,4)
quiv(x,y,1./10.*ampllin2.*cos(philin2),1./10.*ampllin2.*sin(philin2),0);
axis square
title('Linear fit #2');
xlabel('X position (mm)');
ylabel('Y position (mm)');
axis([-350 350 -350 350]);

subplot('Position',[.38 .39 .20 .20])%(3,3,5)
quiv(x,y,1./10.*amplquad1.*cos(phiquad1),1./10.*amplquad1.*sin(phiquad1),0);
axis square
title('Quadratic fit #1');
xlabel('X position (mm)');
ylabel('Y position (mm)');
axis([-350 350 -350 350]);

subplot('Position',[.68 .39 .20 .20])%(3,3,6)
quiv(x,y,1./10.*amplquad2.*cos(phiquad2),1./10.*amplquad2.*sin(phiquad2),0);
axis square
```

```
title('Quadratic fit #2');
xlabel('X position (mm)');
ylabel('Y position (mm)');
axis([-350 350 -350 350]);

subplot('Position',[.08 .09 .20 .20])%(3,3,7)
quiv(x,y,1./10.*amplcub1.*cos(phicub1),1./10.*amplcub1.*sin(phicub1),0);
axis square
title('Cubic fit #1');
xlabel('X position (mm)');
ylabel('Y position (mm)');
axis([-350 350 -350 350]);

subplot('Position',[.38 .09 .20 .20])%(3,3,8)
quiv(x,y,1./10.*amplcub2.*cos(phicub2),1./10.*amplcub2.*sin(phicub2),0);
axis square
title('Cubic fit #2');
xlabel('X position (mm)');
ylabel('Y position (mm)');
axis([-350 350 -350 350]);

subplot('Position',[.68 .09 .20 .20])%(3,3,9)
quiv(x,y,1./10.*amplquar.*cos(phiquar),1./10.*amplquar.*sin(phiquar),0);
axis square
title('Quartic fit');
xlabel('X position (mm)');
ylabel('Y position (mm)');
axis([-350 350 -350 350]);

figure
quiv(x,y,1./10.*amplres.*cos(phires),1./10.*amplres.*sin(phires),0);
axis square
title('Residual Astig3');
xlabel('X position (mm)');
ylabel('Y position (mm)');
axis([-350 350 -350 350]);

%Coma3%
%%%%%%

%Constant part
Z7cst=RComa3(2).*B;
Z8cst=RComa3(1).*B;

%Linear part 1
Z7lin1=RComa3(3).*x+RComa3(4).*y;
Z8lin1=RComa3(3).*y-RComa3(4).*x;

%Linear part 2
```

```
Z7lin2=RComa3(7).*x;
Z8lin2=RComa3(7).*y;

%Quadratic part 1
Z7quad1=RComa3(6).*(x.^2+y.^2);
Z8quad1=RComa3(5).*(x.^2+y.^2);

%Quadratic part 2
Z7quad2=RComa3(9).*(x.^2)+RComa3(8).*x.*y;
Z8quad2=RComa3(9).*x.*y+RComa3(8).*(y.^2);

%Cubic part
Z7cub=RComa3(10).*((x(i)).^3+x.*(y(i)).^2);
Z8cub=RComa3(10).*((y(i)).^3+y.*(x(i)).^2);

%Residual part
Z7res=Z7-Z7cst-Z7lin1-Z7lin2-Z7quad1-Z7quad2-Z7cub;
Z8res=Z8-Z8cst-Z8lin1-Z8lin2-Z8quad1-Z8quad2-Z8cub;

%Creation of the plot with the 7 graphs for Coma3%

figure('Position',[1 29 1024 670],...
  'PaperOrientation','Landscape',...
  'PaperPosition',[0.25 0.25 11.193 7.7677])
subplot('Position',[.08 .69 .20 .20])%subplot(3,3,1)
quiv(x,y,1./1.*Z7,1./1.*Z8,0);
axis square
title(['Original data for Coma3 of ',filename]);
xlabel('X position (mm)');
ylabel('Y position (mm)');
axis([-350 350 -350 350]);

subplot('Position',[.38 .69 .20 .20])%subplot(3,3,2)
quiv(x,y,1./1.*Z7cst,1./1.*Z8cst,0);
axis square
title('Constant fit');
xlabel('X position (mm)');
ylabel('Y position (mm)');
axis([-350 350 -350 350]);

