Optimized active, lightweight space mirrors

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ABSTRACT

Since 1996, a team at the University of Arizona has been designing and fabricating lightweight, active space mirrors. These glass/composite mirrors use a thin flexible substrate for the optical surface and an actuated composite structure for support. We present a design method that yields the best figure correction for the lightest mass by assuming that the substrate's material properties are the limiting parameters. The results are such that the designer decides on a total mass budget and an aperture area, and the algorithm provides the substrate thickness, number of support points, and the mass distribution between the substrate and actuators.

Keywords: Lightweight, space, design, active mirrors, optimized, scaling laws, mirror support

1. INTRODUCTION

Building space mirrors requires that they be lightweight, stowable, and durable. To date, several designs are currently in use. The Hubble Space Telescope (HST) uses a conventional glass mirror that has been aggressively lightweighted into a sandwich geometry. The mirror was figured in the optics shop, and it depends on its structural stability to maintain the surface figure. The HST primary is an example of a passive mirror: the surface figure cannot be changed or manipulated. By contrast, future space telescopes will employ an active mirror design. Active mirrors use a thin, flexible substrate for the reflective surface and an array of position or force actuators to maintain the surface accuracy. An example of an active mirror is the University of Arizona design, shown in Figure 1.

The most important fabrication specifications for space mirrors (excluding cost) are mass and surface quality. For the current generation of space telescopes, the largest portion of the mass budget is taken up by the primary mirror. As a result, there is great interest in making high quality, durable, and lightweight space mirrors.

The mass and surface quality of an active mirror are controlled by two key fabrication parameters: the substrate thickness and the number of support points. For example, the substrate can always be made more stiff by increasing the thickness or the number of support points, but there's a trade-off between the two parameters.

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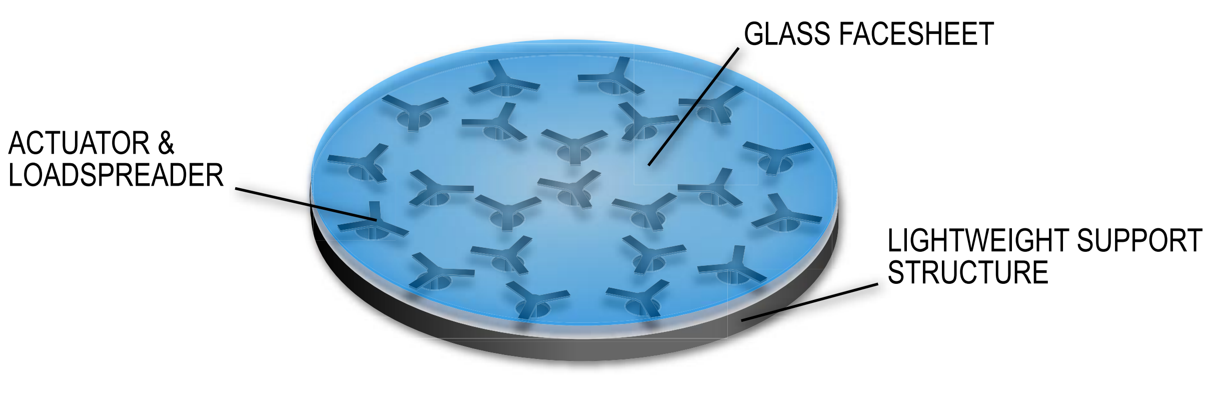

Figure 1. The University of Arizona lightweight space mirror design.

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The membrane can be made thinner (and less massive), but the number of actuators must be increased to compensate for the reduction in substrate stiffness.

In this paper, we derive a relationship between the substrate thickness and the number of actuators that results in the most accurate lightweight mirror. Given a target mass budget for the entire mirror, we have derived the optimum fabrication parameters (membrane thickness, membrane mass, and number of actuators) to produce the lightest, most accurate mirror. The results are based on the assumption that the fundamental limiting factor in surface quality are the CTE and temperature changes across the material. In an effort to concentrate on the overall results, we only briefly describe the mathematics in the body of the paper. All of the derivations are included in the appendices.

