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Parametric model for mirror deflection with axial support

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Abstract: In this study, we verified the effectiveness of the parametric model to estimate the surface RMS due to the mirror deflection. The parametric model based on the 4 empirical equations was derived from the FEA simulations. We can effectively estimate the surface RMS ('total' and 'after power removed') within 8% accuracy using the parametric model.

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1. Introduction

In the early stage of designing opto-mechanical system including flat mirrors quick first-order estimation of the system performance can provide a good starting point. For the efficient estimation of the system performance (i.e. surface RMS), simple analytical model can be used. Nelson[1] developed the simple closed-form formula based on the classical thin plate model[2] in 1982. However, due to neglecting shear effect, the Nelson model does not behave well in the range of small aspect ratio cases. Modern FEA(Finite Element Analysis) tools can simulate more realistic cases with arbitrary mirror geometry and materials at the cost of time. Parametric model based on carefully designed series of simulation runs can fill this gap between the accuracy and time.

2. Simple analytic model for the mirror deflection

One of the most common analytic model is known as Nelson's model. Nelson's theory predicts the surface RMS due to the mirror deflection as below

$$\delta_{\mathrm{rms}} = \gamma_N \frac{q}{D}\left(\frac{A}{N}\right)^2\left[1+2\left(\frac{h}{u}\right)^2\right] \qquad (1)$$

,where γ_N is the support efficiency with N support points, q is the applied force per unit area, D is the flexural rigidity defined as $D = Eh^3/12(1-\nu^2)$, A is the mirror area, h is the thickness of mirror and u is an effective length between support points.

In this paper, we choose the simple three axial point support case (N=3). Then, the mirror deflection is simply governed by 5 parameters: young's modulus(E), Poisson ratio(ν) and density(ρ), aspect ratio(α) and mirror diameter. This model is derived from the shell (thin plate) model, so that it works only for relatively large aspect ratio cases.

3. FEA simulations for the empirical model

Because the Nelson model is only for the thin plate, more realistic deflection calculation can be done using FEA. We used SolidWorks and CosmosWorks to perform series of FEA simulations for various cases. All simulations were carefully designed to explorer a reasonable range of most opto-mechanical systems.

We set 5 independent parameters (Aspect ratio, Mirror diameter, Density, Young's modulus, and Poisson ratio) based on the Nelson model. Each 5 parameters were changed in the FEA model as shown in Table 1.

Table 1. Five independent parameters and its range

Parameter	**Unit**	**Range**
Aspect ratio	N/A	3 ~ 30
Diameter	m	0.25 ~ 2
Young's modulus	GPa	10 ~ 100

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Poisson ratio	N/A	0.1 ~ 0.35
Density	kg/m^3	1000 ~ 3000

The simulated surface RMS results were fitted using polynomial functions to get empirical equations.

3.1 Aspect ratio V.S surface RMS

The effect of the aspect ratio on the surface RMS was investigated. The aspect ratio was changed from 3 to 30 in the FEA model. The surface RMS due to the mirror deflection was calculated. The total surface RMS(f_1) and the surface RMS after removing power(g_1) can be expressed as a function of the aspect ratio as below.

$$f_1(\alpha) = 0.79909(\alpha/10)^2 + 0.18122(\alpha/10) - 0.00637 \quad (2)$$
$$g_1(\alpha) = 0.78881(\alpha/10)^2 + 0.21445(\alpha/10) - 0.01510 \quad (3)$$

3.2 Mirror diameter V.S surface RMS

The effect of the mirror diameter was simulated. The mirror diameter was changed from 0.25m to 2m in the FEA model. The surface RMS due to the mirror deflection was calculated. The total surface RMS(f_2) and the surface RMS after removing power(g_2)can be expressed as a function of the mirror diameter as below.

$$f_2(D) = \left(\frac{D}{1\,m}\right)^2 \quad (4), \qquad g_2(D) = 1.00025\left(\frac{D}{1\,m}\right)^2 \quad (5)$$

