Structure function analysis of mirror fabrication and support errors

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ABSTRACT

Telescopes are ultimately limited by atmospheric turbulence, which is commonly characterized by a structure function. The telescope optics will not further degrade the performance if their errors are small compared to the atmospheric effects. Any further improvement to the mirrors is not economical since there is no increased benefit to performance. Typically the telescope specification is written in terms of an image size or encircled energy and is derived from the best seeing that is expected at the site. Ideally, the fabrication and support errors should never exceed atmospheric turbulence *at any spatial scale*, so it is instructive to look at how these errors affect the structure function of the telescope. The fabrication and support errors are most naturally described by Zernike polynomials or by bending modes for the active mirrors. This paper illustrates an efficient technique for relating this modal analysis to wavefront structure functions. Data is provided for efficient calculation of structure function given coefficients for Zernike annular polynomials. An example of this procedure for the Giant Magellan Telescope primary mirror is described.

Keywords: Annular Zernike Polynomials, Structure Functions, Atmospheric Turbulence

1. INTRODUCTION

The image quality of a distant star is a function of the atmosphere, telescope optics and instrumentation. The goal is to fabricate and support the telescope mirrors so that the performance degradation due to the optics matches the best atmosphere you are statistically likely to see. Any further improvement to the mirrors adds no benefit to performance and is not economical. Therefore, telescope performance is usually limited by atmospheric turbulence. Atmospheric turbulence causes wavefront phase errors on many spatial scales and this is best characterized by a structure function. Structure functions describe the average variance over all pairs of points in the pupil of a given separation and will be defined in the next section. Often, the telescope specifications chosen for the fabrication, support and alignment errors are in terms of a point spread function (PSF), spot size, encircled energy, or modulation transfer function (MTF), but structure functions may also be used. Structure functions were first used as manufacturing specifications for the William Herschel Telescope (WHT) polished by Grubb-Parsons in the 1980s.[1] Other mirrors with polishing or figuring specifications on several spatial scales include the Large Binocular Telescope (LBT),[2] the Giant Magellan Telescope (GMT)[3] and the Discovery Channel Telescope (DCT).[4]

The correlation length, or Fried parameter, is used to describe the strength of atmospheric turbulence and is a single parameter which defines the structure function for atmospheric turbulence.[5] Previously, conversions have been made between the Fried Parameter and structure functions[5] as well as between the Fried parameter and Zernike polynomials.[6] Dai and Mahajan recently wrote a paper discussing imaging through atmospheric turbulence by systems with annular pupils using the Zernike annular polynomials.[7] They found the residual phase structure functions when a certain number of modes are corrected from Kolmogorov turbulence using a deformable mirror. However, this paper does not include a way to convert from a general set of annular Zernike coefficients[8] to a structure function. Telescope polishing and support errors are normally measured using interferometers which output Zernike polynomials. In order to compare these errors to atmospheric errors, there exists a need for an efficient conversion from Zernike polynomials to structure functions, as shown in the following figure.

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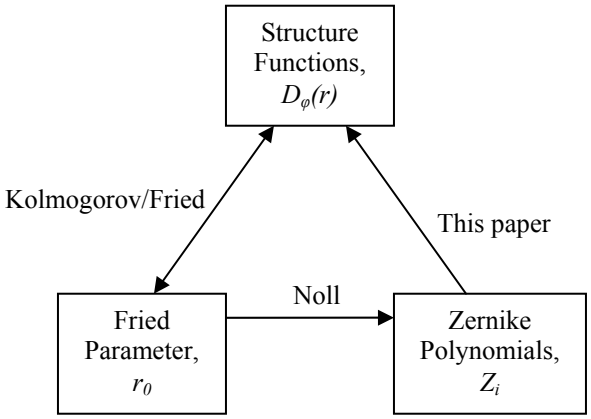

Fig. 1. This paper provides an efficient tool for converting a surface described by Zernike coefficients into a structure function.

1.1 Structure functions

Structure functions statistically define phase differences for pairs of points at different spatial separations. The structure function $D_\phi(r)$ for different spatial separations r is defined as

$$D_\phi(r) = \left\langle [\phi(x+r) - \phi(x)]^2 \right\rangle_x, \tag{1}$$

where φ is the phase at a position x or $x + r$. The angled brackets denote an average over x, which are all points in the pupil. The units of the structure function are waves^2.

Equation 1 can also be expanded into

$$D_\phi(r) = 2\left[\left\langle \phi^2(x) \right\rangle_x - \left\langle \phi(x)\phi(x+r) \right\rangle_x \right], \tag{2}$$

which shows that the structure function is related to the autocorrelation of the phase function. Since autocorrelation and power spectral density (PSD) are related by Fourier Transform, one can think of the structure function as including the same information as a PSD, but in a different form.