subplot('Position',[.68 .69 .20 .20])%subplot(3,3,3)
quiv(x,y,1./1.*Z7lin1,1./1.*Z8lin2,0);
axis square
title('Linear fit #1');
xlabel('X position (mm)');
ylabel('Y position (mm)');
axis([-350 350 -350 350]);

subplot('Position',[.08 .39 .20 .20])%subplot(3,3,4)
```

```
quiv(x,y,1./1.*Z7lin2,1./1.*Z8lin2,0);
axis square
title('Linear fit #2');
xlabel('X position (mm)');
ylabel('Y position (mm)');
axis([-350 350 -350 350]);

subplot('Position',[.38 .39 .20 .20])%subplot(3,3,5)
quiv(x,y,1./1.*Z7quad1,1./1.*Z8quad1,0);
axis square
title('Quadratic fit #1');
xlabel('X position (mm)');
ylabel('Y position (mm)');
axis([-350 350 -350 350]);

subplot('Position',[.68 .39 .20 .20])%subplot(3,3,6)
quiv(x,y,1./1.*Z7quad2,1./1.*Z8quad2,0);
axis square
title('Quadratic fit #2');
xlabel('X position (mm)');
ylabel('Y position (mm)');
axis([-350 350 -350 350]);

subplot('Position',[.08 .09 .20 .20])%subplot(3,3,7)
quiv(x,y,1./1.*Z7cub,1./1.*Z8cub,0);
axis square
title('Cubic fit');
xlabel('X position (mm)');
ylabel('Y position (mm)');
axis([-350 350 -350 350]);

subplot('Position',[.38 .09 .20 .20])%subplot(3,3,8)
quiv(x,y,1./1.*Z7res,1./1.*Z8res,0);
axis square
title('Residual Coma3');
xlabel('X position (mm)');
ylabel('Y position (mm)');
axis([-350 350 -350 350]);

%Ttrefoil5%

%Constant part
clear amplcst phicst
Z9cst=RTrefoil(2).*B;
Z10cst=RTrefoil(1).*B;
amplcst=((Z9cst).^2+(Z10cst).^2).^(1./2);
phicst=1./3.*atan2(Z10cst,Z9cst);

%Linear part
```

```
Z9lin=RTrefoil(3).*y+RTrefoil(4).*x;
Z10lin=-RTrefoil(3).*x+RTrefoil(4).*y;
ampllin=((Z9lin).^2+(Z10lin).^2).^(1./2);
philin=1./3.*atan2(Z10lin,Z9lin);

%Quadratic part
Z9quad=RTrefoil(6).*(y.^2-x.^2)+RTrefoil(5).*2.*x.*y;
Z10quad=-RTrefoil(6).*2.*x.*y+RTrefoil(5).*(y.^2-x.^2);
amplquad=((Z9quad).^2+(Z10quad).^2).^(1./2);
phiquad=1./3.*atan2(Z10quad,Z9quad);

%Cubic part
Z9cub=RTrefoil(7).*(3.*y.^2.*x-x.^3);
Z10cub=RTrefoil(7).*(-3.*x.^2.*y+y.^3);
amplcub=((Z9cub).^2+(Z10cub).^2).^(1./2);
phicub=1./3.*atan2(Z10cub,Z9cub);

%Residual part
clear amplres phires
Z9res=Z9-Z9cst-Z9lin-Z9quad-Z9cub;
Z10res=Z10-Z10cst-Z10lin-Z10quad-Z10cub;
amplres=((Z9res).^2+(Z10res).^2).^(1./2);
phires=1./3.*atan2(Z10res,Z9res);

%Creation of the plot with the 6 graphs for trefoil%

figure
subplot(2,3,1)
quiv(x,y,10./1.*amplZ910.*cos(phiZ910),10./1.*amplZ910.*sin(phiZ910),0);
axis square
title(['Original data for Trefoil5 of ',filename]);
xlabel('X position (mm)');
ylabel('Y position (mm)');
axis([-350 350 -350 350]);

subplot(2,3,2)
quiv(x,y,10./1.*amplcst.*cos(phicst),10./1.*amplcst.*sin(phicst),0);
axis square
title('Constant fit');
xlabel('X position (mm)');
ylabel('Y position (mm)');
axis([-350 350 -350 350]);

subplot(2,3,3)
quiv(x,y,10./1.*ampllin.*cos(philin),10./1.*ampllin.*sin(philin),0);
axis square
title('Linear fit');
xlabel('X position (mm)');
ylabel('Y position (mm)');
```

