Application of geometric dimensioning & tolerancing for sharp corner and tangent contact lens seats

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ABSTRACT

This paper outlines methods for dimensioning and tolerancing lens seats that mate with spherical lens surfaces. The two types of seats investigated are sharp corner and tangent contact. The goal is to be able to identify which seat dimensions influence lens tilt and displacement and develop a quantifiable way to assign tolerances to those dimensions to meet tilt and displacement requirements. After looking at individual seats, methods are then applied to multiple lenses with examples. All geometric dimensioning and tolerancing is according to ASME Y14.5M – 1994.

Keywords: lens mounts, lens seats, geometric dimensioning & tolerancing, ASME Y14.5

1. INTRODUCTION

Geometric dimension and tolerancing (GD&T) is a very powerful language for assigning tolerances to features on a part. It allows for control of a feature's form, size, orientation, and position. Like any language, there can be multiple ways to describe a part's features using GD&T and still get similar results. This paper looks at ways GD&T can be used to describe those dimensions and tolerances important for locating and orientating lenses within a housing. Specifically of interest is mounting one or more lenses with a spherical surface using either a sharp corner or tangent contact seat. It is important to first investigate the seat dimensions that influence tilt and displacement and understand their relationship for a single lens. This is at the core of establishing design intent. Using that information, it's possible to select appropriate GD&T callouts to express that intent. Next, these concepts will be applied to two lens systems with suggestions on configuring systems with three or more lenses.

It should be noted that familiarity with ASME Y14.5M -1994 is assumed. Also, this paper does not explore the effects of form errors. Form is controlled within the context of size, orientation, and position tolerances. Its effects are assumed to be small compared to the other three types of errors. When sizing dimensions, thermal effects due to possible differences in the coefficient of thermal expansion (CTE) of materials used are not considered as are methods of lens retention. Both are important considerations for designing an actual system, but beyond the scope of this paper. Also, while statistical tolerancing methods could have been applied, worse case tolerancing is used. Additionally, all units are in millimeters unless otherwise specified.

2. PERTENANT GD&T INFORMATION

In addition to the general principles laid out it ASME Y14.5M – 1994, there are several key GD&T concepts discussed in this paper the reader should be familiar with. They are:

- Least Material Condition (LMC)
- Zero Positional Tolerance
- Datum Reference Frames
- Datum Shift
- Tapers
- Composite Positional Tolerancing

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Least Material Condition

Material condition modifiers modify the size of a feature's tolerance zone based on the material condition (size) of the feature. The most common material condition modifier used is maximum material condition (MMC). MMC ensures interchangeability between parts while taking full advantage of allowable tolerances. Conceptually similar, but seldom used is the least material condition (LMC) modifier. Typically it ensures a minimum wall thickness or clearance between features. When a feature is produced at its LMC condition (i.e. - largest hole or smallest boss) then it's allowed whatever positional tolerance is called out in its feature control frame. As the size of the feature departs from its LMC condition, then bonus tolerance is applied to its position tolerance on a one-to-one basis (one millimeter of bonus tolerance for one millimeter change in diameter). It's also a very effective way to control the total amount of axial misalignment between features.

Zero Positional Tolerance

Instead of allowing a finite positional tolerance when a feature is at its MMC or LMC, the zero positional tolerance concept states that at either of those conditions (as specified in the feature control frame) the allowable position tolerance is zero. As the feature departs from its specified material condition then bonus tolerance is added to position tolerance on a one-to-one basis. This concept is useful because it allows manufacturing the most flexibility on how tolerances are allocated while still meeting the function of the part.

Datum Reference Frames

A datum reference frame (DRF) is defined by specifying certain features on a part in a certain order. The features used to establish a DRF are called datum features. Different DRFs can be specified using the same datum features but called out in a different order. A DRF is akin to a kinematic mount in that enough datums are specified so that the part becomes kinematically constrained and three mutually orthogonal reference datums are established.

Datum Shift

If a feature of size, such as a hole or boss, is used to establish a DRF, then other features referenced to that DRF can shift as a group by the amount the datum feature of size departs from its specified material condition (either MMC or LMC).

Tapers

Tapers are discussed because of their use in the tangent contact lens seat. ASME Y14.5M – 1994 presents several ways to specify taper. Typical American Standard machine tapers are not useful because of the angle and application of the taper. They are usually meant to mate with another tapered surface and have a large contact area for either driving shaft rotation or controlling coaxially. For the tangent contact lens seat, the tapered surface is making contact with a spherical surface along circular line of contact (see Figure 3). Two other methods of specifying taper are the conical taper symbol or a basic angle as shown in Figure 1.

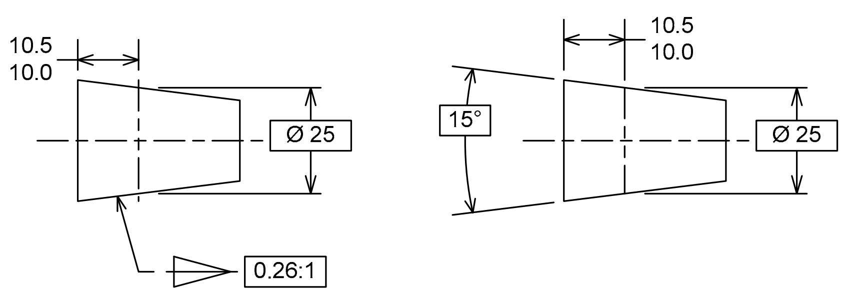

Figure 1. Two equivalent methods for specifying taper. Use of the conical taper symbol is shown on the left and a basic angle is shown on the right.

Taper is defined as change in diameter over a unit length:

$$taper = \frac{d_1 - d_2}{L}$$

Both methods for defining taper shown in Figure 1 are equivalent. The tolerance zone in either case is .066 mm as shown in Figure 2. Note that in defining the taper a basic diameter of 25 mm was used. This is the gauge diameter. The left surface is then located from the gauge diameter with a toleranced dimension.

