An easy way to relate optical element motion to system pointing stability

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ABSTRACT

The optomechanical engineering for mounting lenses and mirrors in imaging systems is frequently driven by the pointing or jitter requirements for the system. A simple set of rules was developed that allow the engineer to quickly determine the coupling between motion of an optical element and a change in the system line of sight. Examples are shown for cases of lenses, mirrors, and optical subsystems. The derivation of the stationary point for rotation is also provided. Small rotation of the system about this point does not cause image motion.

Keywords: Optical alignment, optomechanics, pointing stability, geometrical optics

1. INTRODUCTION

Optical systems can be quite complex, using lenses, mirrors, and prisms to create and relay optical images from one space to another. The stability of the system line of sight depends on the mechanical stability of the components and the optical sensitivity of the system. In general, tilt or decenter motion in an optical element will cause the image to shift laterally. The sensitivity to motion of the optical element is usually determined using computer simulation in an optical design code. If done correctly, the computer simulation will provide accurate and complete data for the engineer, allowing the construction of an error budget and complete tolerance analysis. However, the computer-derived sensitivity may not provide the engineer with insight that could be valuable for understanding and reducing the sensitivities or for troubleshooting in the field. In this paper, a simple method is presented for determining these sensitivities using a few hand calculations. The relationships provide insight to the problem and can be evaluated "on the fly" to help with engineering tradeoffs or evaluating system performance.

The coupling between motion of an optical element and the system performance is important for several types of systems:

1. *Systems that require stability.*
   The calibration of a spectrograph requires stable imaging from the slit to the detector. Optical imaging systems for lithography require accuracy in the image placement of tens of nanometers. Boresight accuracy between optical systems requires stability for each system.

2. *Systems sensitive to image jitter.*
   Vibration of an optical element will cause the line of sight for the system to jitter this can cause decreased acuity for long exposures or limit performance of laser projectors.

This paper provides analysis of the first-order behavior of optical systems, which is adequate to describe small motions. The work presented here builds on well-known principles of first order systems, which are reviewed in Section 2. The effect of motion for lens elements, mirrors, and optical systems is derived in Section 3. In general these motions cause a change in angle and a lateral shift for the paraxial bundle of light that follows. The basic relationships that couple the angular and lateral perturbations with image shift are derived in Section 4. The relationships do not require knowledge of the complete system, only the first order properties of the element that is perturbed, and the numerical aperture for the system. A heuristic explanation and some simple examples are included. Section 5 gives an example and compares the results with a ray trace simulation. Section 6 discusses alternate applications of this work.

2. FIRST ORDER OPTICS

The analysis in this paper uses well-known properties of first order systems, including the representation of the first order behavior with principal planes and the concept of the optical invariant. These are summarized here to define the nomenclature used in the paper. The concepts are explained more fully elsewhere.[1, 2]

The paraxial, or first order behavior of an optical system that has focal length f can be represented using a few cardinal points and some simple rules governing ray propagation. An imaging system can be reduced to two principal planes and two focal points, as shown in Figure 1. Light from the object is incident on the front principal plane and it emerges from the rear principal plane, which is in image space. The mapping between these planes is simple – the ray height y is preserved from one plane to the other and the angle u is changed according to the simple relationship,

$$u' = u - \frac{y}{f}.$$

Eq. 1

For systems in air, the rear focal point is located one focal length f from the rear principal plane and the front focal point is located one focal length from the front focal plane.
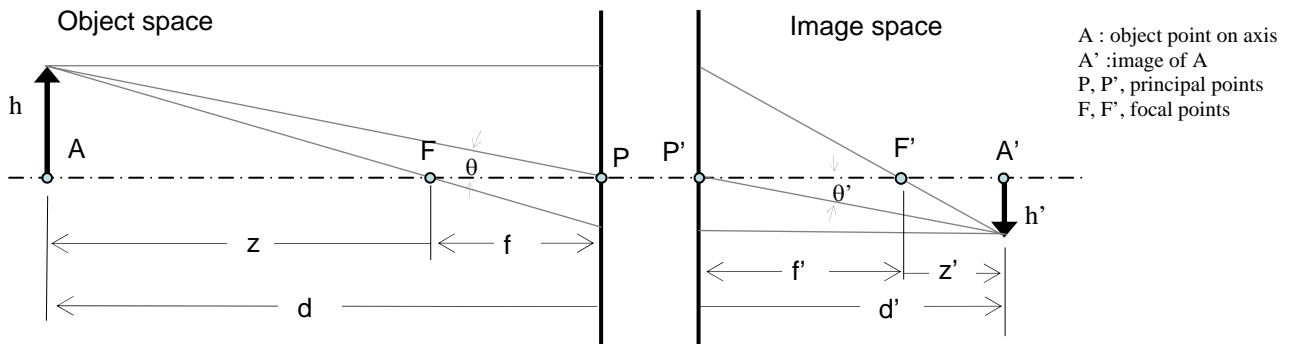
Figure 1. Definition of cardinal points to represent the paraxial performance of an optical system

An object located a distance z from the front focal point will cast an image at z' from the rear focal point, obeying the Newtonian image relation

$$z \cdot z' = f^2$$

Eq. 2

Light rays from an on-axis point at infinity will be parallel to the axis in object space and they will come to focus at the rear focal point in image space. Conversely, light rays that go thought the front focal point will come out parallel to the axis in object space. The nodal points are defined as the points with unit angular magnification. The angle of the light comes out the same as it went in. The nodal points and the principal points coincide for a system in air. The position of an off-axis image can always be found using this simple principle, as shown in Figure 1. The magnification of the image m, defined as the ratio of image height to object height, can then be calculated as