For Kolmogorov turbulence, $D_\phi(r)$ can be written as

$$D_\phi(r) = \left(\frac{\lambda}{2\pi} \right)^2 6.88 \left(\frac{r}{r_0} \right)^{5/3}. \tag{3}$$

r_0 is the correlation length introduced by Fried which depends on wavelength and scales to 6/5 power. (If no wavelength is specified 0.5 μm might be assumed.) Typical values are on the order of centimeters and larger values describe better seeing and sharper telescope images. The following graph shows the structure function for Kolmogorov turbulence for two different values of r_0 (0.1 and 0.2) and demonstrates how smaller values of r_0 lead to larger structure functions values.

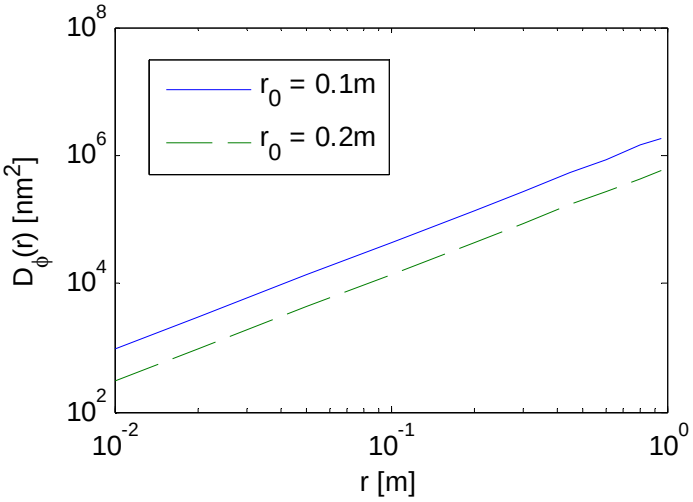

Fig. 2. The structure function for Kolmogorov turbulence for two different values of r_0 (0.1m and 0.2m) for a wavelength of 500nm. Structure functions are normally plotted in log-log space to better see differences at a variety of spatial scales.

1.2 Zernike polynomials

Zernike polynomials are a convenient set of orthonormal basis functions for a unit circle.[8] They are frequently used in optics to describe wavefront errors because many optical elements are circular. However when telescopes have a central obscuration, they are better described by annular Zernike polynomials which are correctly normalized over the annular pupil.[9] Telescope mirrors may have errors from many sources, including bending modes due to changing mirror orientations with respect to gravity or figuring errors from fabrication. Bending modes may be determined using finite element analysis and once those are known, they can be decomposed into Zernike coefficients. During fabrication, figuring errors can be found using interferometers that output surface errors in terms of Zernike coefficients. Active supports are often used to correct for mirror bending modes or low order figuring errors. It is then possible to calculate the leftover shape errors in terms of Zernike polynomials and using the results of this paper, determine the structure function of the telescope.

There are different ways of normalizing and ordering the Zernike coefficients. In this paper, we use the Zernike polynomials used by Noll[6], which are called the Zernike standard polynomials and Zernike annular polynomials in Zemax.[9] Using this definition, the magnitude of the coefficient of a term is equal to the RMS contribution of that term. Annular Zernike polynomials are defined similarly to the standard circular Zernike polynomials, however they have an added dependence on the obscuration ratio, ε, the ratio of the inner radius of the annulus to the outer radius of the annulus which varies between 0 and 1. Circular Zernike polynomials are a special case of the annular Zernike polynomials, where ε = 0. Since the structure function is the average variance over all pairs of points of a given separation in the pupil, a rotation of the surface does not change the structure function. All of the non-axially symmetric Zernike terms come in pairs which only differ by a rotation and hence can be described by one structure function.

Finally, as a note of caution, some interferometer software programs may give you Zernike coefficients for data over an annular region, but these may not actually be properly normalized annular Zernike coefficients. If you use the results of this paper to convert a Zernike polynomial into a structure function, you should be careful to use the same Zernike ordering and normalization as used here.

2. STRUCTURE FUNCTIONS OF THE ZERNIKE POLYNOMIALS

2.1 Algorithm

MATLAB[10] was used to numerically calculate the structure function for the annular Zernike polynomials and the algorithm is as follows:

- The Zernike polynomial surface was created using a matrix of points (201 x 201).