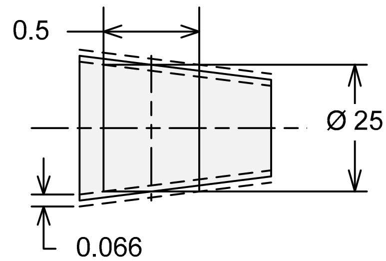

Figure 2. A radial tolerance zone of 66 µm is shown based off the specifications of Figure 1.

Composite Positional Tolerancing

Composite tolerancing allows different tolerances to be applied to the location of a group of features as a whole and to the relative position of each other within the group.

3. DIMENSIONING & TOLERANCING OF INDIVIDUAL LENS MOUNTS

There are three common lens seat configurations in use for convex surfaces:

- Sharp corner
- Tangent Contact
- Spherical Contact (not considered)

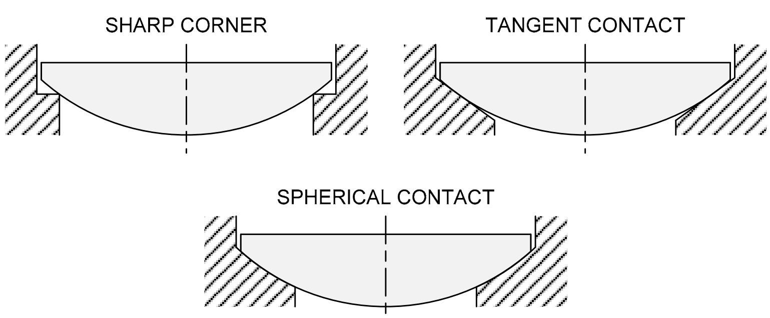

Figure 3. Three types of seat configurations shown.

The sharp corner seat is the easiest to perform a tolerance analysis on and manufacture, however of the three results in the highest stress on the lens. The spherical contact results in the lowest stress, but according to Vukobratovich [2] they are impossible to tolerance and difficult to fabricate. The tangent contact is a happy medium. It results in lower stresses than the sharp corner and is only slightly more difficult to tolerance. This paper will look at methods for dimensioning and tolerancing the sharp corner and tangent contact mounts. Both make a well defined circular line of contact with the convex surface of the lens. Because of this, many of the equations developed for the sharp corner seat also apply to the tangent contact.

For convex lens surfaces, the sag equation is used to determine the distance from the lens vertex to where it makes contact with the seat.

$$z(r) = R - \sqrt{R^2 - r^2} \tag{1}$$

3.1 The Sharp Corner Seat

When a lens is placed into the barrel, one of two situations can occur that determine the order of contact. This is also true for the tangent contact seat.

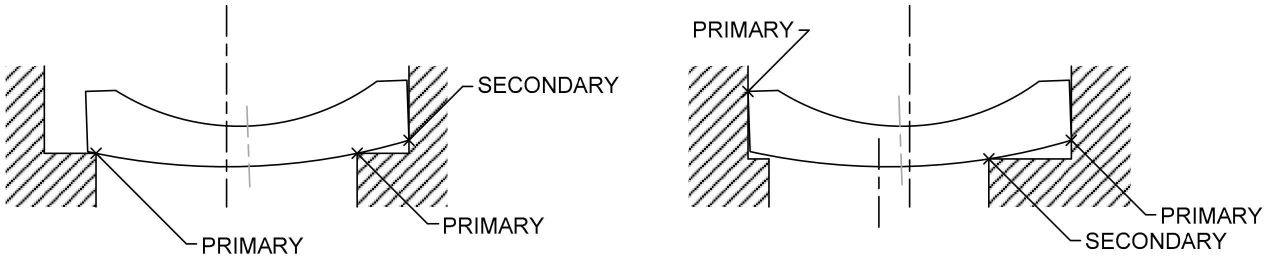

Figure 4. Order of constraint comparison of a lens element mounted on a sharp corner seat. On the left, the lens element is constrained primarily by the seat diameter then by the bore diameter. On the right, the bore diameter provides the primary constraint and the seat secondary contact.

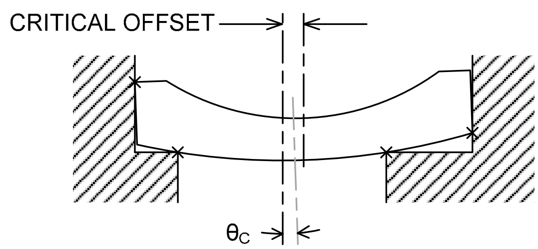

Figure 5. The transition between the two conditions shown in Figure 2. Shown are axial offset and angle at which the lens element is making full contact with the seat and two point contact with the bore.

The critical angle (θ_C) is the angle where the convex surface of the lens is fully seated and both edges just make contact with the side walls of the bore diameter. It is given by equation (2). The associated critical offset of the lens vertex from the axis of the seat diameter is found by equation (3).

$$\theta_C = \tan^{-1}\left(\frac{t_{edge}}{d_{lens}}\right) - \cos^{-1}\left(\frac{d_{bore}}{\sqrt{d_{lens}^2 + t_{edge}^2}}\right) \tag{2}$$

$$\text{CRITICAL OFFSET} = \left(\frac{d_{lens}}{2} + \tan\theta \cdot \sqrt{R_{lens}^2 - \left(\frac{d_{lens}}{2}\right)^2}\right) \cdot \cos\theta - \frac{d_{bore}}{2} \tag{3}$$

Note that equation (3) does not depend on the diameter of the seat. Let's look at an example of a biconvex lens were the critical offset is calculated.

Table 1. Example lens specifications used to calculate critical offset

Specification	**Value**
Lens Diameter (d_{lens})	25.4 mm
Edge Thickness (t_{edge})	2 mm
Lens Radius (R_{lens})	102.4 mm
Bore Diameter (d_{bore})	25.45 mm

From these values, the critical angle is 1.79 degrees and the critical offset is 3.14 mm. The bore is oversized by only 50 µm and yet the axis of the seat diameter can be offset from the bore diameter by up to 3.14 mm and the lens will still locate first on the seat. As the edge thickness of the lens increases, the critical offset decreases, however even with an edge thickness of 10 mm the critical offset is 0.48 mm. This is easily within standard machining tolerances.