$$m \equiv \frac{h'}{h} = \frac{d'}{d} = \frac{z+f}{z'+f'}$$

Eq. 3

While the cardinal points are useful for determining first order properties of an optical system the "rays" may not be physical. It is useful to trace two real rays through the system – a chief ray and a marginal ray. The chief ray starts at an off axis object point and goes through the center of a surface of interest, usually the stop. The marginal ray starts at on object point on axis and goes through the edge of the stop, as shown in Figure 2. For a ray of light leaving surface i, the ray height and the angle with respect to the axis are defined as y_i and θ_i respectively as shown in Figure 2. By convention, the terms with bars over represent the chief ray and the un-barred terms represent the marginal ray.
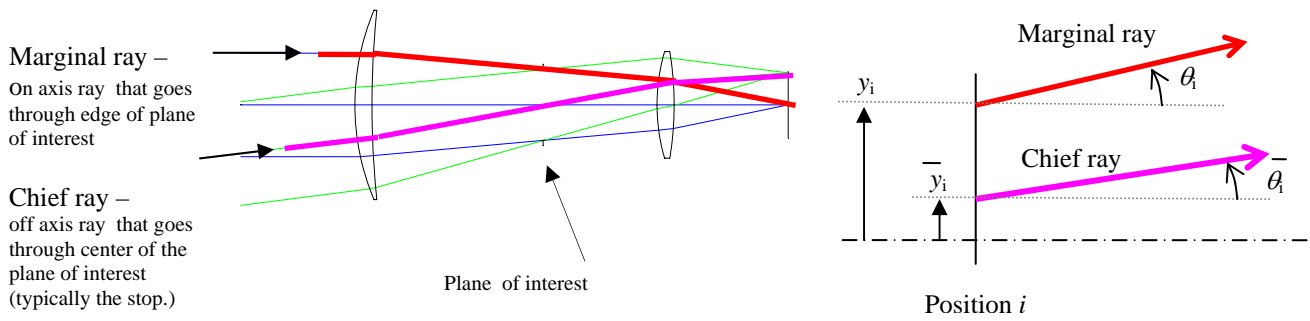
Figure 2. Definition for selection and sign convention for chief and marginal rays.

The effects of different refractive indices can be accommodated by defining the term $u_i = n_i \sin \theta_i$. This equivalent angle u will be used for subsequent analysis in this paper. It is well known that the value computed as

$$I \equiv \overline{u}_i y_i - u_i \overline{y}_i \qquad \text{Eq. 4}$$

is invariant. (The term I is often called the optical invariant.) In fact, this quantity is invariant through the optical system for any two linearly independent rays.[3] This means that the value computed using the ray heights and angles at surface i will have the same value as computed for any other surface.

3. EFFECT OF OPTICAL ELEMENT MOTION

If a mirror, lens, or subsystem moves either laterally (perpendicular to the optical axis) or in tilt, then the line of sight for the optical system will be shifted. For imaging systems, this will result in motion of the image. For a laser projector, the effect will be a deviation in the projected light. Before developing the relationships for the system, the simple effects due to element motion are analyzed in terms of their effect on the light. We start by defining a chief ray that goes through the center of the initial, unperturbed element. As the element is perturbed, the output ray will generally suffer a combination of lateral shift and angular deviation. These are shown in Figure 3. The image shift due to these perturbations is constant across the field, so we only need to know the shift in a single ray to determine the magnitude of the effect.
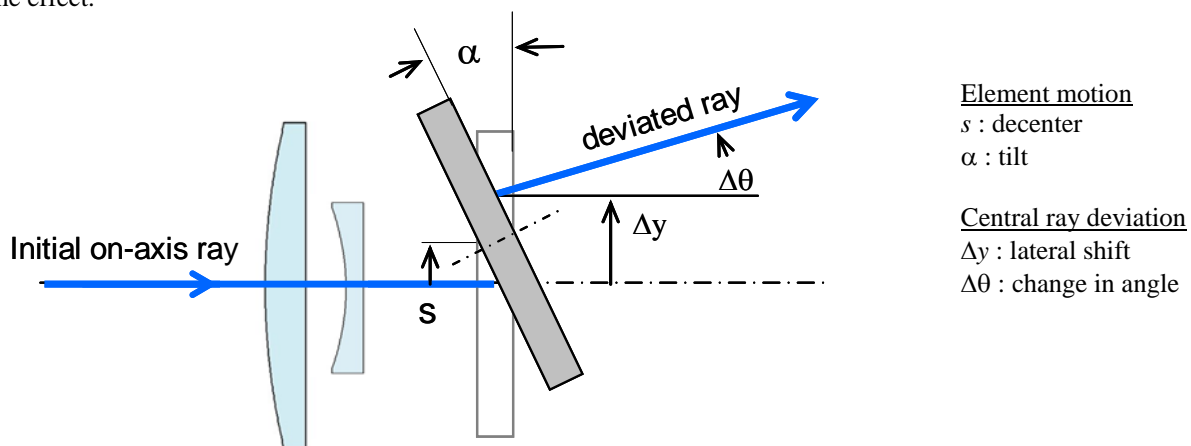
Figure 3. Tilt or lateral shift of an optical element or subsystem will generally cause a combination of angular deviation and lateral shift of a chief ray.