- For each point in the pupil a second point was found a distance *r* away and a determination was made if this point also lies in the pupil.
    - This was repeated for a number of different angles (36)
- The squared difference of the phase was summed for all pairs of points *that both lay in the pupil* a distance *r* apart. The average was found by dividing by the total number of pairs.
- This was repeated for all of the different spatial separations *r* desired to create the structure function $D_\phi(r)$
- This procedure was repeated for each Zernike polynomial (Z = 2...28) and each obscuration ratio (ε = 0, 0.2, 0.4 and 0.6)

2.2 Example and discussion

This algorithm is best understood by using an example. First the Zernike polynomial describing astigmatism with an obscuration ratio of $\varepsilon = 0.4$ was created as matrix of points. Each point of the matrix inside the pupil corresponds to the positions of *x* in Equation 1 and $\varphi(x)$ is the value of the Zernike polynomial at that point. For every point *x* in the pupil, a large number of points (~36) a distance *r* away are found at different angles. The value of the Zernike polynomial at these points is $\varphi(x + r)$. For simplicity, in the figure below, only seven points separated by a distance *r* from the first point are shown.

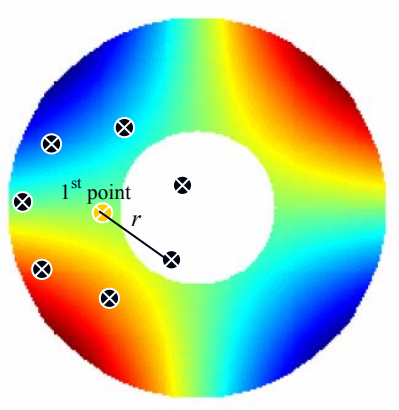

Fig. 3. An example of finding the structure function for the Zernike astigmatism term for an obscuration ratio of ε = 0.4. For the 1st point falling on the surface as shown, seven 2nd points are found.

There are a number of different techniques one could choose to use if the second point falls outside of the pupil. The method chosen should depend on the operation of the mirror. For the case here, we just ignored the second point. In the example above, five of the seven points are in the pupil and are used in the structure function calculation and the other two points are discarded.

This process of finding the average variance for all pairs of points is repeated for all of the different spatial separations *r*. In this paper, the structure functions are found for 10 different values of r between 0.01 and 1, where 1 is the normalized ~~diameter~~ of the pupil. As the value of *r* increases toward 1, the chance of the second point also falling in the pupil becomes increasingly smaller. For larger separations, fewer point pairs are used to describe the structure function so the value of the structure function is not statistically significant. Also, as will be shown in the next section, the structure functions for large spatial separations have some ringing near the edge where $r = 1$. Power spectral density (PSD) plots also have a similar problem which is often addressed by using windowing or filtering.

2.3 Results

The following figure shows the resulting structure functions for all of the annular Zernike polynomials and obscuration ratios listed in the appendix. For convenience, these numbers will be also available online at http://www.optics.arizona.edu/loft/Publications/StructureFunctions.xls. The first Zernike polynomial represents a piston error, which has a structure function of zero since there are no differences in value between any of the points, so it is not included here. For an individual Zernike coefficient, the structure function does not change very quickly with the value

of the obscuration ratio. Therefore, it is reasonable to interpolate the structure functions shown to obtain other obscuration ratios, if desired.