Why is this important? It means that in locating the lens vertex from a reference surface only the location of the seat along the optical axis and the diameter of the seat matter. If instead the bore made primary contact, then the axial position of the seat would also have to be taken into account.

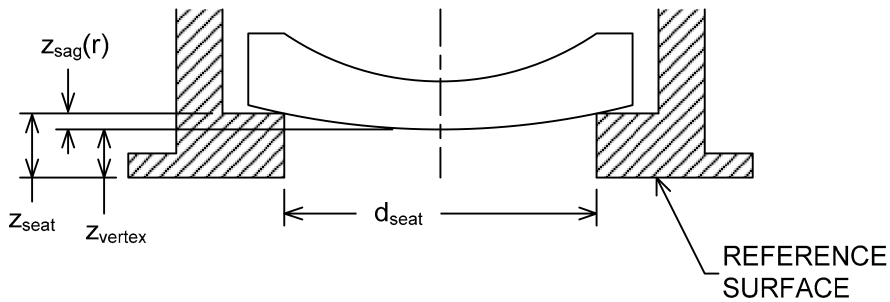

Figure 6. Setup for calculating the distance between the reference surface and lens vertex.

$$z_{vertex} = z_{seat} - z_{sag}(r) \tag{4}$$

$$z_{vertex} = z_{seat} - R_{lens} + \sqrt{R_{lens}^2 - \left(\frac{d_{seat}}{2}\right)^2} \tag{5}$$

If a chamfer is used in place of a sharp corner for the seat, then the chamfer diameter would be substituted for d_{seat} in equation (5).

The size of the bore diameter relative to the seat diameter will determine the axial displacement and tilt of the lens element as show in the left image of Figure 4. The lens element rotates about the center of curvature of the surface that contacts the seat. The amount of rotation is determined by the diameter of the bore, the radius of the lens surface, and the outer diameter of the lens element.

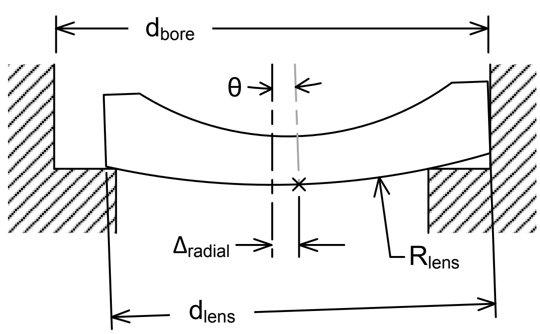

Figure 7. Setup for calculating lens element tilt and displacement.

$$\theta = \sin^{-1}\left(\frac{d_{bore}}{2R_{lens}}\right) - \sin^{-1}\left(\frac{d_{lens}}{2R_{lens}}\right) \tag{6}$$

Equation (6) is valid up to the critical angle calculated in Equation (2). The radial displacement of the lens element for small angles is approximately:

$$\Delta_{\text{radial}} = \theta \cdot R_{lens} \tag{7}$$

$$\Delta_{radial} = \left[\sin^{-1}\left(\frac{d_{bore}}{2R_{lens}}\right) - \sin^{-1}\left(\frac{d_{lens}}{2R_{lens}}\right)\right] \cdot R_{lens} \tag{8}$$

Notice that nowhere is the angle and radial displacement of the lens element dependent on the diameter of the seat or the lens edge thickness. Also notice that for a single lens mount, tilt and radial displacement are tied to each other. This means that if separate requirements are given for both tilt and displacement, then one of the requirements will have excess margin.

Up to this point, it has been assumed that the bore diameter is perfectly aligned coaxially to the seat diameter. As the radial offset of the bore diameter increases (up to the critical offset), it has the result of increasing the effective bore diameter used in equations (6) and (8). This effective bore diameter is the LMC virtual condition of the bore. Both size and position are combined into a single tolerance value. When combined with the zero positional tolerance concept, we arrive at the following shown in Figure 8.

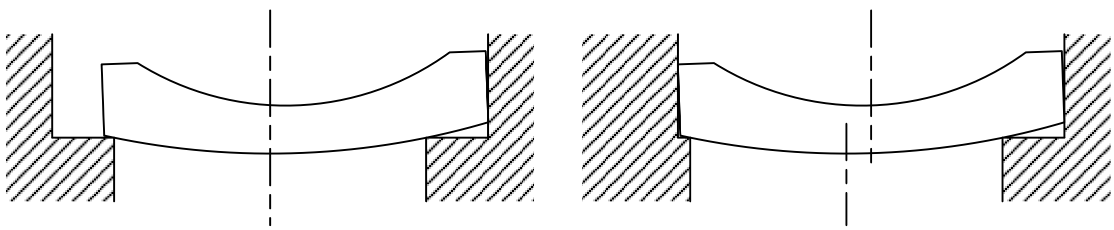

Figure 8. Position comparison showing the lens bore at LMC (left) and MMC (right) when using the LMC modifier with zero positional tolerance .

On the left is the condition where the bore is at its maximum allowable size (least material condition) and therefore is allowed zero positional tolerance from the seat diameter. On the right, the bore diameter is at its minimum size (maximum material condition) and is allowed to shift up to the critical offset found in equation (3). Typically, the allowable tilt is specified from which it's possible to calculate the maximum and minimum bore diameters. Solving equation (6) for d_{bore} results in the maximum bore diameter.

$$d_{max} = 2R_{lens} \cdot \sin\left(\theta + \sin^{-1}\left(\frac{d_{lens}}{2R}\right)\right) \tag{9}$$

Solving equation (2) for d_{bore} results in the minimum bore diameter.

$$d_{min} = \sqrt{d_{lens}^2 + t_{edge}^2} \cdot \cos\left(\tan^{-1}\left(\frac{t_{edge}}{d_{lens}}\right) - \theta\right) \tag{10}$$

Let's revisit the biconvex lens used in the previous example to calculate what the bore diameter and tolerance should be given a tilt and displacement requirement of 0.05 deg and 100 μm.