3.1 Effect of motion for a thin lens

The effect of lens motion can be approximated by ignoring the thickness of the element, treating it as though all of the optical power occurs at the middle of the lens. (Although this is an excellent approximation for most cases, the complete analysis is given below in Section 3.4.) There is no first order effect of tilting the thin lens a small amount. Decentering the lens will cause an angular deviation in the light which is, to first order, constant for every ray across the lens. The effect of translating the lens is easy to see graphically for the central ray, as shown in Figure 4.
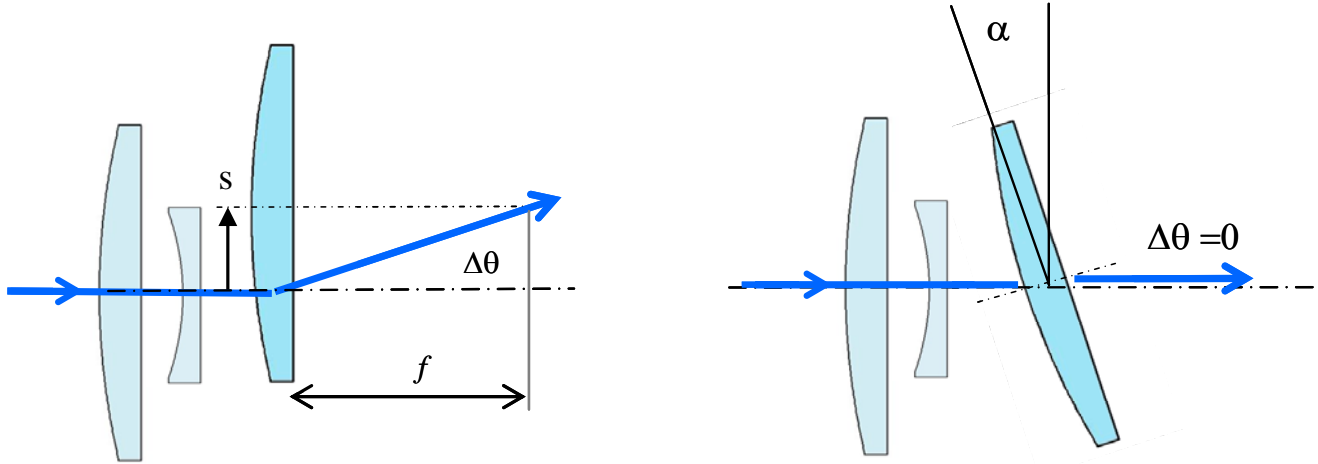
Figure 4. Lateral translation of a thin lens by an amount s causes the transmitted light to be deviated by and angle $\Delta\theta_s$. Tilt of the thin lens has no significant effect. The effect of a thick lens is given below.

For a lens with focal length f, shifted by an amount s, the incident central ray will intercept the shifted lens off its axis, but still parallel to its axis. So the ray will refract and go through the focal point. The angular shift can be determined graphically (assuming the small angle approximation) from Figure 4 or from Eq. 1 as

$$\Delta\theta_s \cong \frac{s}{f} \ .$$

Eq. 5

Note that this does not depend on object or image position because we are only treating the change in the central ray. Also note that this value holds for negative and positive lenses.

3.2 Effect of motion for a mirror

A powered mirror has the same sensitivity to lateral displacement as the lens above, and it also is very sensitive to tilt. These two cases are shown graphically in Figure 5. Upon reflection from a mirror that tilted by an angle α, the reflected light will have an angular change of $\theta_\alpha = 2\alpha$. This is independent of the focal length of the mirror, so it applies for flat mirrors as well as positively and negatively powered ones.
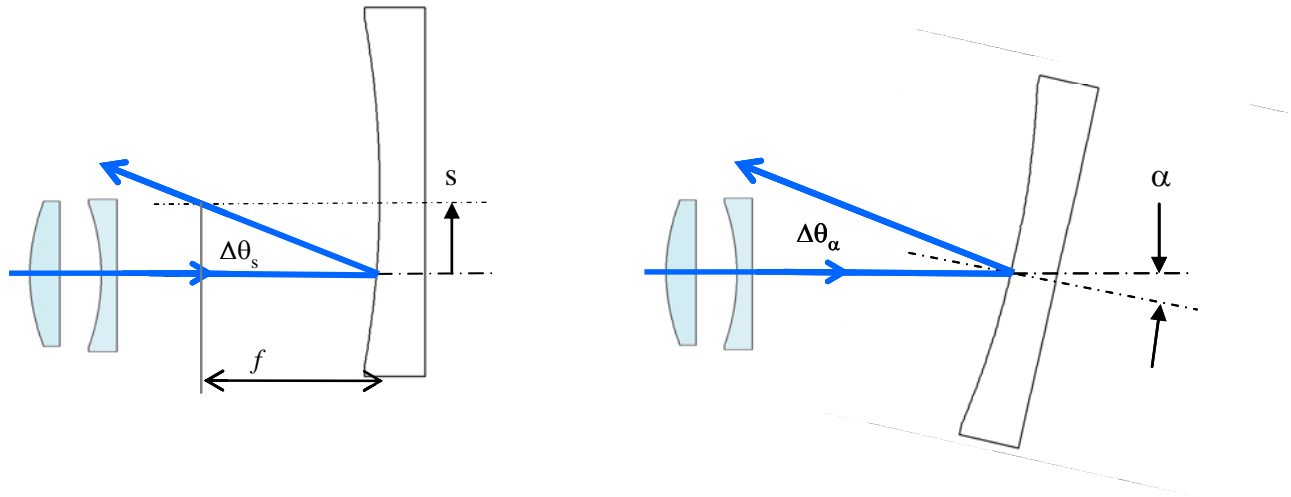
Figure 5. Lateral translation of a mirror s causes the transmitted light to be deviated by and angle $\Delta\theta_s$. Tilt of the mirror by an angle α causes angular deviation of $\Delta\theta_\alpha = 2\,\alpha$.

A linear combination of translation and rotation for a mirror gives

$$\Delta\theta \cong \frac{s}{f} + 2\alpha \;.$$

Eq. 6

There is one combination of translation and rotation which has no effect. This occurs when the mirror tilt is equal to the ratio of the displacement over $2f$. This is of course equivalent to rotating the mirror about its center of curvature.

