Orthonormal rectangular vector polynomials

Presenter: Maham Aftab

Supervisors: Dr. Jim Burge / Dr. Dae Wook Kim

Motivation

Zernike polynomials have been used extensively to analyze wavefronts for circular pupils as they are orthogonal and represent balanced aberrations across such pupils. However, if the pupil is not circular, the polynomials lose their value. The solution is to use polynomials orthogonal across the pupil under consideration.

Rectangular pupils are widely used in optics for example lasers, anamorphic optics etc. We are particularly interested in representing wavefront gradient, which is often what is measured. We derived a set of polynomials that represent vector quantities such as mapping distortion or wavefront gradient in the rectangular domain. Such a polynomial set has been derived for the circular domain*, but not rectangular. Additionally our vector polynomials are easily generalized for any rectangular aspect ratio and make it very easy to move between the scalar and vector domains. The vector polynomials are gradients of the Chebyshev polynomials.

Basics

$$T_{m+1}(x) = 2xT_m(x) - T_{m-1}(x)$$

$$T_0(x) = 1, \qquad T_1(x) = x$$

Chebyshevs in 1-D (recursive)

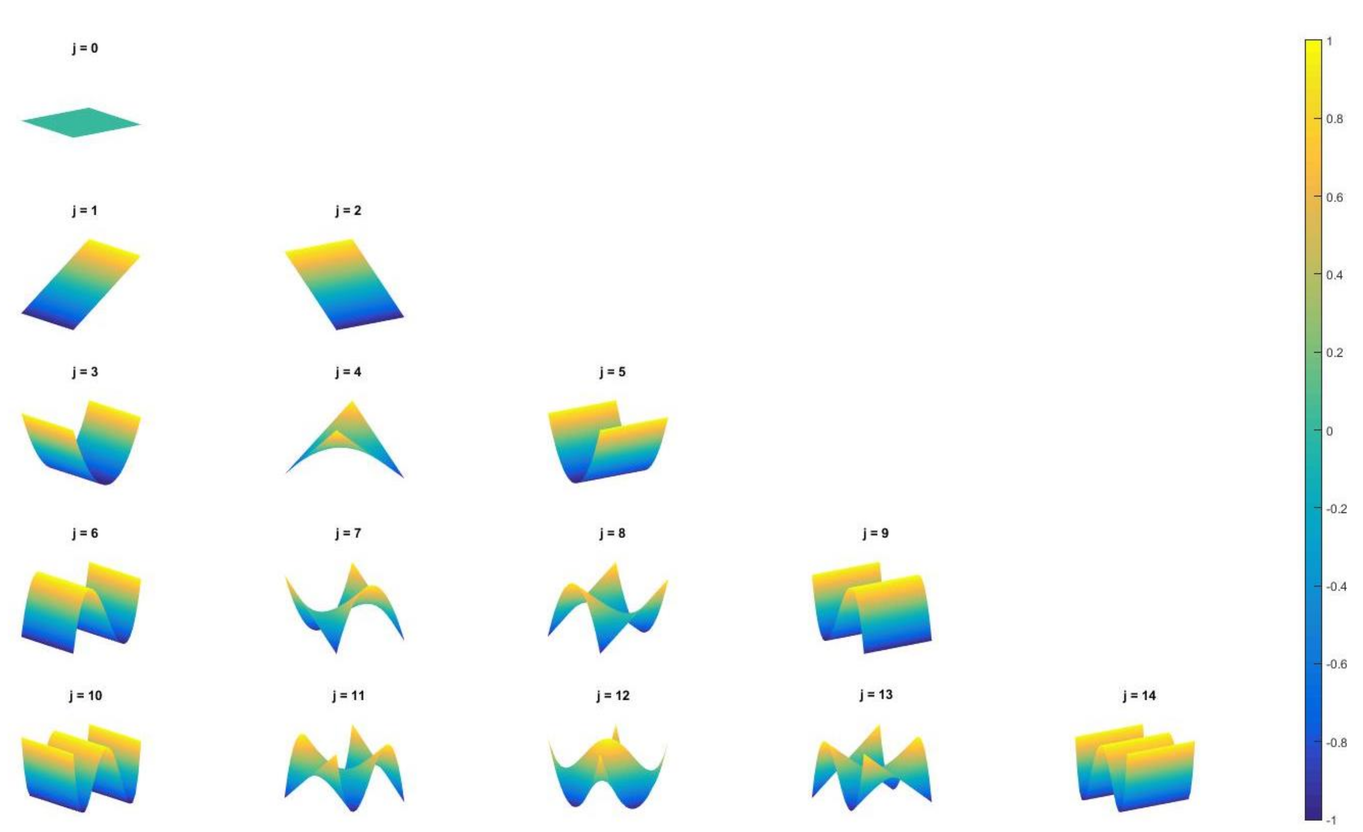