Table 2. Calculation results

Parameter	**Value**
Maximum bore dia.	25.557 mm
Minimum bore dia.	25.402 mm
Displacement due to tilt of 0.05 deg	90 μm

In this case, the angular requirement results in a radial displacement of 90 μm which leaves 10 μm of margin from the requirement. Figure 9 shows how to implement this concept using GD&T. The top image is just the bore diameter related to the seat diameter. The bottom image shows how to then relate the seat diameter to the housing's mating features.

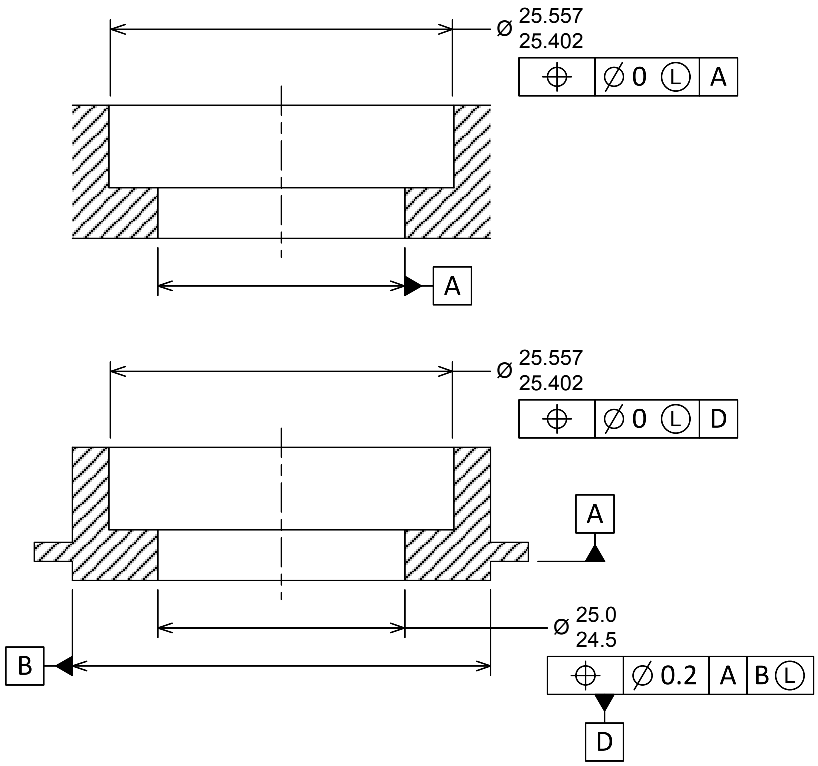

Figure 9. GD&T applied to the sharp corner seat based on the values calculated in Table 2.

Note that in the bottom image the positional tolerance is applied regardless of feature size. This is done because the size of the seat diameter does not affect the radial position of the lens according to equation (8). A datum modifier was used for datum B because we are interested in the total amount of axial misalignment between this part and it's mating part.

Toroidal Contact Lens Seat

Rarely in practice is a true sharp corner produced. More likely it will have either a chamfer or a small radius. If it's an intentional radius, then it is referred to toroidal lens seat. The axial vertex distance as measured from the seat's flat is calculated using equation (11). For an unintended radius, the vertex shift from the sharp corner seat is found using equation (12) (Burge [3]).

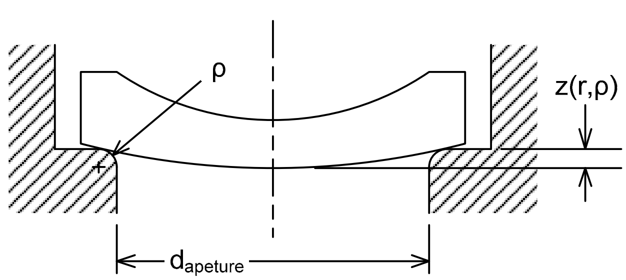

Figure 10. Layout of the toroidal lens seat

$$z = R_{lens} + \rho - \sqrt{(R_{lens} + \rho)^2 - \left(\frac{d_{apeture}}{2} + \rho\right)^2} \quad (11)$$

$$\Delta z \approx \frac{d_{seat}}{2R_{lens}} \cdot \rho \quad (12)$$

3.2 Tangent Contact

For positioning a lens element, the tangent contact seat is similar to the sharp edge seat. Both make circular line contact and therefore all the equations developed for the sharp edge seat also apply to the tangent contact seat.

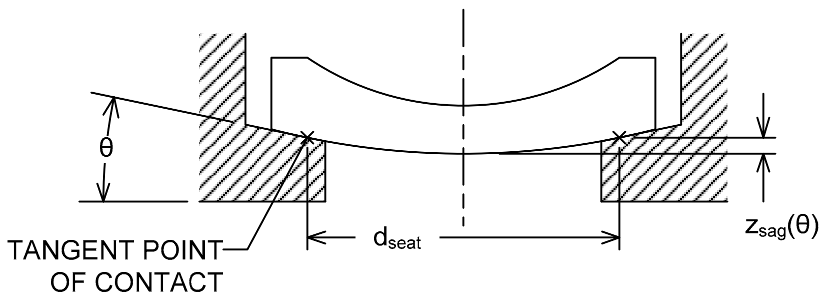

Figure 11. Setup for calculating seat diameter of the tangent contact seat

The diameter of the lens seat is determined by the angle of the seat's taper. Here I've departed from convention for specifying the taper angle which is usually measure from the axis of rotation. I'm specifying the angle from the plane perpendicular from the axis of rotation to make the equations simpler.

$$d_{seat}(\theta) = 2R_{lens} \sin\theta \quad (13)$$

Sag can be calculated from the tangent angle by:

$$z(\theta) = R_{lens} \cdot (1 - \cos\theta) \quad (14)$$

Next, we would like to calculate the distance from the lens surface vertex to some reference surface. This is not as straight forward a calculation as used for the sharp edge seat. An important concept to keep in mind about tapers is that size and position are the same thing.