```
axis([-350 350 -350 350]);

subplot(2,3,4)
quiv(x,y,10./1.*amplquad.*cos(phiquad),10./1.*amplquad.*sin(phiquad),0);
axis square
title('Quadratic fit');
xlabel('X position (mm)');
ylabel('Y position (mm)');
axis([-350 350 -350 350]);

subplot(2,3,5)
quiv(x,y,10./1.*amplcub.*cos(phicub),10./1.*amplcub.*sin(phicub),0);
axis square
title('Cubic fit');
xlabel('X position (mm)');
ylabel('Y position (mm)');
axis([-350 350 -350 350]);

subplot(2,3,6)
quiv(x,y,10./1.*amplres.*cos(phires),10./1.*amplres.*sin(phires),0);
axis square
title('Residual Trefoil5');
xlabel('X position (mm)');
ylabel('Y position (mm)');
axis([-350 350 -350 350]);

%Astig5%

%Constant part
clear amplcst phicst
Z12cst=RAstig5(2).*B;
Z13cst=RAstig5(1).*B;
amplcst=((Z13cst).^2+(Z12cst).^2).^(1./2);
phicst=1./2.*atan2(Z12cst,Z13cst);

%Linear part
clear ampllin philin
Z12lin=RAstig5(3).*y-RAstig5(4).*x;
Z13lin=RAstig5(3).*x+RAstig5(4).*y;
ampllin=((Z13lin).^2+(Z12lin).^2).^(1./2);
philin=1./2.*atan2(Z12lin,Z13lin);

%Quadratic part
clear amplquad phiquad
Z12quad=RAstig5(5).*(y.^2-x.^2);
Z13quad=RAstig5(5).*2.*x.*y;
amplquad=((Z13quad).^2+(Z12quad).^2).^(1./2);
phiquad=1./2.*atan2(Z12quad,Z13quad);
```

```
%Residual part
clear amplres phires
Z12res=Z12-Z12cst-Z12lin-Z12quad;
Z13res=Z13-Z13cst-Z13lin-Z13quad;
amplres=((Z13res).^2+(Z12res).^2).^(1./2);
phires=1./2.*atan2(Z12res,Z13res);

%Creation of the plot with the 4 graphs for astig5%

figure
subplot(2,2,1)
quiv(x,y,1./1.*amplZ1213.*cos(phiZ1213),1./1.*amplZ1213.*sin(phiZ1213),0);
axis square
title(['Original data for Astig5 of ',filename]);
xlabel('X position (mm)');
ylabel('Y position (mm)');
axis([-350 350 -350 350]);

subplot(2,2,2)
quiv(x,y,1./1.*amplcst.*cos(phicst),1./1.*amplcst.*sin(phicst),0);
axis square
title('Constant fit');
xlabel('X position (mm)');
ylabel('Y position (mm)');
axis([-350 350 -350 350]);

subplot(2,2,3)
quiv(x,y,1./1.*ampllin.*cos(philin),1./1.*ampllin.*sin(philin),0);
axis square
title('Linear fit');
xlabel('X position (mm)');
ylabel('Y position (mm)');
axis([-350 350 -350 350]);

subplot(2,2,4)
quiv(x,y,1./1.*amplres.*cos(phires),1./1.*amplres.*sin(phires),0);
axis square
title('Residual Astig5');
xlabel('X position (mm)');
ylabel('Y position (mm)');
axis([-350 350 -350 350]);

%Coma5%

%Constant part
Z16cst=RComa5(2).*B;
Z17cst=RComa5(1).*B;

%Linear part
```

```
Z16lin=RComa5(3).*y;
Z17lin=RComa5(3).*x;

%Residual part
Z16res=Z16-Z16cst-Z16lin;
Z17res=Z17-Z17cst-Z17lin;

%Creation of the plot with the 4 graphs for Coma5%

figure
subplot(2,2,1)
quiv(x,y,1./1.*Z17,1./1.*Z16,0);
axis square
title(['Original data for Coma5 of ',filename]);
xlabel('X position (mm)');
ylabel('Y position (mm)');
axis([-350 350 -350 350]);

subplot(2,2,2)
quiv(x,y,1./1.*Z17cst,1./1.*Z16cst,0);
axis square
title('Constant fit');
xlabel('X position (mm)');
ylabel('Y position (mm)');
axis([-350 350 -350 350]);

subplot(2,2,3)
quiv(x,y,1./1.*Z17lin,1./1.*Z16lin,0);
axis square
title('Linear fit');
xlabel('X position (mm)');
ylabel('Y position (mm)');
axis([-350 350 -350 350]);

subplot(2,2,4)
quiv(x,y,1./1.*Z17res,1./1.*Z16res,0);
axis square
title('Residual Coma5');
xlabel('X position (mm)');
ylabel('Y position (mm)');
axis([-350 350 -350 350]);
```

Appendix I

Zemax® macro to simulate the LSST