$$F_n^m(x,y) = T_m(x)T_n(y)$$

2-D Chebyshev

$$\overrightarrow{G_n^m}(x,y) = \nabla F_n^m = \frac{\partial}{\partial x} F_n^m(x,y)\hat{\imath} + \frac{\partial}{\partial y} F_n^m(x,y)\hat{\jmath}$$

Gradient Polynomials

Surface Plots of 2-D Chebyshevs



Examples and polynomial applications

$$\vec{G_4} = \frac{T_1(y)[T_0(x) - xT_1(x)]}{(1-x^2)}\hat{\imath} + \frac{T_1(x)[T_0(y) - yT_1(y)]}{(1-y^2)}\hat{\jmath}$$

$$\vec{G_5} = 2\frac{T_0(x)[T_1(y) - yT_2(y)]}{(1-y^2)}\hat{\jmath} \qquad \vec{G_6} = 3\frac{T_0(y)[T_2(x) - xT_3(x)]}{(1-x^2)}\hat{\imath}$$

Some examples of what the gradient polynomials look like in terms of the Chebyshev polynomials are given above.
The gradient data (measured or simulated) can be expanded in terms of the gradient polynomials (in the rectangular domain) as:

$$\nabla W(x,y) = \sum_{k=1}^{N} b_k G_k$$ where b_k is the kth expansion coefficient.

Orthonormality

Advantages of orthogonality:

1. Fitting to non-orthogonal basis usually requires more terms
2. Values of coefficients remain unchanged as number of terms used in expansion changes. So coefficients are meaningful.
3. Coefficients can be determined in an easier, faster, more computationally efficient way.

Orthonormality of gradient polynomials:

$$\frac{1}{N}\iint_{-1}^{1} G_j . G_k \, dx\, dy = \delta_{jk}$$

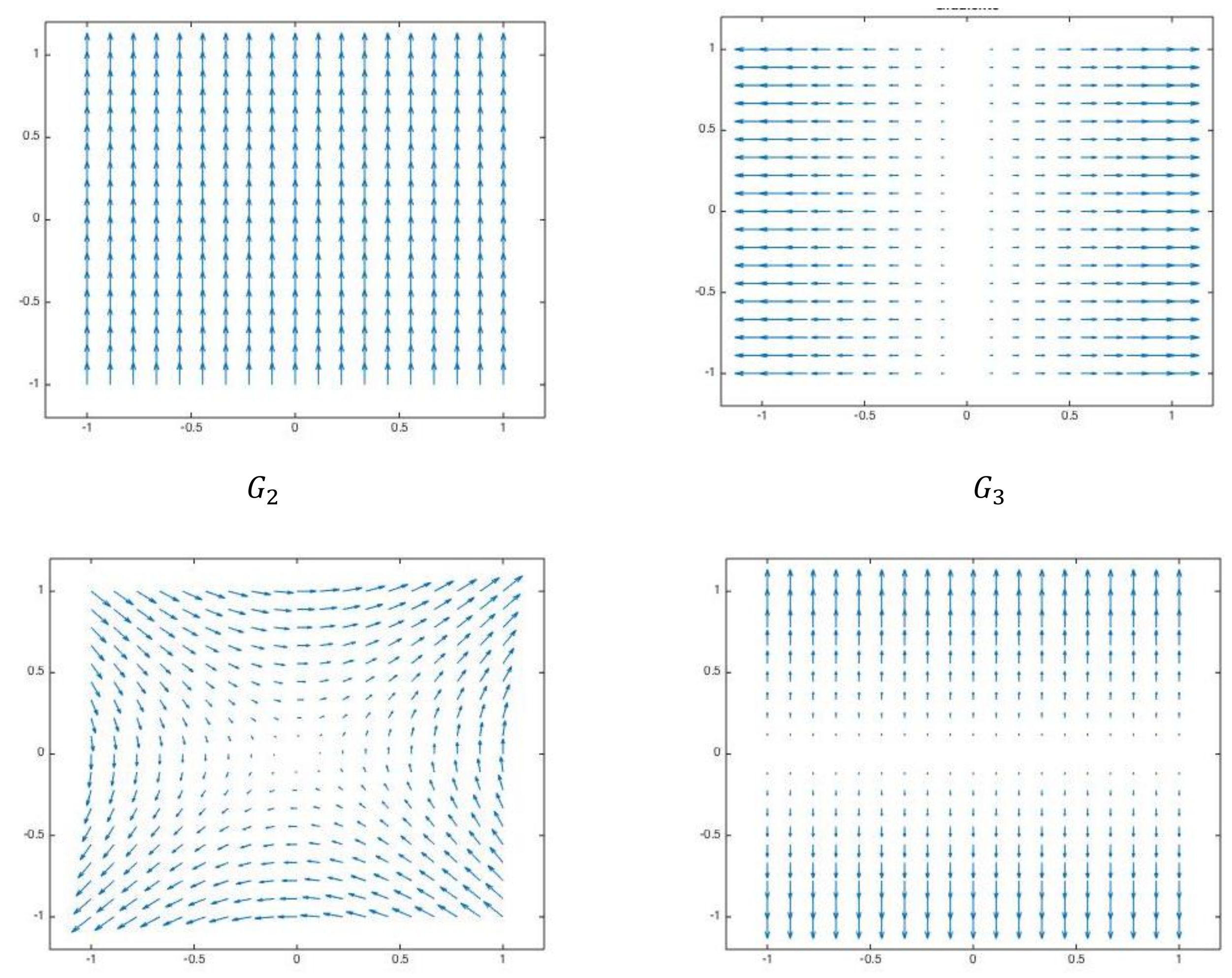
N = normalization factor

Applications

- For the DKIST (solar telescope) project with the LOFT group, we use MFT (Micro Finish Topographer) to measure surface roughness, which is necessary to check certain mirror specifications.
- As the MFT has a rectangular aperture, we should use the rectangular polynomials, instead of the circular ones.
- If we want to get rid of the low order terms, we can simply subtract the corresponding rectangular polynomial terms from the measured data. This corresponds to subtracting the low-order Zernike terms from data measured over a circular aperture.
- Similarly, if we have data from a Shack-Hatmann wavefront sensor or mapping distortion in the rectangular domain, we can use the gradient polynomials to fit the measured data and analyze aberrations.

Results

The following figures show the quiver plots of the x and y components of a few gradient polynomials:



G_2 G_3 G_4 G_5

Future Work

- This polynomial set can be used for analysis of a wide variety of rectangular optics. It is the basis for a paper under progress.
- This work is continued by the determination of a complementary set of vector functions that are defined as the curl of the rectangular basis functions and so consist of rotational terms only.
- The gradient and curl sets together will provide a truly complete basis that can represent any vector data set for a rectangular pupil.

Works Cited

- *1. Chunyu Zhao and James H. Burge, "Orthonormal vector polynomials in a unit circle, Part I: basis set derived from gradients of Zernike polynomials," Opt. Express 15, 18014-18024 (2007)
- 2. Liu F, Robinson BM, Reardon PJ, Geary JM; Analyzing optics test data on rectangular apertures using 2-d chebyshev polynomials. Opt. Eng. 0001;50(4):043609-043609-8. doi:10.1117/1.3569692.
- 3. "Optical Imaging and aberrations part III: wavefront analysis," V. N. Mahajan, pp. 372– 373 (SPIE Press, 2013).