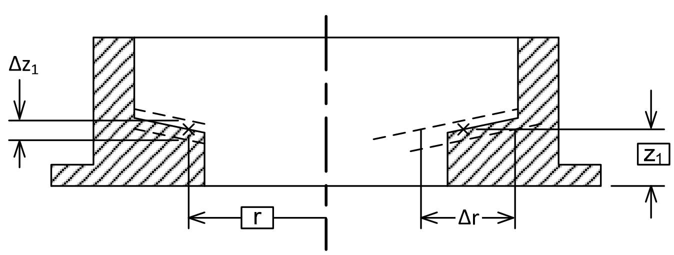

Figure 12. Two methods for specifying taper. On the left-half of the figure, the radial distance (r) is basic and height (z_1) is toleranced. On the right-half, the height (z_1) is basic while the radius (r) is toleranced.

As long as the taper angle and lens radius doesn't change, seat diameter remains constant. It makes sense to tolerance z_1 and keep the seat diameter a basic dimension. This also makes for easy tolerance analysis. The nominal seat diameter found from the nominal design taper will be referred to as the gauge diameter (d_{gauge}).

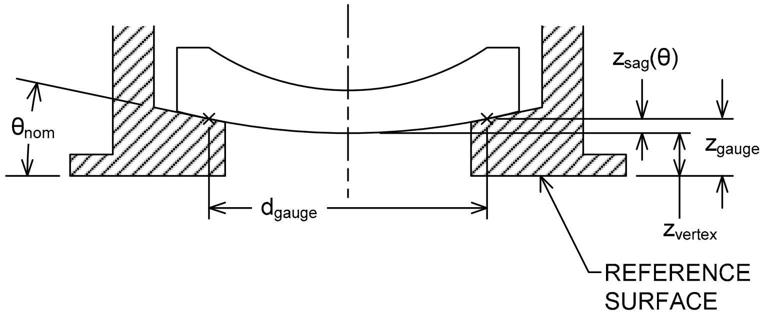

Figure 13. Setup for calculating the distance between the reference surface and lens vertex.

The nominal distance from the vertex to the reference surface is computed the same as in equation (4). The change in vertex position due to a change in taper angle with respect to the reference surface is:

$$\Delta z_{vertex}(\theta) = R_{lens} \cdot [(\sin\theta - \sin\theta_{nom})\tan\theta + \cos\theta - \cos\theta_{nom}] \tag{15}$$

As the taper angle increases, so does the seat diameter and $z_{sag}(\theta)$, however this is offset somewhat because we are still measuring from the reference gauge diameter. The total change (worse case) in vertex position is then:

$$\Delta z_{vertex} = \Delta z_{gauge} + \Delta z_{vertex}(\theta) \tag{16}$$

Using the biconvex lens again as an example, if the gauge diameter is set at 24.5 mm then θ_{nom} is 6.87 deg. For a 1.0 degree angular tolerance, the vertex position shifts axially with respect to the reference surface by only 16 µm. Perhaps a more important reason to control angle is to avoid the situations shown in Figure 14.

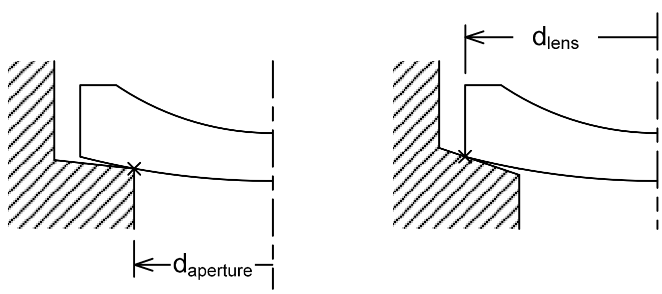

Figure 14. Situation that can occur if the seat diameter is either less than the aperture diameter (left) or greater than the lens diameter (right)

$$\theta_{min} = \sin^{-1}\left(\frac{d_{aperture}}{2R_{lens}}\right) \tag{17}$$

$$\theta_{max} = \sin^{-1}\left(\frac{d_{lens}}{2R_{lens}}\right) \tag{18}$$

If we assign a clear aperture to the biconvex lens example of 22 mm, then θ_{min} is 6.12 deg. Using the 25.4 O.D. of the lens, θ_{max} is 7.12 deg. The angle must me controlled to within 1 deg to avoid losing the tangent contact. The next step is to calculate the width of the tolerance zone to necessary to maintain 1 deg using equation (19).

$$t \approx \left(\frac{d_{lens} - d_{apeture}}{2}\right) \cdot \frac{\tan(\theta_{max} - \theta_{min})}{\cos\theta_{nom}} \tag{19}$$

The width of the tolerance zone (t) is 30 µm. If there was an axial position tolerance of 0.5 mm for the location of the lens vertex (z_{vertex}), then that tolerance alone wouldn't be enough to control the taper angle to within 1 degree.

Next, we will calculate the size of the tolerance zone to limit radial shift to the 90 µm displacement specified in table 2 using equation (20). This results in a 11 µm wide tolerance zone which is definitely the most restrictive tolerance so far. Figure 15 illustrates what's going on. For shallow angles, the taper can easily slide left and right. The taper surfaces shown on the right image resemble wobble if rotated about the bore diameter.