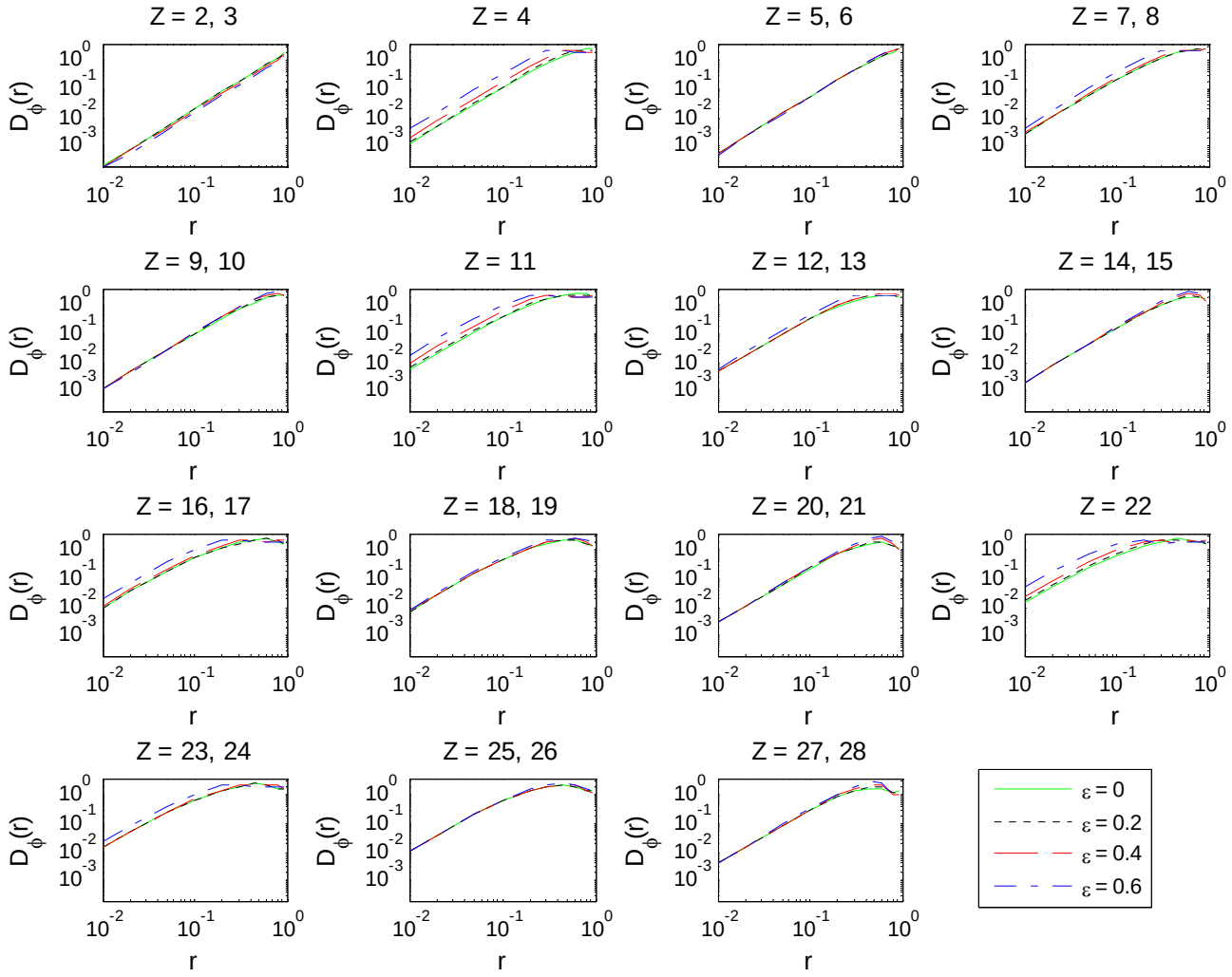

Fig. 4. Structure functions for obscuration ratios of 0, 0.2, 0.4 and 0.6 for annular Zernike polynomials 2-28. (The structure function for Zernike polynomial 1 (piston) is zero for all values of r.)

2.4 Finding the total structure function for a surface described by Zernike polynomials

To find the total structure function, one must add up the scaled individual structure functions for each of the Zernike polynomials that describe the surface. To scale the structure function, multiply by the squared value of the coefficient describing the Zernike term (which is the rms value of the term). For example, if the scaling coefficient is c, then the total phase (assuming one Zernike term) is $\phi_{total}(x) = c\phi(x)$ and the total structure function can be found as follows:

$$D_{\phi_{total}}(r) = \left\langle \left[c\phi(x+r) - c\phi(x)\right]^2 \right\rangle_x = \left\langle c^2\left[\phi(x+r) - \phi(x)\right]^2 \right\rangle_x = c^2 D_\phi(r). \quad (4)$$

Therefore, the structure function will have the units of c^2 which may be waves^2 or nm^2 or other distance unit squared. Now, instead assume the phase is composed of two different Zernike terms $\phi(x) = \phi_1(x) + \phi_2(x)$ and expand the structure function definition as follows:

$$D_\phi(r) = \left\langle \left[\phi(x+r) - \phi(x)\right]^2 \right\rangle_x = \left\langle \left[\left(\phi_1(x+r) + \phi_2(x+r)\right) - \left(\phi_1(x) + \phi_2(x)\right)\right]^2 \right\rangle_x. \quad (5)$$

Collect the like Zernike polynomial terms together to find:

$$D_\phi(r) = \left\langle \left[\left(\phi_1(x+r) - \phi_1(x)\right) + \left(\phi_2(x+r) - \phi_2(x)\right) \right]^2 \right\rangle_x . \tag{6}$$

Expand the square and separate each term into its own average over the pupil to find:

$$D_\phi(r) = \left\langle \left[\left(\phi_1(x+r) - \phi_1(x)\right) \right]^2 \right\rangle_x + 2\left\langle \left[\left(\phi_1(x+r) - \phi_1(x)\right) \right]\left[\left(\phi_2(x+r) - \phi_2(x)\right) \right] \right\rangle_x + \left\langle \left[\left(\phi_2(x+r) - \phi_2(x)\right) \right]^2 \right\rangle_x . \tag{7}$$

The middle term in the above equation is equal to zero because all of the terms in its expansion cancel:

$$\left\langle \phi_1(x+r)\phi_2(x+r) \right\rangle_x - \left\langle \phi_1(x)\phi_2(x+r) \right\rangle_x - \left\langle \phi_1(x+r)\phi_2(x) \right\rangle_x + \left\langle \phi_1(x)\phi_2(x) \right\rangle_x = 0 . \tag{8}$$