3.3 Effect of motion of a plane parallel plate

The tilt of a plane parallel plate, such as a window, with thickness t by an amount α causes a lateral shift Δy as shown in Figure 6. Of course the lateral translation of a plane parallel plate has no optical effect.


$$\Delta y \cong \frac{\alpha t\,(n-1)}{n}$$

$$\cong \frac{t}{3}\alpha \quad \text{for n = 1.5}$$

Plane parallel plate
thickness t
index n
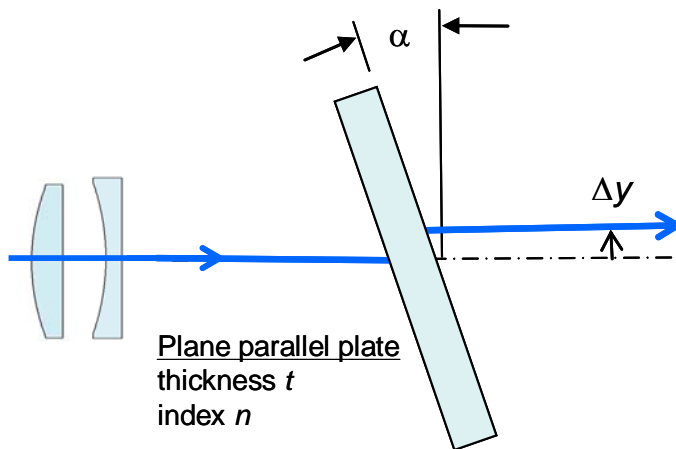
Figure 6. Tilt of a plane parallel plate causes a lateral deviation that depends on the thickness and refractive index of the plate.
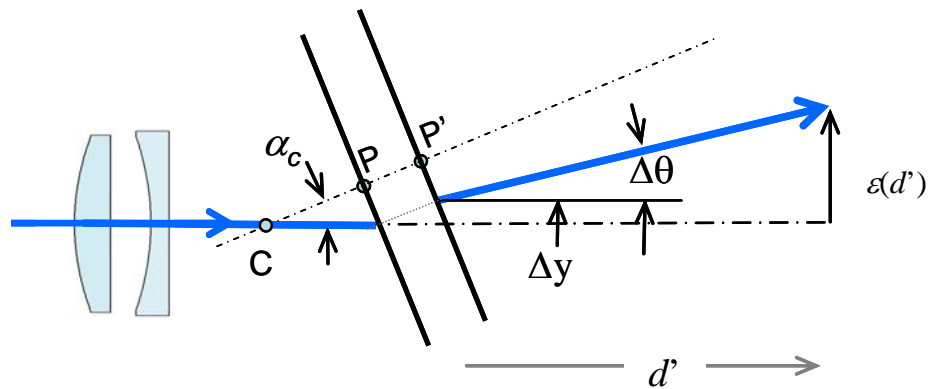
3.4 Effect of motion of an optical system

A thick lens or any general set of powered optical elements can be treated using principal planes as described above. The effect of motion of such an optical system is more complicated, but it can be calculated directly. If we decompose general motions into a combination of pure lateral translation and rotation about the front principal point P, the relationships can be obtained graphically, as in Figure 7.
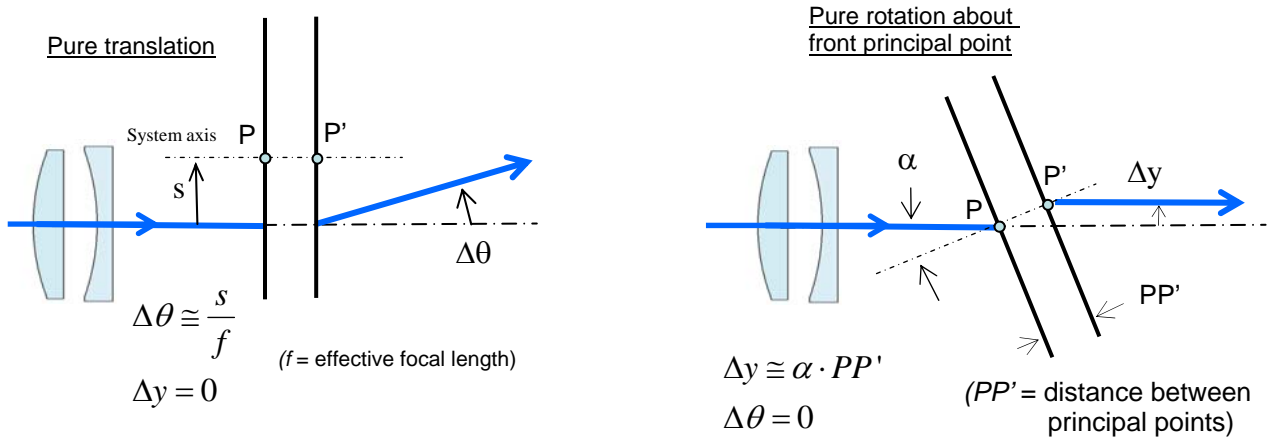
Figure 7. General relationship for the motion of an optical system represented by principal planes.

Rotation about a general point C can be evaluated as a linear combination of translation and rotation about the principal point. Consider rotating an angle α_C about a general point C, which is located distance *CP* from the front principal point as shown in Figure 8. For small angles, the rotation about this point causes an effective system translation of $s = \alpha_C\,CP$. The combined effect of rotation and translation causes the central ray to deviate by

$$\Delta y \cong \alpha_c \cdot PP'$$

$$\Delta\theta \cong \alpha_c \frac{CP}{f}$$

Eq. 7

Figure 8. Relationship for the motion of an optical system represented by principal planes as it is rotated about an arbitrary point C.

For the case where the image is located a distance *d'* from the rear principal point, the image motion ε due to the optical system rotating an amount α_c about point *C* is determined by the combined effect:

$$\varepsilon \cong \alpha_c \cdot PP' + \alpha_c \frac{CP}{f} d' \; .$$

Eq. 8

The effect of tilting a real plano-convex thick lens making an image from infinity was examined. A 50 mm diameter lens, 200 mm focal length with 6.2 mm center thickness was used. The front principal point *P* lies on the vertex of the curved surface and *PP'* is 2.1 mm. We rotate about the middle so *CP* = -3.1 mm, and we evaluate the motion at focus so *d'* = *f*. Evaluating Eq. 8, the sensitivity of image motion to lens rotation is only 1 µm/mrad. This sensitivity was corroborated by direct simulation using Zemax[4] ray trace software. This sensitivity is very small indeed. A 1 µm image shift caused by 1 mrad lens tilt corresponds to an angle of only 0.005 mrad in object space, so the effect is only 0.5% of the lens tilt. The sensitivity to lens decenter is 1 µm/µm, which is much more important.