```
! LSSTperturb.zpl
! Perturbs inital case of LSST design
! Uses uniform distribution given by P-V variations:
! Assuming Focal plane is fixed, relative misalignments with P-V :
! Decenter (in mm) gives dx, dy for SM, TM, Corr.
! Tilt  (degrees) gives tx,  ty for SM, TM, Corr.
! Regis Tessieres - September 24th 2003

Decenter = 1
Tilt = 0.08

! Program start
print "Macro LSSTperturb.ZPL in progress"

path$ = "c:\LSST\MONTE"

FOR i=1,100,1
    FORMAT 1.0
    infile$ = path$"\LSSTperturb"i".zmx"
    outfile$ = path$"\LSSTmodel"i".txt"
    FORMAT 8.6
    LOADLENS "C:\LSST\LSST_Monte.zmx"
    GOSUB PERTURB
    SAVELENS infile$
    GOSUB MERIT
    GOSUB CALCULATIONS
NEXT

print " All done :))))))))))))"

END

! subroutines

!-------------------------------------------------------
SUB PERTURB
! Perturb the system

!Secondary
!---------
 msurf = 5
! Decenter
 PV = Decenter
```

```
 GOSUB random
 PARM 1,msurf,RV
 GOSUB random
 PARM 2,msurf,RV
! Tilt
 PV = Tilt
 GOSUB random
 PARM 3,msurf,RV
 GOSUB random
 PARM 4,msurf,RV

!Tertiary
!--------
 msurf = 11
! Decenter
 PV = Decenter
 GOSUB random
 PARM 1,msurf,RV
 GOSUB random
 PARM 2,msurf,RV
! Tilt
 PV = Tilt
 GOSUB random
 PARM 3,msurf,RV
 GOSUB random
 PARM 4,msurf,RV

!CORRECTOR
!---------
 msurf = 15
! Decenter
 PV = Decenter
 GOSUB random
 PARM 1,msurf,RV
 GOSUB random
 PARM 2,msurf,RV
! Tilt
 PV = Tilt
 GOSUB random
 PARM 3,msurf,RV
 GOSUB random
 PARM 4,msurf,RV
return
!------------------------------------------------------------
SUB MERIT
!Calculation of the weighted RMS Spot Radius

UPDATE ALL
OUTPUT "C:\LSST\Original_Distribution.txt" APPEND
```

```
PRINT 1000*MFCN()
OUTPUT SCREEN

RETURN
!------------------------------------------------------------

SUB CALCULATIONS
! Calculate the zernike coefficients for 48 points in the field

!Definition of pi to use it later on
pi=4*ATAN(1)

!First set of field points (Only 12 field points can be defined at a time)
NUMFIELD 12
!fix the number of field and define each field

FTYP=0
FLDX 1 = 0
FLDY 1 = 1.5
FWGT 1 = 1
FLDX 2 = -1
FLDY 2 = 1
FWGT 2 = 1
FLDX 3 = -0.75
FLDY 3 = 1
FWGT 3 = 1
FLDX 4 = -0.5
FLDY 4 = 1
FWGT 4 = 1
FLDX 5 = -0.25
FLDY 5 = 1
FWGT 5 = 1
FLDX 6 = 0
FLDY 6 = 1
FWGT 6 = 1
FLDX 7 = 0.25
FLDY 7 = 1
FWGT 7 = 1
FLDX 8 = 0.5
FLDY 8 = 1
FWGT 8 = 1
FLDX 9 = 0.75
FLDY 9 = 1
FWGT 9 = 1
FLDX 10 = 1
FLDY 10 = 1
FWGT 10 = 1
FLDX 11 = -1
FLDY 11 = 0.5
```

```
FWGT 11 = 1
FLDX 12 = -0.75
FLDY 12 = 0.5
FWGT 12 = 1


!Update the windows
UPDATE ALL

GOSUB ZERN
!Go to the subroutine Zern to calulate and store the zernike coefficients