Therefore the total structure function is the sum of the individual structure functions of each of the Zernike terms:

$$D_\phi(r) = D_{\phi_1}(r) + D_{\phi_2}(r) . \tag{9}$$

In summary, if the total phase is comprised of individual terms, each scaled by some factor, as in the following equation:

$$\phi(x) = c_1\phi_1(x) + c_2\phi_2(x) + c_3\phi_3(x) + \ldots , \tag{10}$$

then the total structure function may be found as the sum of the individual structure functions, each scaled by the square of the coefficient as shown in the following equation:

$$D_\phi(r) = c_1^2 D_{\phi_1}(r) + c_2^2 D_{\phi_2}(r) + c_2^2 D_{\phi_2}(r) + \ldots . \tag{11}$$

3. GIANT MAGELLAN TELESCOPE EXAMPLE

This section describes an example using an 8.4m segment of the Giant Magellan Telescope.[3] During the analysis of the optical test, Zernike coefficients were found which describe the expected errors due to misalignments of the test optics. To find the total structure function, each individual structure function was scaled by the squared value of the Zernike coefficient. It is convenient to think of the phase errors in terms of difference in surface height (units of distance) during manufacturing. Therefore, the square root of the structure function was found. Finally, since this example involves surface errors for a mirror, the structure function needs to be multiplied by 2 because phase errors are doubled upon reflection. The calculation for this example is shown in the appendix. The following figure shows the structure function for the optical test errors along with the structure function for the mirror specification.

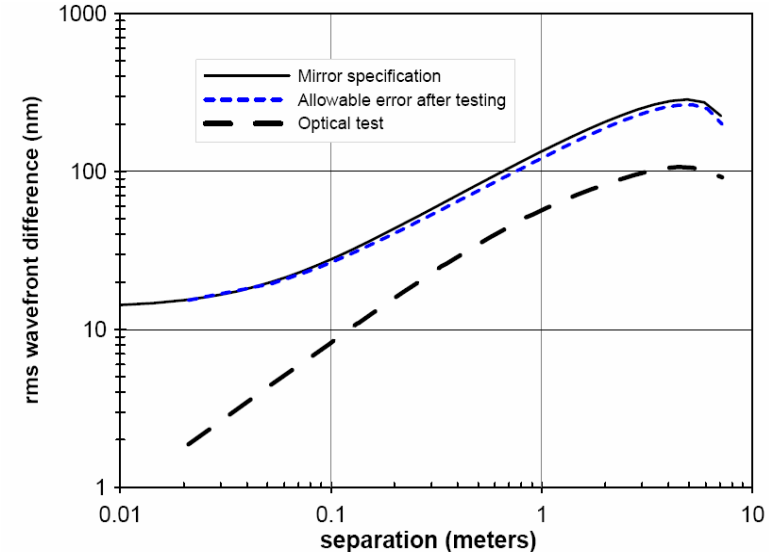

Fig. 5. Structure function analysis for a GMT segment shows the expected optical test errors and the mirror specification structure function.[3]

The mirror specification was derived from an atmospheric structure function with r_0 = 92 cm (large compared to the typical atmospheric coherence length) and has two modifications, shown by the following equation:

$$Mirror\ Specification = \sqrt{D(r)} = \sqrt{\left(\frac{\lambda}{2\pi}\right)^2 6.88\left(\frac{r}{r_0}\right)^{\frac{5}{3}}\left[1 - 0.975\left(\frac{r}{D}\right)^{\frac{1}{3}}\right] + 2\sigma^2}\ . \quad (12)$$

In Equation 12, the wavelength is λ = 500 nm, the Fried parameter is r_0 = 92 cm (at a 500 nm wavelength) and the mirror diameter is D = 8.4 m. The first modification is a term to reduce the allowed error on large scales and corresponds to the removal of the average wavefront tilt across the aperture of diameter D, as in the case of a telescope with rapid active guiding. The second modification is the $2\sigma^2$ term, which is a relaxation at small spatial separations for scattering losses. σ may be found from the equation for scattering losses of 1.5%:

$$L = 1 - e^{-(2\pi\sigma/\lambda)^2} = 1.5\%\ . \quad (13)$$

The curve labeled "Allowable error after testing" comes from subtracting in quadrature the optical test from the mirror specification curve. This example shows that the optical test errors are small compared to the mirror figuring specification, so there is still room in the budget for errors on the telescope mirror itself.

4. CONCLUSION

This paper offers an efficient way to convert Zernike polynomials into structure functions to be able to compare a mirror surface to atmospheric turbulence at all spatial scales. The technique used to calculate the structure function is described and the results are provided in the appendix and online.