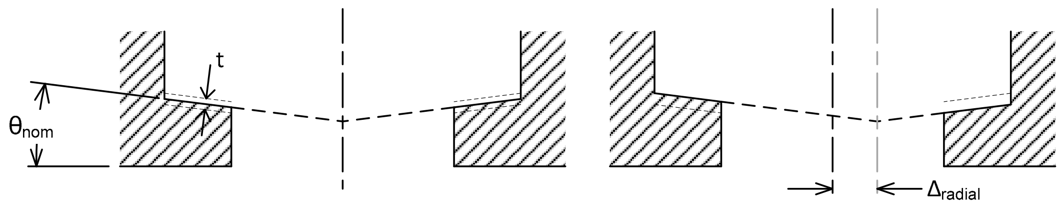

Figure 15. The effects of tolerance zone size on radial shift on the tangent contact seat

$$t = \Delta_{radial} \sin \theta_{nom} \tag{20}$$

The next figure shows how to dimension and tolerance the tangent contact lens seat. The top image meets the 0.5 mm vertex requirement. The middle image adds the 1 degree taper angle requirement. Finally, the bottom image adds the 90 µm radial shift requirement so all three requirements are included.

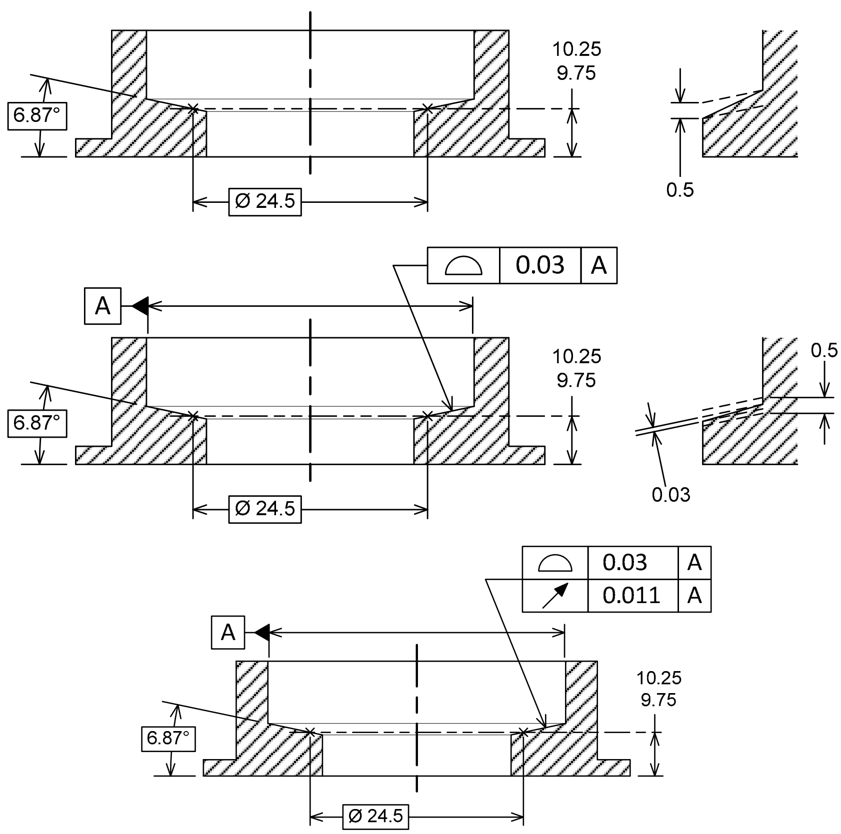

Figure 16. Dimensions shown on the tangent contact seat. The top image relies on the 0.5mm tolerance used to locate the gauge diameter to control form and orientation. The middle image uses profile tolerance to control the form and orientation of taper to within 30 µm. The bottom image controls wobble (using runout) to 11 µm.

Note that in the bottom two images of Figure 16 the taper is orientated to the bore diameter. Also, profile is not modifiable by material condition. The aperture diameter doesn't figure into the position or orientation for the lens. Figure 17 shows how to relate the tangent contact seat to the housing's mating features.

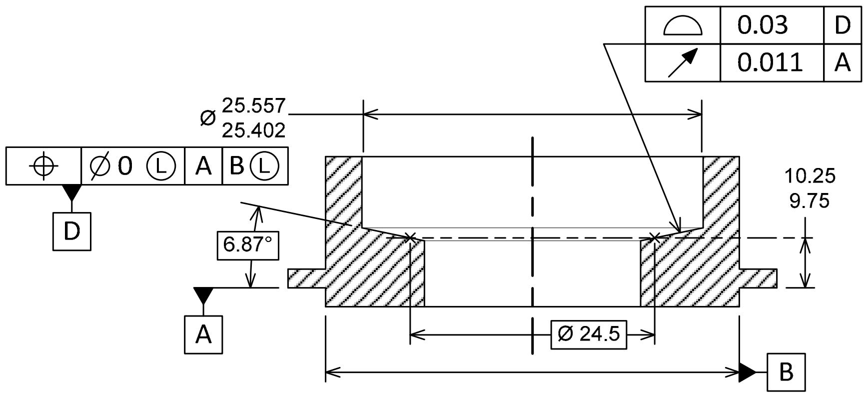

Figure 17. Dimensions shown on the tangent contact seat relative to external housing features.

As the bore diameter's size decreases, it's allowed additional positional tolerance relative to datum B. It also gets bonus tolerance, as the diameter of datum B gets larger. This maintains the overall allowable axial misalignment.

4. DIMENSIONING & TOLERANCING OF A SYSTEMS OF LENSES

4.1 Two lens system

For mounting two lenses relative to each other, let's look at three different configurations shown in Figure 18. Sharp corner seats are shown, but the methods are also applicable to tangent contact seats.