3.3 Material density V.S surface RMS

The effect of the mirror material density was simulated. The material density was changed from 1000 kg/m^3 to 3000kg/m^3in the FEA model. The surface RMS due to the mirror deflection was calculated. The total surface RMS(f_3) and the the surface RMS after reomving power(g_3) can be expressed as a function of the material density as below.

$$f_3(\rho) = g_3(\rho) = \frac{\rho}{1000\ \mathrm{kg}/m^3} \quad (6)$$

3.4 Young's modulus and Poisson ratio V.S surface RMS

The effect of the Young's modulus and Poisson ratio was simulated. Because these two parameters are coupled we performed series of simulation for various combinations. The Young's modulus was changed from 10GPa to 100GPa. The Poisson ratio was varied from 0.1 to 0.3. The surface RMS as a function of these two parameters was calculated. The total surface RMS(f_4) and the surface RMS after removing power(g_4) can be expressed as a function of the Young's modulus and Poisson ratio as below.

$$f_4(E,\nu) = -0.0036 + 1.0065\left(\frac{10\ \mathrm{GPa}}{E}\right) + 0.0037\left(\frac{\nu}{0.1}\right) - 0.000015\left(\frac{10\ \mathrm{GPa}}{E}\right)\left(\frac{\nu}{0.1}\right) \quad (7)$$
$$g_4(E,\nu) = -0.0053 + 0.9914\left(\frac{10\ \mathrm{GPa}}{E}\right) + 0.0056\left(\frac{\nu}{0.1}\right) + 0.0005\left(\frac{10\ \mathrm{GPa}}{E}\right)^2 - 0.0013\left(\frac{\nu}{0.1}\right)^2 - 0.0115\left(\frac{10\ \mathrm{GPa}}{E}\right)\left(\frac{\nu}{0.1}\right) \quad (8)$$

We get the 8 empirical equations (4: original surface RMS, 4: surface RMS after power removed) for the surface RMS due to the mirror deflection of a flat mirror with three axial supports.

4. Parametric model for mirror deflection

The surface RMS for an arbitrary set of parameters will be expressed as

$$\text{Total surface RMS: } w(\alpha, D, \rho, E, \nu) = f_1(\alpha)\cdot f_2(D)\cdot f_3(\rho)\cdot f_4(E,\nu)\cdot w_0 \quad (9)$$
$$\text{Surface RMS after power removed: } \mathrm{X}(\alpha, D, \rho, E, \nu) = g_1(\alpha)\cdot g_2(D)\cdot g_3(\rho)\cdot g_4(E,\nu)\cdot \mathrm{X}_0 \quad (10)$$

,where $w_0(= 5.160\mu m)$ and $\mathrm{X}_0(= 0.481\mu m)$ is the reference point for the surface RMS when $f_1(\alpha = 10) = f_2(D = 1\,m) = f_3(\rho = 1000) = f_4(E = 10\mathrm{GPa}, \nu = 0.1) = g_1(\alpha = 10) = g_2(D = 1\,m) = g_3(\rho = 1000) = g_4(E = 10\mathrm{GPa}, \nu = 0.1) \cong 1$.

Equation (9) and (10) assume that the total surface RMS is a product of the four *f*-functions. This assumption is valid, at least for the first order estimation. However, these functions may need correction in complicate non-linear fashion, so that regression analysis may result in better parametric model. This will be studied in other papers in the future.

We performed 17 case studies for various sets of 5 parameters (Aspect ratio, Mirror diameter, Density, Young's modulus, and Poisson ratio) to verify the parametric model, equation (9) and (10). The simulation sets and results are shown in Table 2.