The relationship above can be used to determine the stationary point for rotation. The stationary point is defined such that small system rotation about this point does not cause image motion. The stationary is found by setting Eq. 8 to zero and solving for *CP*$_{stationary}$, the position of the stationary point relative to the front principal point:

$$CP_{stationary} = -\frac{f}{d'} \cdot PP' \; .$$

Eq. 9

Evaluating Equation 9 for some interesting cases:
- For a thin lens, *PP'* = 0, and the stationary point occurs at *CP* = 0, rotating about the principal points.
- For an object at infinity, *d'* = *f*, so *CP*$_{stationary}$ = -*PP'*, which means that the stationary point occurs at the rear principal point. This principle is used for finding the principal point with a nodal slide[5].
- The stationary point depends not only on the optical system, but also on the object and image positions.
- A real biconvex "thick lens" operating at 1:1 conjugates has its stationary point in the middle. (*CP* = -0.5 *PP'* and *d'* = 2*f* .)

4. COUPLING OF OPTICAL ELEMENT MOTION TO SYSTEM LINE OF SIGHT

The relationships between optical element motion and the light are given above. The resulting effect on image motion in an optical system is derived here using the optical invariant. This provides a shortcut, analogous to using energy conservation for mechanical problems. The solution depends only on the first order ray height and slopes at the element of interest and the numerical aperture (or focal ratio) at the image. There is no need to perform analysis through the intermediate optics.

The optical invariant used here is defined so the chief ray goes through the center of the element of interest, not the system aperture stop. Also, the optical system following element *i* is included, but any optics that precede this element are ignored. This is done so that the "field angle" at element *i* can be taken as the small angle due to the element misalignment. (For the on-axis object point, there is no field angle for the preceding optics.) Table 1 shows the evaluation of the optical invariant at surface *i* and at the final image. The following definitions, shown in Figure 9, have been applied:

D_i = diameter of on-axis beam at surface *i* (= 2 y_i)   *NA* numerical aperture for system

$NA_i = u_i$ = equivalent numerical aperture for the light after element *i* F_n working focal ratio for system

$\Delta\theta_i$ change in angular deviation for light immediately after element *i* ε = resulting image shift.

Δy_i change in lateral deviation for light immediately after element *i*

Table 1. Evaluation of optical invariant at surface *i* and at final image plane

	At surface *i*	At image plane *N*
u : marginal ray angle	$u_i = NA_i$ = numerical aperture at *i*	$u_N = NA = 1/2F_n$
y : marginal ray height	$y_i = D_i / 2$ defined by beam footprint	0
$\bar{u}$: chief ray angle	$\bar{u}_i = \Delta\theta_i$ due to element motion	$\bar{u}_N$
$\bar{y}$: chief ray height	$\bar{y}_i = \Delta y_i$ due to element motion	$\bar{y} = \Delta y_N \equiv \varepsilon$, image motion
$I \equiv \bar{u}_i y_i - u_i \bar{y}_i$: optical invariant	$\dfrac{D_i}{2}\Delta\theta_i - NA_i\Delta y_i$	$NA \cdot \varepsilon = \dfrac{\varepsilon}{2F_n}$
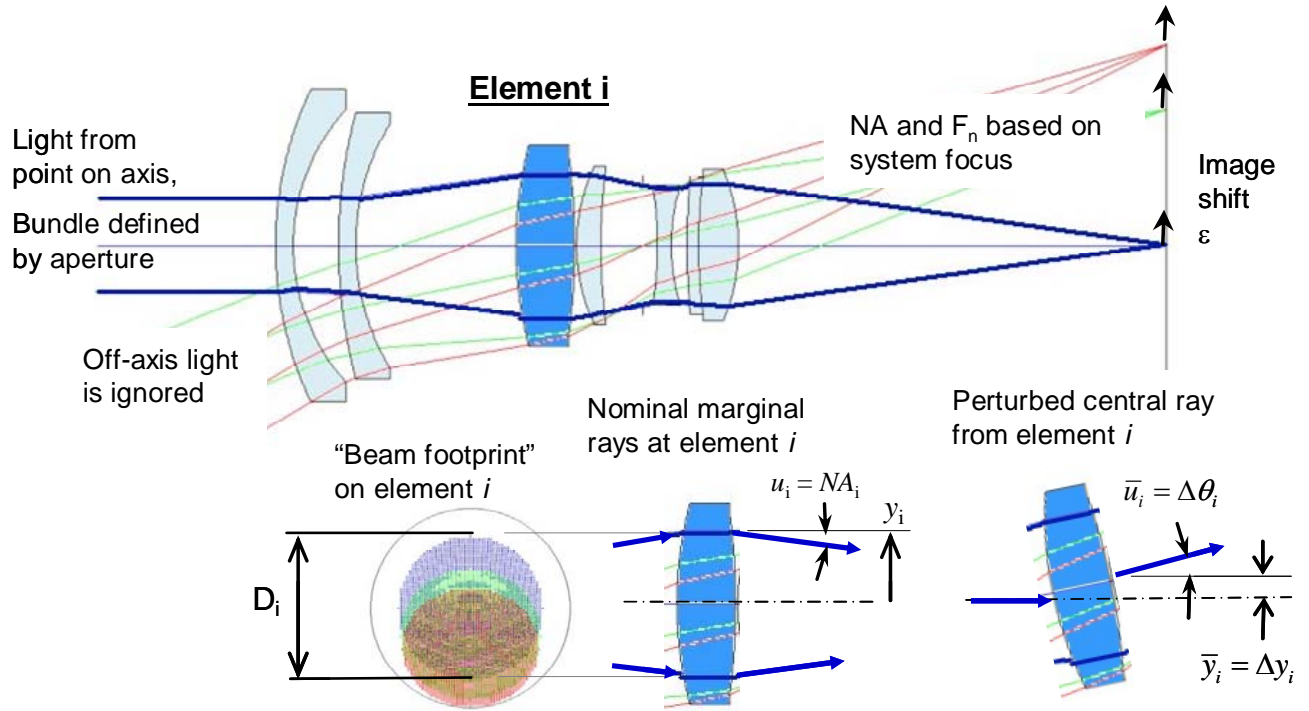
Figure 9. Definitions for system analysis relating image shift to element motion