FTYP=0
FLDX 1 = -0.5
FLDY 1 = 0.5
FWGT 1 = 1
FLDX 2 = -0.25
FLDY 2 = 0.5
FWGT 2 = 1
FLDX 3 = 0
FLDY 3 = 0.5
FWGT 3 = 1
FLDX 4 = 0.25
FLDY 4 = 0.5
FWGT 4 = 1
FLDX 5 = 0.5
FLDY 5 = 0.5
FWGT 5 = 1
FLDX 6 = 0.75
FLDY 6 = 0.5
FWGT 6 = 1
FLDX 7 = 1
FLDY 7 = 0.5
FWGT 7 = 1
FLDX 8 = -1.5
FLDY 8 = 0
FWGT 8 = 1
FLDX 9 = -1
FLDY 9 = 0
FWGT 9 = 1
FLDX 10 = -0.75
FLDY 10 = 0
FWGT 10 = 1
FLDX 11 = -0.5
FLDY 11 = 0
FWGT 11 = 1
FLDX 12 = -0.25
FLDY 12 = 0
FWGT 12 = 1
```

```
!Update the windows
UPDATE ALL
GOSUB ZERN
!Go to the subroutine Zern to calulate and store the zernike coefficients

FTYP=0
FLDX 1 = 0
FLDY 1 = 0
FWGT 1 = 1
FLDX 2 = 0.25
FLDY 2 = 0
FWGT 2 = 1
FLDX 3 = 0.5
FLDY 3 = 0
FWGT 3 = 1
FLDX 4 = 0.75
FLDY 4 = 0
FWGT 4 = 1
FLDX 5 = 1
FLDY 5 = 0
FWGT 5 = 1
FLDX 6 = 0
FLDY 6 = -1.5
FWGT 6 = 1
FLDX 7 = 1.5
FLDY 7 = 0
FWGT 7 = 1
FLDX 8 = -1
FLDY 8 = -0.5
FWGT 8 = 1
FLDX 9 = -0.75
FLDY 9 = -0.5
FWGT 9 = 1
FLDX 10 = -0.5
FLDY 10 = -0.5
FWGT 10 = 1
FLDX 11 = -0.25
FLDY 11 = -0.5
FWGT 11 = 1
FLDX 12 = 0
FLDY 12 = -0.5
FWGT 12 = 1

!Update the windows
UPDATE ALL
GOSUB ZERN
!Go to the subroutine Zern to calulate and store the zernike coefficients

FTYP=0
```

```
FLDX 1 = 0.25
FLDY 1 = -0.5
FWGT 1 = 1
FLDX 2 = 0.5
FLDY 2 = -0.5
FWGT 2 = 1
FLDX 3 = 0.75
FLDY 3 = -0.5
FWGT 3 = 1
FLDX 4 = 1
FLDY 4 = -0.5
FWGT 4 = 1
FLDX 5 = -1
FLDY 5 = -1
FWGT 5 = 1
FLDX 6 = -0.75
FLDY 6 = -1
FWGT 6 = 1
FLDX 7 = -0.5
FLDY 7 = -1
FWGT 7 = 1
FLDX 8 = -0.25
FLDY 8 = -1
FWGT 8 = 1
FLDX 9 = 0
FLDY 9 = -1
FWGT 9 = 1
FLDX 10 = 0.25
FLDY 10 = -1
FWGT 10 = 1
FLDX 11 = 0.5
FLDY 11 = -1
FWGT 11 = 1
FLDX 12 = 0.75
FLDY 12 = -1
FWGT 12 = 1

!Update the windows
UPDATE ALL
GOSUB ZERN
!Go to the subroutine Zern to calulate and store the zernike coefficients


RETURN



!----------------------------------------------------------------
SUB random
```

```
! returns RV
! random value from uniform distribution with P-V range PV
RV = rand(PV)-PV/2.
RETURN
!---------------------------------------------------------------
!Subroutine to get the Zernike polynomial and store them into a file
SUB ZERN