2. SUPPORTING A MEMBRANE WITH A DISCRETE NUMBER OF POINTS

Unlike conventional mirrors that derive their stiffness from their thickness, the surface quality for a thin*, active mirror isn't determined by the structural geometry of the membrane. Instead, the design uses an array of force or position actuators to correct for any localized figure errors. These figure errors can be from several sources: self-weight deflection (gravity), temperature gradients across the material, or fabrication (polishing) errors. Whatever the cause, all of these sources cause a strain in the membrane.

For the purposes of this analysis, we will assume that errors in the membrane are caused either by discrete temperature differences (hot and cold spots on the membrane) or patches with different CTEs combined with a temperature change. These effects cause errors in the membrane at various spatial frequencies, and we can fix the errors that are larger in scale than the actuator spacing.

Figure 2 shows what happens locally when one of these effects causes an error in the membrane. If a region on the membrane expands, it pushes against the area around it and a "blister" forms. Of course, if this occurs over several actuator lengths, we can use them to remove this error from the surface figure.

In order to fix the blisters, the actuators must exert a force on the membrane. Because the membrane is supported by a discrete number of points, the surface will consist of local bumps (or holes) over every actuator. Obviously, these bumps will affect the surface quality of the membrane, and we can quantify this effect by using a relationship that Nelson developed[1] that describes the rms surface error of a plate that is supported by N points:

$$\delta_{\rm rms} = 0.0012 \frac{P}{D} \left(\frac{A}{N} \right)^2 . \tag{1}$$

*We define "thin" as having an aspect ratio of over 100. That is, the diameter is at least 100 times that of the thickness.

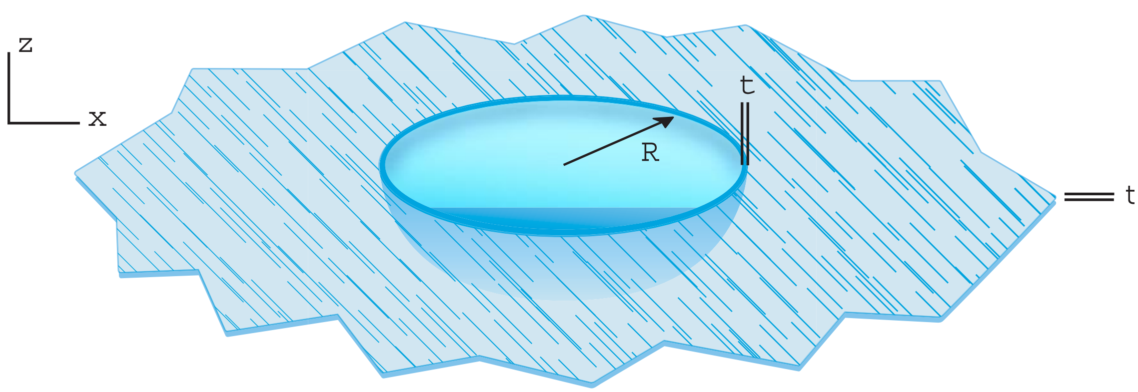

Figure 2. If a region on the membrane expands, it pushes against the area around it and creates a blister.

This relationship is a function of the force per unit area, P, applied by N actuators, and it is the starting point for our derivation. A is the total plate area, and N is the number of support points. D is the modulus of rigidity, and it depends strongly on the geometry of the membrane. This relationship is discussed in more detail in Appendix A, Equation 6. Equation 1 has two key assumptions associated with it:

- The substrate is a thin shell.
- The actuators are arranged in a triangular, periodic pattern. A triangular geometry is more effective at correcting the surface error than a hexagonal, circular, or rectangular geometry.[1]