Equating the values for the optical invariant in Table 1, the following solution is obtained:

$$\varepsilon = F_n D_i \Delta\theta_i - \frac{NA_i}{NA}\Delta y_i \qquad \text{Eq. 10}$$

where $\Delta\theta_i$ and Δy_i give the deviations of the central ray as described above, and as calculated in Section 3.

This result appears to depend on the size of the stop, but it does not. The values of D_i, F_n, NA_i, and NA all depend on the stop size, but the quantity $F_n D_i$ and NA_i/NA normalize out the stop size. If the stop is reduced by a factor of two then D_i would reduce by a factor of two and F_n would increase by this same factor so the product is unchanged. Likewise, both NA and NA_i would decrease by this same factor so the ratio is unchanged.

4.1 Effect of angular deviation

For almost all cases, the effects of angular deviation of the light will dominate the image shift. Since the total shift is a linear sum, we can look separately at the effect of angle by simply setting the other term, Δy_i to zero. The resulting relation is very simple and it requires only knowledge of D_i, the size of the "beam footprint," and the final focal ratio.

Some examples for the case of angular deviation are

- For a system which contains only a single lens or mirror in translation, the product $F_n D_i$ is simply equal to the focal length f, giving the expected result that $\varepsilon = f \Delta \theta$. This makes the relationship easy to remember!

- For a lens or mirror element with focal length f_i moving laterally an amount s_i, the image shift will be

$$\varepsilon = F_n \frac{D_i}{f_i} s_i = \frac{F_n}{f_i / D_i} \cdot s_i \ . \qquad \text{Eq. 11}$$

This simple result gives the magnification of image motion to lateral motion is simply the ratio of the system working focal ratio to the effective f-number of the element, defined by the ratio the focal length to the diameter of the on axis beam footprint. (Note that this relationship does not depend on the numerical aperture of the light that is illuminating the element, nor does it depend on the size of the element.)

- For a mirror that is rotated by a small angle α_i, the relationship is reduced to

$$\varepsilon = 2 F_n D_i \cdot \alpha_i \ . \qquad \text{Eq. 12}$$

So the "lever arm" which relates image motion to rotation of the mirror is simply $2 F_n D_i$, which is exactly the value one would expect for a mirror that is in the converging beam in image space with no other elements between the mirror and the image. This does not depend on the mirror focal length, and it works for flats. Be careful when evaluating the beam footprint for tilted mirrors. The value for D_i should be determined by the size of the bundle of light, not the elongated projection on the tilted mirror.

The simple relationship in Eq. 12 should make sense to somebody who works in the area of optical testing. The "tilt" of an optic is usually measured in fringes. The effect of the tilt on the interferogram does not depend on which optic caused the tilt, but only by the number of fringes introduced. Given the same number of tilt fringes, the tilt angle of a mirror with 10 cm diameter beam footprint would be twice that of a 20 cm mirror.

4.2 Effect of pure translation

The effect of pure translation for the light is evaluated using Eq. 10, where the angular deviation $\Delta \theta_i$ is set to zero. The light from the element will generally form an image, either virtual or real. The optics between this element and the final focus can be thought of as relaying this image. The magnification of this relay can be calculated as the ratio of the numerical apertures, exactly as Eq. 10. Some interesting conclusions are

- The effect of tilting a window is $\varepsilon \cong -\dfrac{t(n-1)}{n} \dfrac{NA_i}{NA} \alpha$ which does not depend on the beam footprint.

- A tilted window in collimated space has no effect ($NA_i = 0$).
- For lenses, the beam translation effect due to lens tilt is typically only a few percent as large as the effect on angular deviation caused by element decenter.
- If this small effect is to be evaluated, the analysis must include the position of the pivot point relative to the principal point to include the angular effect. Both terms in Eq. 7 must be included in the analysis.