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5. APPENDIX

5.1 Structure functions tables

Table 1. Structure Functions for ε = 0 (Standard circular Zernike polynomials)

separation / Zernike	0.01	0.02	0.05	0.1	0.2	0.3	0.45	0.6	0.8	0.95
2, 3	0.0002	0.0008	0.0051	0.0203	0.0811	0.1825	0.4106	0.7300	1.2977	1.8301
4	0.0012	0.0047	0.0284	0.1077	0.3843	0.7669	1.4230	2.0482	2.6497	2.8417
5, 6	0.0006	0.0024	0.0145	0.0563	0.2106	0.4421	0.8911	1.4066	2.0974	2.5478
7, 8	0.0027	0.0106	0.0617	0.2190	0.6881	1.2143	1.8976	2.3521	2.5923	2.5147
9, 10	0.0012	0.0047	0.0281	0.1055	0.3696	0.7248	1.3098	1.8303	2.2510	2.2995
11	0.0057	0.0220	0.1202	0.3863	1.0176	1.5711	2.2062	2.6336	2.6358	2.1011
12, 13	0.0048	0.0185	0.1038	0.3455	0.9649	1.5365	2.1169	2.3561	2.2117	1.8740
14, 15	0.0020	0.0077	0.0454	0.1651	0.5426	0.9956	1.6117	1.9765	1.9675	1.7081
16, 17	0.0096	0.0361	0.1856	0.5441	1.2524	1.8151	2.4337	2.5898	2.0366	1.6108
18, 19	0.0074	0.0282	0.1521	0.4770	1.2015	1.7613	2.1937	2.1905	1.7541	1.4825
20, 21	0.0029	0.0114	0.0661	0.2327	0.7189	1.2346	1.7821	1.8969	1.5458	1.3211
22	0.0153	0.0557	0.2644	0.6946	1.4384	2.0484	2.5174	2.3262	1.9105	1.7803
23, 24	0.0142	0.0520	0.2529	0.6832	1.4345	2.0166	2.4917	2.2689	1.6242	1.6145
25, 26	0.0105	0.0396	0.2054	0.6085	1.3958	1.9067	2.1503	1.9046	1.4153	1.4971
27, 28	0.0041	0.0158	0.0896	0.3063	0.8896	1.4278	1.8199	1.6689	1.2409	1.3592
29, 30	0.0218	0.0771	0.3394	0.8161	1.6124	2.2237	2.4129	2.0941	1.7724	1.9963
31, 32	0.0194	0.0697	0.3200	0.8038	1.5882	2.1683	2.3703	1.8712	1.4874	1.8811
33, 34	0.0141	0.0522	0.261	0.7323	1.5455	1.9825	2.0142	1.5918	1.3283	1.7796
35, 36	0.0054	0.0208	0.1158	0.3839	1.0466	1.5620	1.7405	1.3934	1.1843	1.6658
37	0.0307	0.1047	0.4213	0.9332	1.8088	2.3344	2.3048	1.8950	2.0117	2.3825

Table 2. Structure Functions for ε = 0.2

separation / Zernike	0.01	0.02	0.05	0.1	0.2	0.3	0.45	0.6	0.8	0.95
2, 3	0.0002	0.0008	0.0049	0.0198	0.0791	0.1779	0.4002	0.7115	1.2650	1.7840
4	0.0013	0.0053	0.0323	0.1233	0.4460	0.8995	1.6576	2.2736	2.3508	2.3725
5, 6	0.0006	0.0024	0.0146	0.0571	0.2172	0.4634	0.9452	1.4891	2.1605	2.5398
7, 8	0.0027	0.0104	0.0608	0.2161	0.6800	1.2000	1.8888	2.4134	2.7144	2.5936
9, 10	0.0012	0.0047	0.0284	0.1073	0.3826	0.7641	1.4054	1.9712	2.3200	2.2173
11	0.0065	0.0248	0.1364	0.4409	1.1704	1.7926	2.3249	2.3882	2.4032	2.4088
12, 13	0.0048	0.0186	0.1042	0.3491	0.9847	1.5772	2.1582	2.3954	2.3351	1.9073
14, 15	0.0020	0.0077	0.0458	0.1678	0.5620	1.0511	1.7365	2.1478	2.0013	1.5471
16, 17	0.0098	0.0365	0.1879	0.5490	1.2513	1.8015	2.4539	2.6439	2.2012	1.7041
18, 19	0.0074	0.0283	0.1533	0.4848	1.2415	1.8473	2.2975	2.2406	1.8060	1.4780
20, 21	0.0029	0.0114	0.0666	0.2366	0.7449	1.3049	1.9248	2.076	1.5386	1.1179
22	0.0173	0.0629	0.3000	0.7947	1.6503	2.2214	2.246	2.0247	2.4011	1.7549
23, 24	0.0142	0.0521	0.2539	0.6879	1.4426	2.0209	2.5265	2.3803	1.7615	1.5837
25, 26	0.0106	0.0397	0.2071	0.6189	1.4460	2.0106	2.2843	1.9552	1.3940	1.4688
27, 28	0.0041	0.0158	0.0904	0.3115	0.9225	1.5085	1.9675	1.8303	1.2059	1.1581