Equation 1 is difficult to apply to our situation because we usually don't know how much pressure each actuator must exert to fix a blister. To that end, we have derived an expression for P that depends on more appropriate variables. Because we want to focus on the practical conclusions to be drawn from this relationship, we will only show the results here. A detailed derivation is included the Appendix A. Our expression for P is as follows:

$$P = \frac{2t\left[0.36(\frac{12}{t^3}D(1-\nu^2))\right]\Delta(\alpha\, T)}{R},$$

and we can substitute this into Equation 1 to get a new expression for $\delta_{\rm rms}$:

$$\delta_{\rm rms} = \frac{0.01(1-\nu^2)\,\Delta(\alpha\, T)}{R\, t^2}\left(\frac{A}{N}\right)^2. \tag{2}$$

For the sake of clarity, here are the variables that are used in Equation 2:

$\boldsymbol{\nu}$	Poisson's ratio: $-\frac{\epsilon_{\rm transverse}}{\epsilon_{\rm longitudinal}}$, ϵ represents the strain: $\frac{\Delta l}{l}$
$\boldsymbol{\Delta(\alpha\, \mathrm{T})}$	The change in CTE (α) and/or temperature (T) across the material
R	Semi-diameter of the blister
t	Shell thickness
A	Shell area
N	Number of actuators

Equation 2 represents an important conclusion. We now have an expression for RMS surface error that depends on the two material properties responsible for causing the error: temperature (T) and CTE (α) differences. They are grouped together in the parenthesis because it is assumed that these effects will occur concurrently with each another. Also note that, unlike Equation 1, Equation 2 is now in terms of three fabrication parameters: t, A, and N, the thickness, shell area and number of actuators, respectively. Most important, these parameters are all dependent on mass.

A fundamental relationship for thin mirrors is contained within Equation 2. The variables for shell thickness and number of actuators are both in the denominator: more actuators or a thicker shell result in a smaller residual error. Also, there is a direct tradeoff between shell thickness and number of actuators. For example, if the shell thickness decreases by half, then the number of actuators must double to maintain the same surface quality.

Finally, it's worth noting that Equation 2 does not contain Young's modulus, E. (It falls out of the derivation. See Appendix A.) This implies that the designer does not gain anything by choosing a stiffer material: in theory, rubber is just as acceptable as glass or metal, given our initial assumptions. Choosing a stiffer material will require more force from the actuators to remove the blisters, but the membrane will also be less likely to initially distort in the first place. These two effects cancel each other out, and E is not a factor in our algorithm.

3. OPTIMIZING THE SYSTEM FOR THE SMALLEST MASS

Equation 2 yields some insightful information, but it doesn't provide a solution for building an optimized mirror. Practically speaking, the system mass is the biggest driving factor in designing lightweight space mirrors. All three of the fabrication parameters (t, A, and N) depend on mass, so we can optimize Equation 2 to find the optimum fabrication parameters for the smallest mass.

If we express t and N in terms of mass, we can take the derivative – with respect to the substrate mass – of Equation 2, set it equal to zero, and find the mass condition that minimizes δ_{rms}. This procedure is shown in detail in Appendix B. When t and N are expressed in terms of mass, Equation 2 can be written as follows:

$$\delta_{\mathrm{rms}} = \frac{0.01\,(1-\nu^2)\Delta(\alpha\,T)\rho^2 A^2}{R\left(\frac{m_{sub}}{A}\right)^2\left(\frac{m-m_{sub}}{m_{act}}\right)^2}, \tag{3}$$

where m_{sub} is the mass of the substrate, m_{act} is the mass of each actuator, and m is the total mass ($m = m_{sub} + Nm_{act}$). When we take the derivative of Equation 3 and set it equal to zero, we find the following relationship:

$$(4m_{sub} - 2m)(m_{sub} - m) = 0.$$

The relationship is stunningly simple! The solution where $m_{sub} = m$ is trivial; the other solution is the important one. This relationship states that the minimum surface error will occur when the shell mass, m_{sub}, makes up one half of the mass budget. In other words, the optimum correction occurs when

$$\text{substrate mass} \quad = \quad \text{actuator mass.}$$

This result yields an important fabrication relationship for building the lightest mirror with the best surface performance. Here are the basic relationships:

$$\begin{aligned} t &= \frac{m_{sub}}{A\rho} = \frac{Nm_{act}}{A\rho} \\ m_{sub} &= Nm_{act} = \frac{m}{2}. \end{aligned} \tag{4}$$