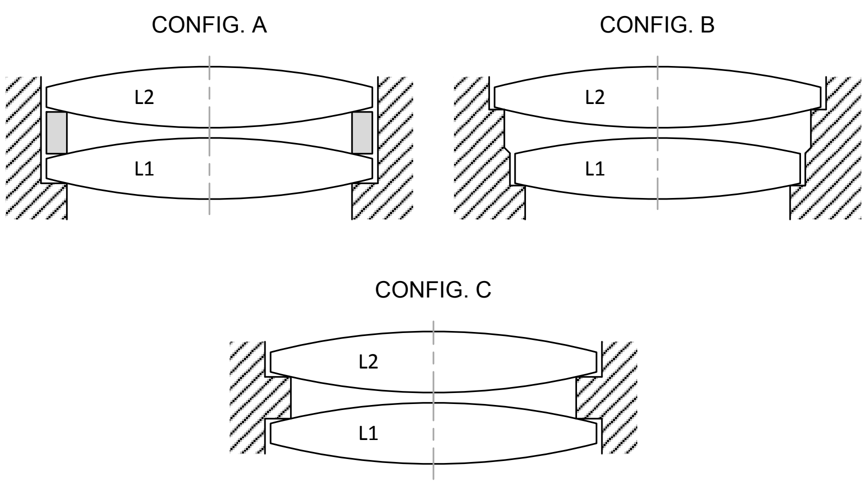

Figure 18. Three possible ways of mounting two lenses.

In config. A, both lenses share a common bore diameter and are separated by a spacer ring. Config. B features separate seat and bore diameters for each lens with the first lens (L1) having a smaller diameter than the second (L2). The last configuration, Config. C, shares a common seat diameter. The bore diameters can be either the same size or different sizes. Config. C can only be used with two lenses while the other configurations can be adapted to three or more lenses. Figure 19 shows how the lenses can shift in each of the configurations.

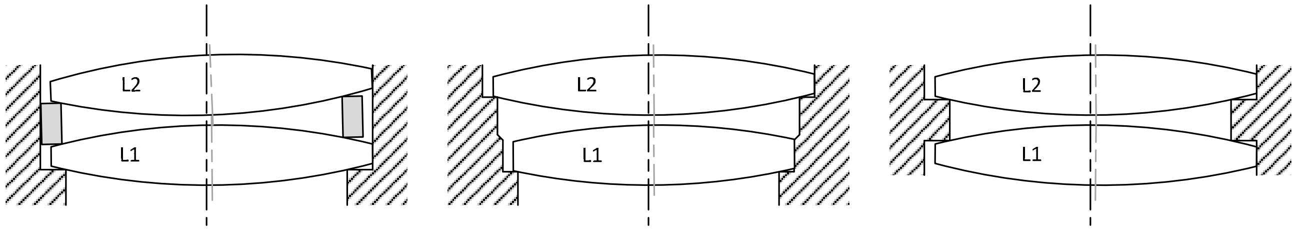

Figure 19. Possible tilts and displacements of the three mounting configurations.

Of the three configurations, config. A is probably the most difficult to analyze. The use of a spacer ring increases the number of variables necessary to calculate tilt. Lens element L1 shifts as described in equations (6) & (8), however the possible tilt of L2 depends on the cumulative tilts of L1 and the spacer ring. On the positive side, the vertex spacing between L1 and L2 is controlled only by the thickness and inner diameter of the spacer ring. Radial displacement of both lenses is controlled by the size and position of the bore diameter as shown in Figure 9.

Config. B completely decouples the two seats. Where lenses in config. A shared a common bore diameter the two lenses in config. B each have their own seat and bore diameters. Any radial displacement between the two seat diameters can be controlled using coaxiality. Its drawback is that the thickness tolerance of L1 is figured into the vertex distance between the two lenses. Config. C share a common seat diameter so radial displacement between the two is based only on the amount each is allowed to tilt. It is dimensioned very similar to Figure 9 as shown in Figure 20. In this instance, both bore diameters are the same.

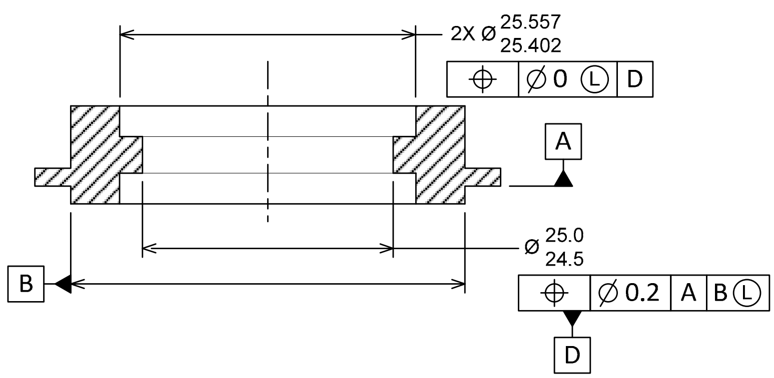

Figure 20. Method of dimensioning config. C two lens mount.

Dimensioning configuration B becomes a little more complex. In Figure 21, both seat diameters are given the same positional tolerance of 0.2 mm. This means that the axis of each can be offset by 0.1 mm from the diameter established by datum B plus any bonus tolerance due to datum shift. Their axis to each other can be off by up to 0.2 mm. Datum shift doesn't apply between features that reference the same DRF.

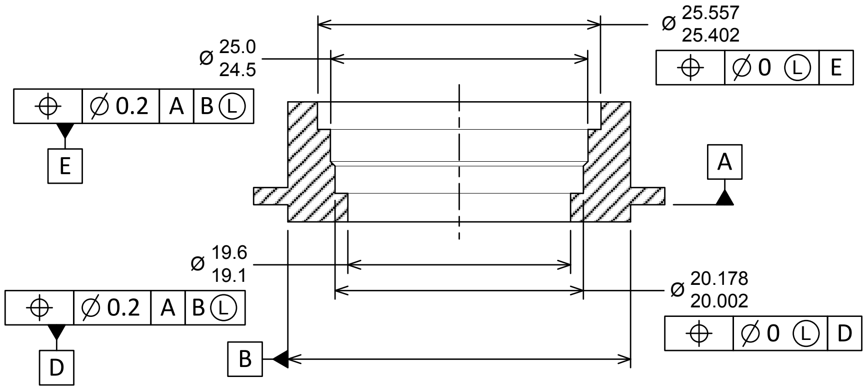

Figure 21. Method of dimensioning config. B two lens mount.