Table 3. Structure Functions for ε = 0.4

separation / Zernike	0.01	0.02	0.05	0.1	0.2	0.3	0.45	0.6	0.8	0.95
2, 3	0.0002	0.0007	0.0044	0.0177	0.0709	0.1595	0.3588	0.6379	1.1342	1.5991
4	0.0020	0.0078	0.0474	0.1816	0.6616	1.3308	2.3442	2.4288	2.0790	1.9598
5, 6	0.0006	0.0023	0.0144	0.0569	0.2205	0.4805	1.0230	1.6793	2.4490	2.7114
7, 8	0.0029	0.0115	0.0675	0.2422	0.7784	1.4064	2.2441	2.4218	2.4561	2.7073
9, 10	0.0012	0.0047	0.0286	0.1097	0.4029	0.8327	1.6308	2.3825	2.7276	2.3687
11	0.0094	0.0361	0.1969	0.6281	1.5903	2.2256	2.3001	2.0871	2.0804	2.1481
12, 13	0.0049	0.0188	0.1060	0.3561	1.0098	1.6295	2.3253	2.6004	2.5434	2.1720
14, 15	0.0020	0.0078	0.0464	0.1725	0.5979	1.1651	2.0774	2.6587	2.3050	1.5267
16, 17	0.0118	0.0442	0.2283	0.6719	1.5499	2.1669	2.3416	2.1186	2.3049	2.3383
18, 19	0.0074	0.0284	0.1546	0.4928	1.2754	1.9099	2.4719	2.6229	1.9958	1.4385
20, 21	0.0029	0.0115	0.0675	0.2436	0.7946	1.4549	2.3373	2.5801	1.6385	0.9750
22	0.0250	0.0906	0.4251	1.0923	2.0541	2.2915	1.8137	2.2579	2.1175	2.2626
23, 24	0.0152	0.0559	0.2723	0.7372	1.5493	2.1641	2.4516	2.2704	2.1892	1.7957
25, 26	0.0106	0.0399	0.2095	0.6334	1.5113	2.1325	2.4871	2.2969	1.3991	1.3049
27, 28	0.0041	0.0159	0.0916	0.3207	0.9847	1.6854	2.4076	2.2459	1.1334	1.0145

Table 4. Structure Functions for ε = 0.6

separation / Zernike	0.01	0.02	0.05	0.1	0.2	0.3	0.45	0.6	0.8	0.95
2, 3	0.0002	0.0006	0.0038	0.0151	0.0605	0.1361	0.3062	0.5444	0.9677	1.3640
4	0.0040	0.0157	0.0956	0.3646	1.2885	2.3494	2.2833	2.1202	2.0532	2.0253
5, 6	0.0005	0.0022	0.0136	0.0539	0.2122	0.4700	1.0225	1.7048	2.6520	3.2281
7, 8	0.0046	0.0182	0.1077	0.3932	1.2950	2.2728	2.2636	2.1566	2.1566	2.1992
9, 10	0.0012	0.0046	0.0283	0.1101	0.4188	0.8952	1.7943	2.6609	3.3079	3.1439
11	0.0189	0.0713	0.3755	1.1101	2.2457	2.2696	2.0592	2.0458	2.0368	2.0409
12, 13	0.0061	0.0237	0.1349	0.4609	1.3571	2.2319	2.2680	2.2726	2.3470	2.3333
14, 15	0.0020	0.0077	0.0469	0.1786	0.6513	1.3374	2.4553	3.1624	2.8820	1.9058
16, 17	0.0207	0.0773	0.3927	1.1161	2.2224	2.2815	2.0292	2.0131	2.0217	2.0516
18, 19	0.0083	0.0316	0.1731	0.5563	1.4699	2.2700	2.4674	2.5123	2.3325	1.9107
20, 21	0.0029	0.0116	0.0692	0.2567	0.8921	1.7482	2.8825	3.0843	1.8688	0.8927
22	0.0494	0.1741	0.7523	1.6735	2.2676	1.7834	2.1091	2.0535	2.0465	2.0497
23, 24	0.0230	0.0844	0.4102	1.1068	2.1803	2.3288	2.0897	2.0823	2.0999	2.1110
25, 26	0.0111	0.0419	0.2213	0.6736	1.6194	2.3424	2.5918	2.4458	1.7593	1.3326
27, 28	0.0041	0.0160	0.0939	0.3389	1.1171	2.0708	2.9940	2.5572	1.0472	0.9326