With these relationships in place, we can outline a procedure for designing a mirror:

1. Determine the mass budget and substrate diameter. This sets the values for the total mass, m, and the area of the substrate, A.
2. Calculate the mass budget for the substrate:
$$m_{sub} = \frac{m}{2}.$$
3. Calculate the substrate thickness:
$$t = \frac{m_{sub}}{A\rho}.$$
4. Use the thickness to determine the total number of actuators from the mass of each actuator: (We assume that the actuator mass is a predetermined constant.)
$$Nm_{act} = tA\rho.$$

Notice that the relationship between the number and mass of each support point is an independent variable. However, mirror designers usually have a working actuator design in mind when they design an active mirror so they can use this value to calculate the number of required support points. As the actuator mass decreases, more actuators can be included in the design.

Finally, it's important to notice that this design scheme does not include the mass of the support structure. The support structure's design depends on the system dynamics so we do not consider it in this analysis. When we discuss the "mass budget," we refer only to the elements that maintain the reflective surface: the thin membrane and the actuators.

4. PRACTICAL EXAMPLE: 2-M MIRROR FOR USE IN GEOSYNCHRONOUS ORBIT

As a practical example, let's put the four step procedure to work by calculating the parameters for a hypothetical two meter mirror for use at geosynchronous orbit. Geosync orbit is useful for Earth-imaging situations because the satellite remains fixed over the same position as the Earth rotates. Let's assume that an areal density of $5\ \frac{\mathrm{kg}}{\mathrm{m}^2}$ is the nominal areal density required for this application.[2] Applying the areal density over a two meter mirror, we find that the mass budget for the glass and the actuators is as follows:

$$\begin{aligned} \text{total mass} &= \text{areal density} \times \text{aperture area} \\ m &= 5\frac{\mathrm{kg}}{\mathrm{m}^2} \times (1\mathrm{m})^2\pi \\ &= 5\pi\ \mathrm{kg} \\ m &\sim 16\ \mathrm{kg}. \end{aligned}$$

Now that we know the target mass and mirror diameter, we can calculate the mass budget for the substrate:

$$\begin{aligned} m_{sub} &= \frac{m}{2} \\ &= \frac{16\ \mathrm{kg}}{2} = 8\ \mathrm{kg}. \end{aligned}$$

Let's assume that we're going to use Corning's ULE[†] as our substrate material. ULE has a density of 2210 $\frac{\mathrm{kg}}{\mathrm{m}^3}$. Using step three, we can calculate the substrate thickness:

$$\begin{aligned} t &= \frac{m_{sub}}{A\rho} \\ &= \frac{8\ \mathrm{kg}}{(\pi\ \mathrm{m}^2)(2210\frac{\mathrm{kg}}{\mathrm{m}^3})} \\ t &= 1.2\ \mathrm{mm}. \end{aligned}$$

Finally, we can use step four to calculate the required number of actuators. If we know *a priori* that we can build 50 gram actuators ($m_{act} = 50$ g), then we can calculate how many support points we will need:

[†]This glass has a very low coefficient of thermal expansion; standard-grade ULE has a CTE of 15 $\frac{\mathrm{ppb}}{^{\circ}\mathrm{C}}$. By comparison, BK-7 has a CTE of 7000 $\frac{\mathrm{ppb}}{^{\circ}\mathrm{C}}$. This glass is usually a good choice for space applications.

$$
\begin{aligned}
t &= \frac{N m_{act}}{A\rho} \\
N &= \frac{tA\rho}{m_{act}} \\
&= \frac{(.0012\ \mathrm{m})(\pi\ \mathrm{m}^2)(2210\ \frac{\mathrm{kg}}{\mathrm{m}^3})}{0.050\ \mathrm{kg}} \\
N &= 167\ \text{actuators.}
\end{aligned}
$$

It's important to note that all of the values obtained via this procedure are reasonable and capable of being fabricated! The University of Arizona has created 50 cm Zerodur substrates less than 1 mm thick.[3] We have designed and built actuators that are less than 40 grams.[4] The numbers generated in this example are certainly possible using existing fabrication methods.