Table 2. Various set of simulations and results verifying the parametric model

Case #	Material	Parameters					FEA model		Nelson model		Parametric model			
		D	α	ρ	E	ν	RMS(μm)		RMS (μm)	Relative error to FEA	RMS(μm)		Relative error to FEA	
		m	N/A	kg/m^3	GPa	N/A	total	power removed	total	total	total	power removed	total	power removed
1	ULE	2	3	2210	67	0.17	0.081	0.081	0.069	15%	0.081	0.074	1%	7.7%
2	Fused silica	2	3	2203	72	0.16	0.075	0.075	0.065	14%	0.075	0.069	1%	7.6%
3	Borosilicate	2	3	2230	63	0.2	0.088	0.087	0.074	16%	0.086	0.080	2%	8.7%
4	Zerodur	2	30	2530	91	0.24	4.429	4.156	4.392	1%	4.255	4.013	4%	3.4%
5	ULE	2	30	2210	67	0.17	5.288	4.902	5.370	-2%	5.196	4.787	2%	2.3%
6	Fused silica	2	30	2203	72	0.16	4.908	4.542	4.998	-2%	4.823	4.443	2%	2.2%
7	Borosilicate	2	30	2230	63	0.2	5.661	5.276	5.696	-1%	5.546	5.126	2%	2.8%
8	ULE	2	10	2210	67	0.17	0.674	0.652	0.611	9%	0.655	0.612	3%	6.1%
9	Fused silica	2	10	2203	72	0.16	0.625	0.604	0.568	9%	0.608	0.568	3%	5.9%
10	Borosilicate	2	10	2230	63	0.2	0.724	0.702	0.648	10%	0.699	0.655	3%	6.7%
11	ULE	1.5	3	2210	67	0.17	0.046	0.046	0.033	28%	0.045	0.042	2%	8.2%
12	Fused silica	1.5	3	2203	72	0.16	0.043	0.042	0.031	28%	0.042	0.039	1%	7.9%
13	Zerodur	1.5	30	2530	91	0.24	2.427	2.305	2.466	-2%	2.393	2.257	1%	2.1%
14	ULE	1.5	30	2210	67	0.17	2.897	2.719	3.014	-4%	2.923	2.693	-1%	1.0%
15	Fused silica	1.5	30	2203	72	0.16	2.689	2.519	2.806	-4%	2.713	2.499	-1%	0.8%
16	Borosilicate	1.5	30	2230	63	0.2	3.102	2.926	3.198	-3%	3.120	2.883	-1%	1.5%
17	Beryllium	1.5	30	1844	303	0.07	0.537	0.494	0.570	-6%	0.519	0.470	3%	4.8%

The difference between the Nelson model and the FEA model shows more than 25% errors, especially for the small aspect ratio cases such as case 11 and 12. On the other hand, the difference between the parametric model and FEA model shows up to 5% errors in all ranges. Also, not like the Nelson model, we were able to estimate the surface RMS after the power is removed. In this case the difference error was up to 8%, which is still pretty good for the first order estimation. (Fig. 1.)

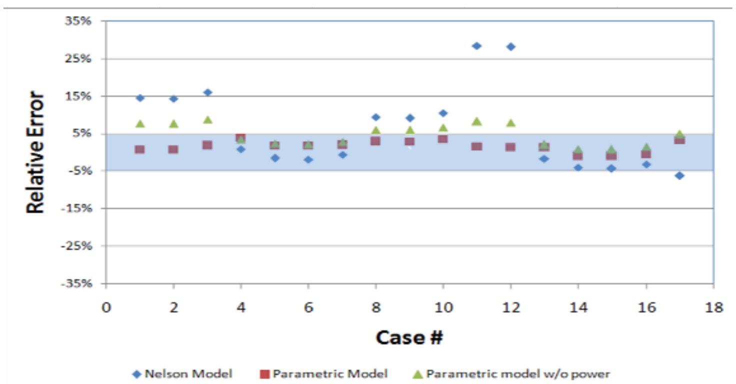

Fig. 1. Relative error comparison between Nelson's model and parametric models. (Shade indicates ±5% error)

5. Conclusion

In this study, we verified the effectiveness of the parametric model to estimate the surface RMS due to the mirror deflection. Nelson model, which is an analytic solution, was well worked for the large aspect ratio mirror cases. However, for the small aspect ratio cases, we needed to perform FEA which takes significant efforts and time. The parametric model based on the 4 empirical equations was derived from the FEA simulations. We can effectively estimate the surface RMS ('total' and 'after power removed') due to the mirror deflection within 8% accuracy using the parametric model.

6. References

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