If it's necessary to have greater lens-to-lens coaxiality, then the two seat diameters can be toleranced using a composite feature control frame as shown in the top view of Figure 22. The upper feature control frame controls the axial position of both diameters as a pattern relative to datums A and B while the lower feature control frame controls the axis of each diameter relative to each other. Since the two seats have different diameters, it's important that they are identified with a note. It this case, each seat diameter is noted with 'CODE A' and a note is added to the composite feature control frame 'TWO COAXIAL HOLES CODED A'.

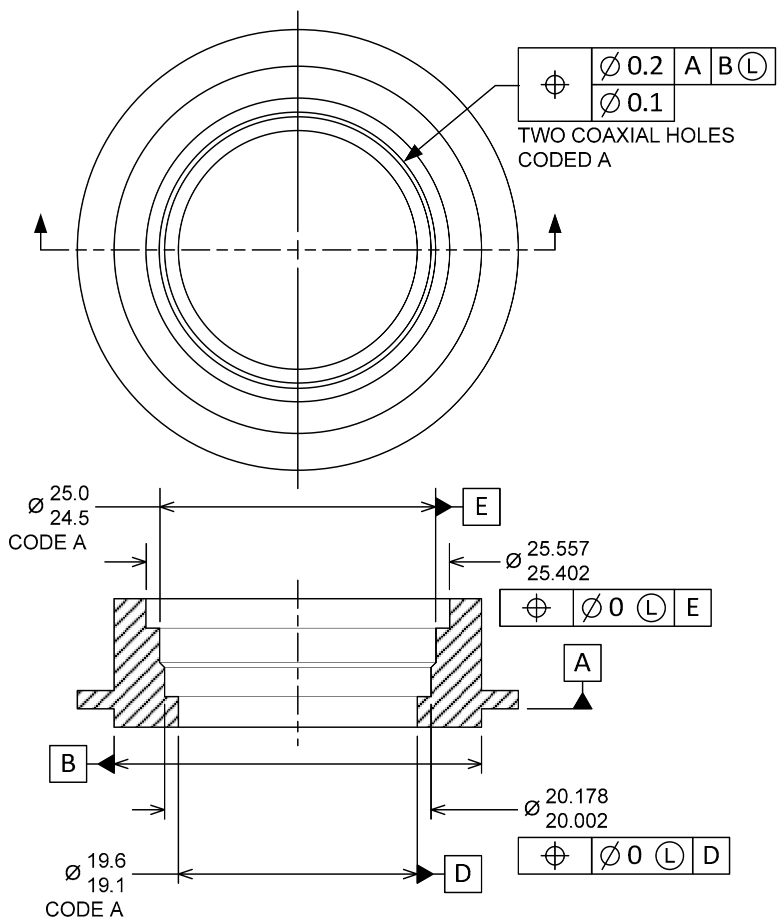

Figure 22. Additional coaxial control of the two seat diameters 'CODED A' through the use of composite positional tolerance.

4.2 More than two lenses

When mounting three or more lenses the three basic configurations presented in Figure 18 can be scaled or mixed and matched. Shown in Figure 23 are just a few of the possibilities. It's worth noting that as additional lenses are added to config. A, tolerances affecting tilt start stacking up in a way that's difficult to control. Config. C can be used only once per non-separable part/assembly. Careful attention needs to be paid to how the lenses are assembled.

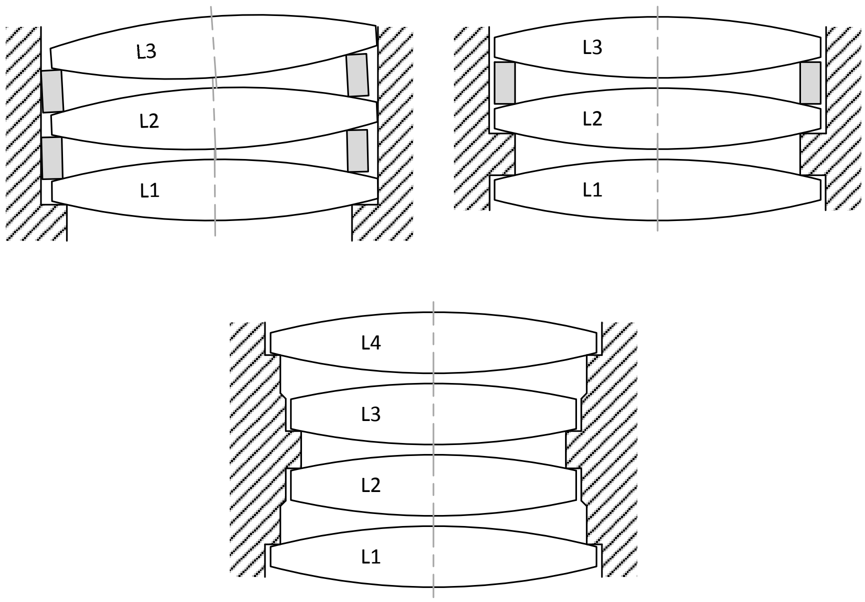

Figure 23. Multiple ways the different configurations of Figure 18 can be combined together for three or more lenses.

5. CONCLUSION

This paper has demonstrated several key concepts for mounting spherical lenses using sharp corner and tangent contact lens seats. First, that spherical lenses make primary contact with the lens seat and are orientated by the bore. Tilt and displacement are functions of bore size and position relative to the seat diameter not the size of the seat. For the tangent contact seat, taper angle has little effect on axial vertex position while radial offset is highly sensitive to the width of the taper tolerance zone. When dealing with multiple lenses or relating a lens to the housing's external mating features, the least material condition modifier is useful for controlling total axial misalignment. Composite positional tolerancing is also useful for refining lens-to-lens coaxiality.

Again, it's also worth pointing out for designing an actual system, thermal effects should be considered and if statistical tolerancing methods were used, it's possible to gain additional tolerance.

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