5. CONCLUSION

We presented a set of scaling laws that give the lightest mirror with the best surface. Our derivation shows that the optimum mirror performance occurs when the total actuator mass is equal to the substrate mass. We derived this solution assuming that there are discrete patches on the surface that have different temperature and CTEs, but we have also looked what happens when linear temperature or CTE gradients occur in the material. Our preliminary results show that the same design rules fall out of the derivation, and this work will be discussed in a future article.

Finally, it's worth mentioning that mass, alone, does not determine the outcome of a mirror design. The cost and time spent fabricating a mirror are certainly important components. However, if one had a similar set of equations (cost or time as a function of mass and surface quality), the same procedure could be used to find the optimum solution.

APPENDIX A. PRESSURE EXPRESSION FOR δ_{RMS}

In an ideal situation, once the actuator has been activated, it should be able to remove all of the surface error caused by the strain. In reality, however, a small error remains. In 1982, Nelson presented a relationship that describes that describes the rms surface error of a plate supported by N points:

$$\delta_{\mathrm{rms}} = 0.0012 \frac{P}{D} \left(\frac{A}{N}\right)^2 . \tag{5}$$

This relationship is a function of the force per unit area, P, applied by the actuators. A is the total plate area and N is the number of support points. D is the modulus of rigidity, which is given by this relationship:

$$D = \frac{Et^3}{12(1-\nu^2)}, \tag{6}$$

where t is the shell thickness, ν is Poisson's ratio ($\frac{\epsilon_{\mathrm{trans}}}{\epsilon_{\mathrm{long}}}$), and E is Young's modulus.

The expression for δ_{rms} will be more helpful if P is expressed in terms of something more tangible than the pressure applied by the actuators. The following derivation for P generates an expression that depends on the shell thickness, blister size, and the stress.

Figure 2 shows the blister that results when a local strain exists in a glass shell. The stress, σ, is defined as the force per unit area. The stress is all located in the annular boundary (with area A_{bndry}) around the blister. Thus, the force in the z direction is

$$
\begin{aligned}
F_z &= \sigma A_{\mathrm{bndry}} \\
&= \sigma(2\pi Rt) \\
&= 2\pi\sigma Rt.
\end{aligned}
$$

If we use an actuator to correct this blister, the actuator will apply a pressure over the area A_{blstr}:

$$
\begin{aligned}
F_z &= PA_{\mathrm{blstr}} \\
&= P\pi R^2
\end{aligned}
$$

If we assume that we apply enough pressure such that the system is in static equilibrium, then the two forces shown above are equal:

$$
\begin{aligned}
2\pi\sigma Rt &= P(\pi R^2) \\
P &= \frac{2\sigma t}{R}. \qquad (7)
\end{aligned}
$$

The expression for P in Equation 7 contains the reactive stress, σ. This quantity is not easily measured, but we can replace it with other quantities. Using finite element modeling, we developed an empirical expression for σ:

$$
\sigma = 0.36E\Delta(\alpha T),
$$

where $\Delta(\alpha T)$ represents a change in either α or T across the blister.[‡] We can substitute this equation for σ into Equation 7 to get a new expression for P:

$$
\begin{aligned}
P &= \frac{2\sigma t}{R} \\
&= \frac{2t(0.36E\Delta(\alpha T))}{R} \qquad (8)
\end{aligned}
$$

Notice that Equation 8 now expresses pressure P as a function of the two possible sources of error: changes in temperature, patches of different CTEs, or both.

Finally, we can rewrite Equation 6 to create an expression for Young's modulus, E:

$$
E = \frac{12}{t^3}D(1-\nu^2),
$$

and we can substitute this into 8:

$$
\begin{aligned}
P &= \frac{2t(0.36E\Delta(\alpha T))}{R} \qquad (9) \\
&= \frac{2t\left[0.36(\frac{12}{t^3}D(1-\nu^2))\right]\Delta(\alpha T)}{R}. \qquad (10)
\end{aligned}
$$

[‡]This calculation applies to thin shells, where $\frac{\text{diameter}}{\text{thickness}} \sim 10^4$.