OUTPUT outfile$ APPEND

FOR j=1,12,1
    GETZERNIKE 17,1,j,3,1,0
    PRINT TANG(FLDX(j)*pi/180)*10496.5," ",TANG(FLDY(j)*pi/180)*10496.5," ",
    PRINT VEC1(14)*550/sqrt(6)," ",
    PRINT VEC1(13)*550/sqrt(6)," ",VEC1(16)*550/sqrt(8)," ",
    PRINT VEC1(15)*550/sqrt(8)," ",
    PRINT VEC1(19)*550/sqrt(8)," ",VEC1(18)*550/sqrt(8)," ",
    PRINT VEC1(20)*550/sqrt(10)," ",
    PRINT VEC1(21)*550/sqrt(10)," ",VEC1(22)*550/sqrt(12)," ",
    PRINT VEC1(23)*550/sqrt(12)
NEXT

OUTPUT screen
RETURN
```

Appendix J

MATLAB® ROUTINE TO CALCULATE THE EXPECTED PERTURBATIONS

```
function Rlsf=LSF(filename)
%Routine to calculate the expected perturbations
%Can be use in an other program to calulate the perturbations
%of several models

format long %double precision
D= importdata(filename,'\t');
nbpoints=size(D);
nbpoints=nbpoints(1);

%read the data in the file%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%initialization
Z56=[];
Z5=[];
Z6=[];
Z78=[];
Z7=[];
Z8=[];
Z910=[];
Z9=[];
Z10=[];
Z1213=[];
Z12=[];
Z13=[];
Z1617=[];
Z16=[];
Z17=[];
Z=[];
M=[];
B=zeros(10*nbpoints,12);

for i=1:nbpoints

    %Store the values for third order astigmatism
    z5(i)=D(i,3);
    z6(i)=D(i,4);

    %Store the values for third order coma
    z7(i)=D(i,5);
```

```
    z8(i)=D(i,6);

    %Store the values for trefoil
    z9(i)=D(i,7);
    z10(i)=D(i,8);

    %Store the values for fifth order astigmatism
    z12(i)=D(i,9);
    z13(i)=D(i,10);

    %Store the values for fifth order coma
    z16(i)=D(i,11);
    z17(i)=D(i,12);

    %Store the field positions
    x(i)=D(i,1);
    y(i)=D(i,2);

    %Vector Z with all the zernike coefficients
    Y=[z5(i) z6(i) z7(i) z8(i) z9(i) z10(i) z12(i) z13(i) z16(i) z17(i)]';
    Z=[Z;Y];
% Creation of a Matrix %
%%%%%%%%%%%%%%%%%%%%%%%%%

    %First column
    a11=416.4.*x(i);
    a21=2069-416.4.*y(i);
    a31=-177507+0.165.*y(i)-0.0119.*((x(i)).^2+(y(i)).^2)+0.466.*y(i);
    a41=-0.165.*x(i)+0.466.*x(i);
    a51=-0.181.*y(i)+0.0054.*((y(i)).^2-(x(i)).^2);
    a61=-0.181.*x(i)+0.0108.*x(i).*y(i);
    a71=31.3+2.1.*y(i);
    a81=-2.1.*x(i);
    a91=0;
    a101=2364.25;

    %Second column
    a12=-416.4.*y(i);
    a22=-2069-416.4.*x(i);
    a32=-0.165.*y(i)+0.466.*y(i);
    a42=177507+0.165.*x(i)+0.0119.*((x(i)).^2+(y(i)).^2)+0.466.*x(i);
    a52=0.181.*y(i)+0.0108.*x(i).*y(i);
    a62=0.181.*x(i)-0.0054.*((y(i)).^2-(x(i)).^2);
    a72=-31.3+2.1.*x(i);
    a82=2.1.*y(i);
    a92=-2364.25;
    a102=0;

    %Third column
```

```
a13=3.793.*y(i);
a23=3.793.*x(i);
a33=0;
a43=-1755.24;
a53=0;
a63=0;
a73=0.054.*x(i);
a83=0.054.*y(i);
a93=60.64;
a103=0;

%Fourth column
a14=3.793.*x(i);
a24=-3.793.*y(i);
a34=-1755.24;
a44=0;
a54=0;
a64=0;
a74=-0.054.*y(i);
a84=0.054.*x(i);
a94=0;
a104=60.64;

%Fifth column
a15=-130.6.*x(i)-0.000083.*((x(i)).^3+x(i).*(y(i)).^2);
a25=-5164+130.6.*y(i)+0.0036.*((x(i)).^2+(y(i)).^2)+...
0.000083.*((y(i)).^3+y(i).*(x(i)).^2);
a35=-186150-0.454.*y(i)+0.0277.*((x(i)).^2+(y(i)).^2)-...
1.12.*y(i)+0.0464.*(y(i)).^2;
a45=0.454.*x(i)-1.12.*x(i)+0.0464.*x(i).*y(i);
a55=0.545.*y(i)-0.0065.*((y(i)).^2-(x(i)).^2);
a65=0.545.*x(i)-0.013.*x(i).*y(i);
a75=-229+0.62.*y(i);
a85=-0.62.*x(i);
a95=0;
a105=-4315;

