# **Parametric Tool Influence Function Size Optimization**

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**Abstract:** To further increase the efficiency of deterministic optical fabrication, a parametric Tool Influence Function optimization method was developed to assist in the optimal tool size selection process based on any given surface error map. © 2021 The Author(s)

## 1. Introduction

Over the past few decades optical fabrication has become a much more deterministic process. It is well known that certain tool sizes have a limit to the feature sizes they can correct, and there is a trade-off between processing time and residual surface error. Typically, larger tools are favored because they remove material faster, but, while any given tool can correct features larger than the tool, they cannot correct features that are smaller than the tool footprint very well. The question we pose, then, is, with a given surface, how can an optician choose the set of tool sizes that will complete the part in the shortest amount of time. Lin, et al. [1] derived a method to determine how well a certain TIF can correct certain spatial frequencies, but they make no attempt to tie this to the resulting run time. Wang, et al. [2, 3] proposed a procedure to calibrate a specific tool to determine how well it can correct features smaller than the tool, but this requires multiple fabrication runs and metrology to feed back to the results and is specifically looking at the smoothing efficiency of a tool, which we will not consider here.

## 2. Fourier Decomposition of Target Surface Removal Map

The power spectral density (PSD) of a mean-removed surface error map is simply a band-limited breakdown of the surface variance contribution of each spatial frequency in the measurement. A newly defined term, the Encircled Error (EE) of a surface error map, defined the same way as the encircled energy of a point source function, then specifies how much of the error is due to spatial frequencies lower than a given frequency. We define the frequency which occurs at 80% EE the Characteristic Frequency (CF) of the surface. Figure 1 shows these concepts on a synthetically generated surface error map (i.e., target removal map).





#### 3. Encircled Correctability calibrated by Reference TIF

According to Preston's equation for material removal, to perfectly correct an error mode represented by a single sinusoidal frequency in the shortest amount of time, a tool influence function (TIF) shape must be chosen to match the error shape, namely one period of a sinusoid with the same frequency as the error. We term this frequency the Characteristic Frequency (CF) of the TIF. A TIF size smaller than the error can correct the error perfectly, but will take longer to do so, while a TIF size larger than the error cannot correct all the error without relying on smoothing. In this way, a TIF with a given CF is the most efficient TIF to correct that frequency.

Lin, et al. [1] discussed how the amplitude frequency spectrum, given by the Fourier Transform, of a TIF is a measure of the correctability of that TIF for localized errors of the same spatial frequencies. We take this analysis further by defining the Encircled Correctability (EC), which is simply an encircled energy calculation of the PSD

of a TIF. We postulate that these Fourier methods can be used to determine the CF of any TIF. We can simplify this analysis by considering the 1-D line TIF (analogous to the ring TIF [4]), which considers a raster motion path of the TIF on the surface, calculated simply by summing the rows or columns of the TIF matrix. To avoid time-consuming simulations for each possible TIF shape, this technique is calibrated by finding where in the EC curve the CF occurs in the reference TIF, which we found to be at 91.8% EC as shown in Figure 2. Now we can take any TIF shape and find its CF by simple Fourier methods.



Fig. 2. (a) Reference line TIF defined with a frequency of 0.05 mm<sup>-1</sup>. (b) 1-D PSD of line TIF. (c) 1-D Encircled correctability of line TIF showing a characteristic frequency of 0.05 mm<sup>-1</sup> occurring at 91.8% EC.

### 4. Statistical Simulation Results and Conclusion

The characteristic frequency ratio (CFR), defined by the ratio of the CF of a TIF and the CF of a surface, can be used as a measure to select a TIF size for a given surface error map. We ran a series of Monte Carlo simulations on randomly generated surface error maps using two common TIF shapes - Gaussian and Orbital TIFs. Each TIF shape was sized according to a CFR of 0.2 to 1 for a random surface error map and the RIFTA dwell time algorithm [5] was used to calculate the run time and RMS residual error vs. CFR value. These results were then normalized, averaged, and fit to curves to provide a useful means for selecting a TIF size according to a given surface error map. The figure of merit (FoM) equation is expressed in Eq. 1 with visual trends shown in Figure 3, where the portion within the first square brackets is the "E" term, the portion within the second square brackets is the "T" term, and the "W" represents an optional weighting factor.



Fig. 3. Figure of Merit for choosing TIF size based on average values (with standard deviation error bars) from the Monte Carlo simulations, where "E" is the normalized residual RMS error and "T" is the normalized run time.

If the PRR vs. TIF size can be characterized, a means of determining the optimal TIF size can be formulated to assist the optician substantially.

#### 5. References

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