5.2 Sample structure function calculation from GMT example

Table 5. Calculation of expected optical test errors (dotted black line in Figure 5). The bold numbers are used in the plot.

	separation	0.01	0.02	0.05	0.1	0.2	0.3	0.45	0.6	0.8	0.95
	~~8.4m~~ * separation	**~~0.084~~**	**~~0.168~~**	**~~0.42~~**	**~~0.84~~**	**~~1.68~~**	**~~2.52~~**	**~~3.78~~**	**~~5.04~~**	**~~6.72~~**	**~~7.98~~**
Zernike term	Zernike coefficients	Structure functions (for ε = 0), scaled by the square of the coefficients									
1	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0	0	0
4	0.05	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.01
5	0.12	0.00	0.00	0.00	0.00	0.00	0.01	0.01	0.02	0.03	0.04
6	0.79	0.00	0.00	0.01	0.04	0.13	0.28	0.56	0.88	1.31	1.59
7	7.03	0.13	0.52	3.05	10.81	33.97	59.95	93.68	116.12	127.97	124.14
8	0.93	0.00	0.01	0.05	0.19	0.59	1.05	1.64	2.03	2.24	2.17
9	0.33	0.00	0.00	0.00	0.01	0.04	0.08	0.14	0.20	0.24	0.25
10	0.28	0.00	0.00	0.00	0.01	0.03	0.06	0.10	0.14	0.18	0.18
11	5.57	0.18	0.68	3.72	11.97	31.53	48.67	68.35	81.59	81.66	65.09
12	5.63	0.15	0.59	3.29	10.97	30.62	48.76	67.18	74.78	70.19	59.47
13	0.84	0.00	0.01	0.07	0.24	0.68	1.09	1.50	1.67	1.56	1.33
14	0.19	0.00	0.00	0.00	0.01	0.02	0.03	0.06	0.07	0.07	0.06
15	0.07	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.01	0.01	0.01
16	3.65	0.13	0.48	2.48	7.26	16.72	24.23	32.48	34.57	27.18	21.50
17	25.45	6.22	23.4	120.2	352.3	810.9	1175	1576	1677	1319	1043
18	4.04	0.12	0.46	2.49	7.80	19.65	28.81	35.88	35.83	28.69	24.25
19	4.70	0.16	0.62	3.36	10.53	26.53	38.89	48.43	48.36	38.73	32.73
20	0.06	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.01	0.00	0.00
21	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
22	10.48	1.68	6.11	29.0	76.2	157.9	224.9	276.3	255.3	209.7	195.4
23	2.04	0.06	0.22	1.05	2.85	5.98	8.40	10.38	9.45	6.77	6.73
24	13.62	2.63	9.65	46.9	126.8	266.2	374.2	462.3	421.0	301.4	299.6
25	0.44	0.00	0.01	0.04	0.12	0.27	0.37	0.42	0.37	0.28	0.29
26	0.92	0.01	0.03	0.17	0.51	1.18	1.61	1.81	1.60	1.19	1.26
27	0.45	0.00	0.00	0.02	0.06	0.18	0.28	0.36	0.33	0.25	0.27
28	0.26	0.00	0.00	0.01	0.02	0.06	0.09	0.12	0.11	0.08	0.09
29	4.67	0.47	1.68	7.39	17.8	35.1	48.4	52.5	45.6	38.6	43.5
30	0.68	0.01	0.04	0.16	0.38	0.75	1.04	1.13	0.98	0.83	0.93
31	0.59	0.01	0.02	0.11	0.28	0.56	0.76	0.84	0.66	0.52	0.66
32	0.56	0.01	0.02	0.10	0.25	0.50	0.68	0.75	0.59	0.47	0.59
33	0.15	0.00	0.00	0.01	0.02	0.04	0.05	0.05	0.04	0.03	0.04
34	0.17	0.00	0.00	0.01	0.02	0.05	0.06	0.06	0.05	0.04	0.05
35	0.09	0.00	0.00	0.00	0.00	0.01	0.01	0.02	0.01	0.01	0.01
36	0.15	0.00	0.00	0.00	0.01	0.02	0.04	0.04	0.03	0.03	0.04
37	2.23	0.15	0.52	2.09	4.64	8.98	11.59	11.45	9.41	9.99	11.83
	structure function D(r)	12.13	45.07	225.81	642.07	1449.2	2099.6	2744.4	2818.7	2268.9	1937.0
	2*sqrt(D(r))	**7.0**	**13.4**	**30.1**	**50.7**	**76.1**	**91.6**	**104.8**	**106.2**	**95.3**	**88.0**