5. EXAMPLE FOR AN IMAGING SYSTEM

These relationships were corroborated by direct simulation of an imaging system using Zemax[4] optical design code. A Cooke triplet that is provided with the program was modeled and perturbed. A layout of the optical system and the key parameters are given in Figure 10.
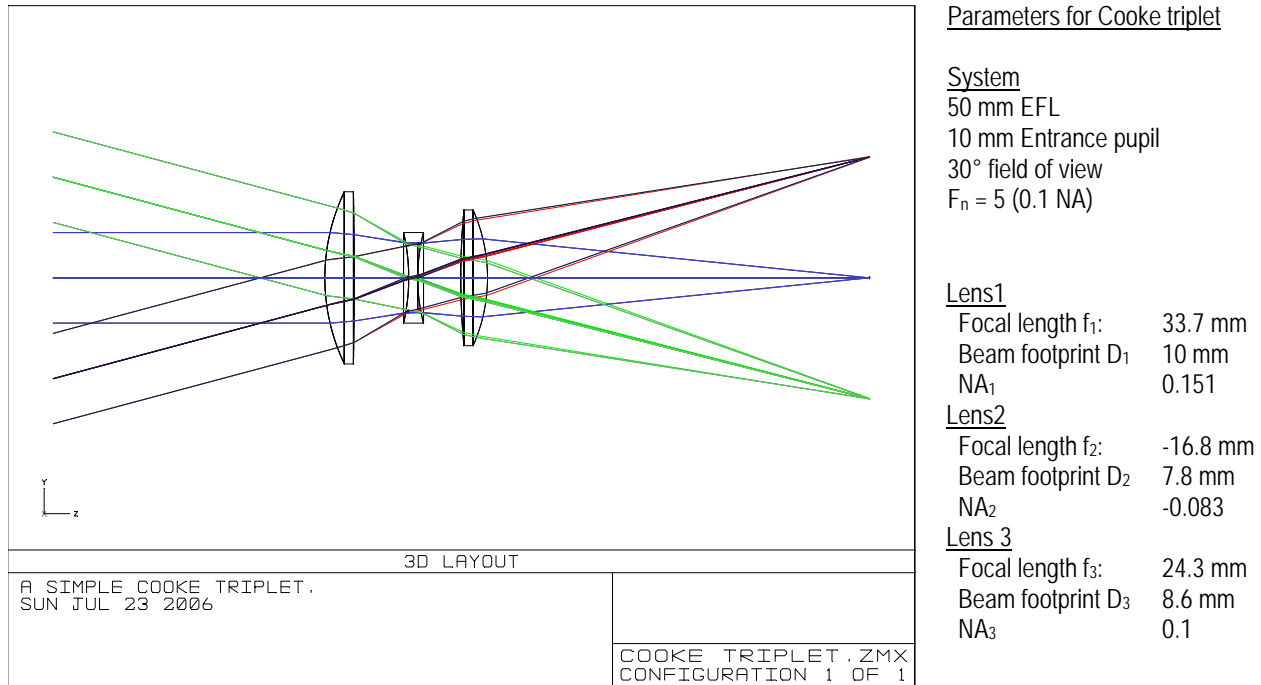
Parameters for Cooke triplet

System
50 mm EFL
10 mm Entrance pupil
30° field of view
F_n = 5 (0.1 NA)

Lens1
  Focal length f_1:   33.7 mm
  Beam footprint D_1 10 mm
  NA_1 0.151
Lens2
  Focal length f_2:   -16.8 mm
  Beam footprint D_2 7.8 mm
  NA_2 -0.083
Lens 3
  Focal length f_3:   24.3 mm
  Beam footprint D_3 8.6 mm
  NA_3 0.1

3D LAYOUT

A SIMPLE COOKE TRIPLET.
SUN JUL 23 2006

COOKE TRIPLET.ZMX
CONFIGURATION 1 OF 1

Figure 10. Cooke triplet used for evaluation of image motion.

As the individual elements were perturbed in Zemax[4], the image motion and the degradation of the image quality were evaluated. First, a large shift is shown to illustrate the concept. Lens 2 was shifted by 1 mm and all images were seen to shift by 2.3 mm as predicted using Eq. 11 with f_2/D_2 = 2.15 for the f/5 system. This large misalignment also caused significant aberrations that varied somewhat with field position, but the image shift was constant across the field, as seen in Figure 11.
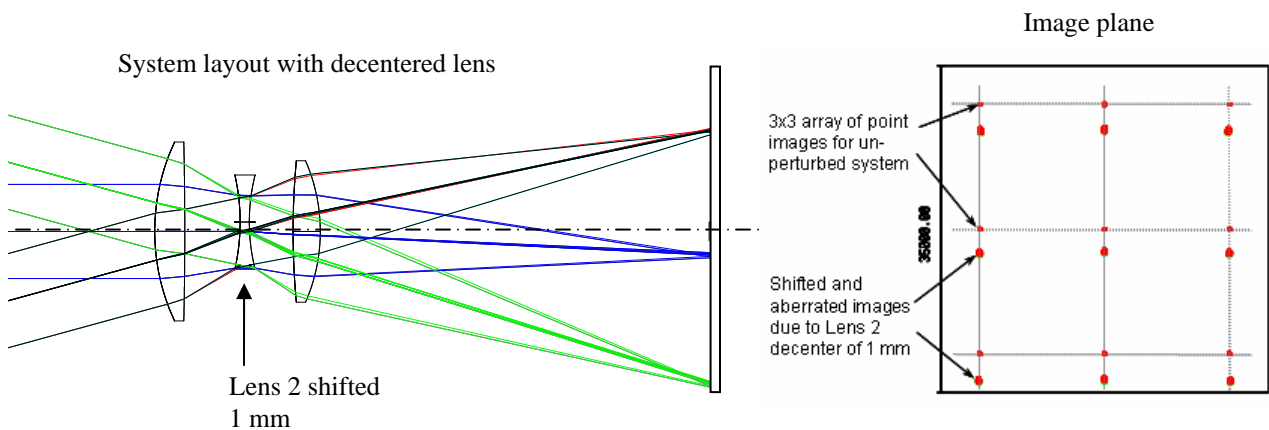
Image plane

System layout with decentered lens

3x3 array of point
images for un-
perturbed system

Shifted and
aberrated images
due to Lens 2
decenter of 1 mm

Lens 2 shifted
1 mm

Figure 11. Image shift across the 35 mm focal plane due to a 1 mm decenter for Lens 2.