%Sixth column
a16=130.6.*y(i)+0.000083.*((y(i)).^3+y(i).*(x(i)).^2);
a26=5164+130.6.*x(i)-0.0036.*((x(i)).^2+(y(i)).^2)+...
0.000083.*((x(i)).^3+x(i).*(y(i)).^2);
a36=0.454.*y(i)-1.12.*y(i)-0.0464.*x(i).*y(i);
a46=186150-0.454.*x(i)-0.0277.*((x(i)).^2+(y(i)).^2)-...
1.12.*x(i)-0.0464.*(x(i)).^2;
a56=-0.545.*y(i)-0.013.*x(i).*y(i);
a66=-0.545.*x(i)+0.0065.*((y(i)).^2-(x(i)).^2);
a76=229+0.62.*x(i);
a86=0.62.*y(i);
a96=4315;
```

```
a106=0;

%Seventh column
a17=-0.8997.*y(i);
a27=-0.8997.*x(i);
a37=0.00053.*x(i).*y(i);
a47=-1273.7+0.0002936.*((x(i)).^2+(y(i)).^2)+0.00053.*(x(i)).^2;
a57=-0.000298.*x(i).*y(i);
a67=0.000149.*((y(i)).^2-(x(i)).^2);
a77=0.025.*x(i);
a87=0.025.*y(i);
a97=-25.88;
a107=0;

%Eighth column
a18=-0.8997.*x(i);
a28=0.8997.*y(i);
a38=-1237.7+0.0002936.*((x(i)).^2+(y(i)).^2)+0.00053.*(y(i)).^2;
a48=0.00053.*x(i).*y(i);
a58=-0.000149.*((y(i)).^2-(x(i)).^2);
a68=-0.000298.*x(i).*y(i);
a78=-0.025.*y(i);
a88=0.025.*x(i);
a98=0;
a108=-25.88;

%Nineth column
a19=-38.35.*x(i)+0.000084.*((x(i)).^3+x(i).*(y(i)).^2);
a29=-38+38.35.*y(i)-0.000084.*((y(i)).^3+y(i).*(x(i)).^2);
a39=869+0.006797.*((x(i)).^2+(y(i)).^2)-0.02109.*(y(i)).^2;
a49=-0.02109.*x(i).*y(i);
a59=-0.00609.*((y(i)).^2-(x(i)).^2);
a69=-0.01218.*x(i).*y(i);
a79=1.27.*y(i);
a89=-1.27.*x(i);
a99=0;
a109=0;

%Tenth column
a110=38.35.*y(i)-0.000084.*((y(i)).^3+y(i).*(x(i)).^2);
a210=38+38.35.*x(i)-0.000084.*((x(i)).^3+x(i).*(y(i)).^2);
a310=0.02109.*x(i).*y(i);
a410=-869-0.006797.*((x(i)).^2+(y(i)).^2)+0.02109.*(x(i)).^2;
a510=-0.01218.*x(i).*y(i);
a610=0.00609.*((y(i)).^2-(x(i)).^2);
a710=1.27.*x(i);
a810=1.27.*y(i);
a910=0;
a1010=0;
```

```
%Eleventh column
a111=-0.705.*y(i);
a211=-0.705.*x(i);
a311=-0.000635.*x(i).*y(i);
a411=160.56-0.0002345.*((x(i)).^2+(y(i)).^2)-0.000635.*(x(i)).^2;
a511=-0.00034.*x(i).*y(i);
a611=0.00017.*((y(i)).^2-(x(i)).^2);
a711=-0.0824.*x(i);
a811=-0.0824.*y(i);
a911=0;
a1011=0;

%Twelveth column
a112=-0.705.*x(i);
a212=0.705.*y(i);
a312=160.56-0.0002345.*((x(i)).^2+(y(i)).^2)-0.000635.*(y(i)).^2;
a412=-0.000635.*x(i).*y(i);
a512=0.00017.*((y(i)).^2-(x(i)).^2);
a612=0.00034.*x(i).*y(i);
a712=0.0824.*y(i);
a812=-0.0824.*x(i);
a912=0;
a1012=0;