Now that we have a new expression for P, we can substitute this into Equation 5, the starting pointing in this derivation:

$$\begin{aligned}
\delta_{\mathrm{rms}} &= 0.0012\,\frac{P}{D}\left(\frac{A}{N}\right)^2 \\
&= 0.0012\,\frac{\left(\frac{2t(0.36(\frac{12}{t^3}D(1-\nu^2)))\Delta(\alpha\,T)}{R}\right)}{D}\left(\frac{A}{N}\right)^2 \\
\delta_{\mathrm{rms}} &= \frac{0.01(1-\nu^2)\,\Delta(\alpha\,T)}{R\,t^2}\left(\frac{A}{N}\right)^2. \qquad (11)
\end{aligned}$$

Equation 11 now contains a tangible fabrication parameter that depends on mass: the substrate thickness t.

APPENDIX B. FINDING A MINIMUM OF δ_{RMS} BY TAKING A DERIVATIVE.

The system mass is the biggest driving factor in designing lightweight mirrors for use in space. All three of the fabrication parameters $(t, A, \text{and } N)$ depend on mass, so we can optimize Equation 2 to find the optimum fabrication parameters for the smallest mass. To do this, we first need to express t and N in terms of mass.

First, let's express the shell thickness, t, in terms of m_{sub}, the mass of the shell. We can do this by starting with two well-known relationships:

$$\begin{aligned}
V &= A\,t \\
\rho &= \frac{m_{sub}}{V},
\end{aligned}$$

where V is the volume of the shell and ρ is the density. If we combine these two relationships, we can get an expression for the thickness in terms of the mass of the shell:

$$t^2 = \left(\frac{m_{sub}}{A}\right)^2\frac{1}{\rho^2}. \qquad (12)$$

This result is squared because the thickness term in Equation 2 is squared.

Now, let's express the number of actuators in terms of the shell mass. The total mass is equal to the following expression

$$\begin{aligned}
\text{total mass} &= \#\text{ actuators (mass at each support point)} + \text{shell mass} \\
m &= N m_{act} + m_{sub},
\end{aligned}$$

where m_{act} is the actuator mass. Solving this equation for N yields

$$N = \frac{m - m_{sub}}{m_{act}}. \qquad (13)$$

Finally, we can substitute Equations 12 and 13 into Equation 11:

$$\delta_{\mathrm{rms}} = \frac{0.01\,(1-\nu^2)\Delta(\alpha\,T)\rho^2A^2}{R\left(\frac{m_{sub}}{A}\right)^2\left(\frac{m-m_{sub}}{m_{act}}\right)^2}. \qquad (14)$$

Now that we have the residual surface rms error expressed in terms of mass, we can take a derivative with respect to m_{sub}, set it equal to zero, and find the condition that minimizes the mass.

The dependent variable in Equation 14 is m_{sub}. It will be easier to take the derivative if we just concentrate on the parts that include m_{sub}. With this in mind, let's rewrite Equation 14 without all of the constants and then take the derivative and set it equal to zero :

$$\begin{aligned}
\delta_{\text{rms}} &\propto \frac{1}{m_{sub}^2 (m - m_{sub})^2} \\
&\propto \frac{1}{m_{sub}^2 (m^2 - 2m\, m_{sub} + m_{sub}^2)} \\
&\propto \frac{1}{m_{sub}^2\, m^2 - 2m\, m_{sub}^3 + m_{sub}^4)} \\
\frac{d\delta_{\text{rms}}}{dm_{sub}} &\propto \frac{0 - (2m_{sub}\, m^2 - 6m\, m_{sub}^2 + 4m_{sub}^3)}{(m_{sub}^2\, m^2 - 2m\, m_{sub}^3 + m_{sub}^4)^2} \\
0 &= 4m_{sub}^3 - 6m\, m_{sub}^2 - 2m^2\, m_{sub} \\
(4m_{sub} - 2m)(m_{sub} - m) &= 0
\end{aligned}$$

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