The lenses were then shifted by smaller amounts to allow a quantitative comparison. The relationships developed above were evaluated and compared with the direct simulation for all 3 lens elements. The simulation matches the analytical prediction extremely well, as seen in Figure 12. This graph shows simulated image position with the rms image size represented with the error bars.
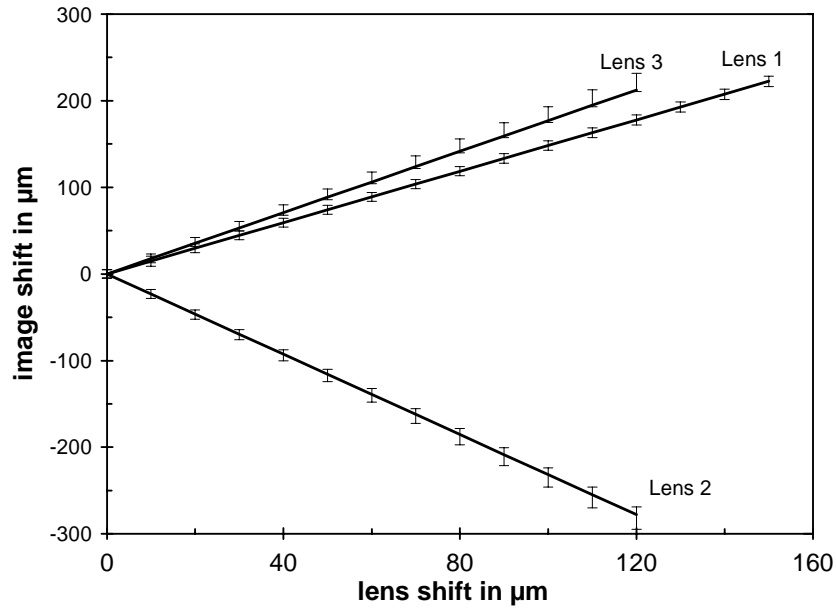
Figure 12. Comparison of analytic prediction of image shift (solid lines) with computer simulation for element decenter. The data points from the simulations are represented with error bars sized according to the rms image blur on axis.

The effect of tilting the lenses was also evaluated and compared with the Zemax[4] simulation, shown in Figure 13. In some cases, the aberrations are larger than the image tilt. In order to make a good comparison, the image position was defined by the chief ray, not by the centroid.
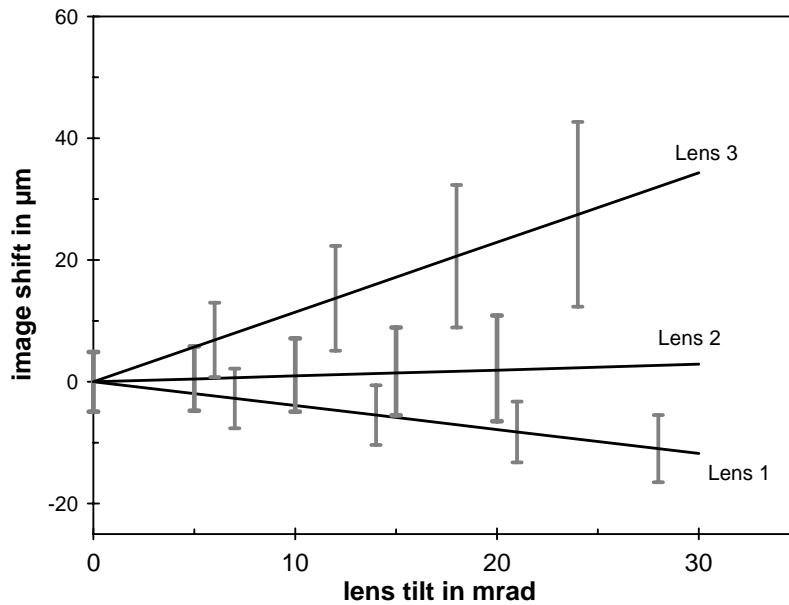
Figure 13. Comparison of analytic prediction of image shift (solid lines) with computer simulation for tilted elements. The data points from the simulations are represented with error bars sized according to the rms radius of the on axis point spread function.

6. FURTHER DISCUSSION

There are some system variations that require some modifications to these relations. A simple modification allows calculation of line of sight stability for infinite conjugate systems, and a change of variables allows calculation of pupil stability. Also, these relations can be used to determine image blur due to slope aberrations across the pupil from optical surface irregularities.

The relationships are derived above for imaging systems where the light comes to focus with a finite f-number. The analysis works the same for infinite conjugate systems where the angular change in line of sight θ_{LOS} can be determined by replacing $\dfrac{\varepsilon}{F_n}$ with $\theta_{LOS} \cdot D_0$ where D_0 is the beam diameter in collimated space. Since the optical invariant was used for the derivation, θ_{LOS} can be evaluated equally well in object space or image space as long as the appropriate beam size D_0 is used.

In cases where the pupil shift is important, the same relationships can be applied where the pupil image is transferred from one space to another. Whether the pupil image is virtual or real, it can be transferred according to the linear imaging relationships and the shift of this pupil image due to element motion can be calculated in the same way as the image shift above. When performing these calculations, a stop for the pupil imaging optics must be assumed, and the values for F_n and *NA* must be taken appropriately for this case. As described above, the solution does not depend on the choice of the stop.

Another application of the relationships can be used to determine the coupling between wavefront aberrations across the pupil and image blur for an optical system. A wavefront aberration, possibly due to an optical surface imperfection, will cause the light to propagate with slope errors across the beam footprint. When these rays intercept the focal plane, they will be shifted from the ideal focus due to the aberration in the same way that the central ray was found to shift. Applying the methods above, the mapping between wavefront slope aberration and image error is

$$\varepsilon(\rho_x, \rho_y) = F_n D_i \Delta \theta_i (\rho_x, \rho_y)$$

Eq. 13

where $\varepsilon(\rho_x, \rho_y)$ represents image aberration
        $\Delta\theta(\rho_x, \rho_y)$ represents the wavefront slope variation across the beam footprint.
        (ρ_x, ρ_y) are the pupil coordinates over the beam footprint with diameter D_i.
        F_n is the final system focal ratio

For example, a 20 mm *f*/10 lens with 2 µm PV astigmatism in one surface will cause wavefront variations of 1 µm PV, which has slope variations of ±100 µrad. If this is used in an optical system where the beam footprint sees the full surface, the image degradation due to the astigmatism in the lens would give a point spread function with 20 µm radius.

7. CONCLUSION

An easy method of relating image motion to tilt and decenter of optical elements was developed. This is directly applicable to systems that require image stability, such as spectrographs, laser projectors, photonic couplers, and systems that require boresight. These relationships can be applied for any element in an optical system without requiring a complete simulation of the system – only a few simple parameters must be known. These same relationships can be applied to determine image degradation from surface irregularities.

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