M1=[a11 a12 a13 a14 a15 a16 a17 a18 a19 a110 a111 a112];
M2=[a21 a22 a23 a24 a25 a26 a27 a28 a29 a210 a211 a212];
M3=[a31 a32 a33 a34 a35 a36 a37 a38 a39 a310 a311 a312];
M4=[a41 a42 a43 a44 a45 a46 a47 a48 a49 a410 a411 a412];
M5=[a51 a52 a53 a54 a55 a56 a57 a58 a59 a510 a511 a512];
M6=[a61 a62 a63 a64 a65 a66 a67 a68 a69 a610 a611 a612];
M7=[a71 a72 a73 a74 a75 a76 a77 a78 a79 a710 a711 a712];
M8=[a81 a82 a83 a84 a85 a86 a87 a88 a89 a810 a811 a812];
M9=[a91 a92 a93 a94 a95 a96 a97 a98 a99 a910 a911 a912];
M10=[a101 a102 a103 a104 a105 a106 a107 a108 a109 a1010 a1011 a1012];
A=[M1;M2;M3;M4;M5;M6;M7;M8;M9;M10];
B((1+10*(i-1)):(10+10*(i-1)),:)=[M1;M2;M3;M4;M5;M6;M7;M8;M9;M10];
M=[M;A];

end

Rlsf=M\Z;
```

REFERENCES

[1] R.A. Buchroeder, "Tilted Component Optical Systems", Ph.D. Dissertation, University of Arizona, Tucson, Arizona, 1976.

[2] R.V. Shack and K.P. Thompson, "Influence of alignment errors of a telescope system on its aberration field", Proc. SPIE, v.251, p.146, 1980

[3] K.P. Thompson, "Aberrations Fields in Tilted and Decentered Optical Systems", Ph.D. Dissertation, University of Arizona, Tucson, Arizona, 1980.

[4] T.S. Turner, Jr., "Vector aberration theory on spreadsheet-analysis of tilted and decentered systems", SPIE, v.1752, p.184, 1992

[5] M. Rimmer, "Analysis of Perturbed Lens Systems", Applied Optics, v.9, n.3, p.533, March 1970

[6] M.J. Fehniger, "Alignment of a full aperture system test of a Cassegrain telescope", SPIE, v.251 Optical alignment, p.21, 1980

[7] R. Bhatia, "Is the zero-coma condition sufficient?", Telescopio Nazionale Galileo, Tech. Rep., No.43, 1995

[8] D.J. Schroeder, *Astronomical Optics*, p.112, Academic Press., 1987

[9] B.A. McLeod, "Collimation of Fast Wide-Field Telescopes", PASP, 108, p.217, 1996

[10] Ph. Gitton and L. Noethe, "Tuning of the NTT Alignment", The Messenger, 92, 15-18, 1998

[11] R.N. Wilson, *Reflecting Telescope Optics I*, p.260, Springer-Verlag, 1996

[12] D.J. Schroeder, *Astronomical Optics*, p.132, Academic Press., 2000

[13] V.N. Mahajan, *Optical Imaging and Aberrations*, Part I, p.437, SPIE, 1998

[14] R.N. Wilson and B. Delabre, "Concerning the Alignment of Modern Telescopes: Theory, Practice, and Tolerances Illustrated by the ESO NTT", PASP, 109, p.53, 1997

[15] K.P. Thompson, "A graphic approach to the analysis of perturbed optical systems", SPIE, v.237 International Lens Design Conference, p.127, 1980

[16] J.R. Rogers, "Techniques and tools for obtaining symmetrical performance from tilted component systems", Optical Engineering, 39(7), p.1776, 2001

[17] M. Born and E. Wolf, *Principle of Optics*, Pergamon Press, 1964

[18] D. Malacara, *Optical Shop Testing*, Wiley, 1978

[19] W. J. Smith, *Modern Optical Engineering: The Design of Optical Systems*, McGraw-Hill, 2000

[20] W. T. Welford, *Aberrations of Optical Systems*, Adam Hilger, 1986

[21] L.G. Seppala, "Improved optical design for the Large Synoptic Survey Telescope (LSST)", Proc. SPIE Int. Soc. Opt. Eng., v.4836, p.111, 2002

[22] Talk presented by C. Claver and J.H. Burge at the Astronomical Telescopes and Instrumentation at Waikola, Hawaï, SPIE 22-28 August 2002