Invited Paper

Determination of off-axis aberrations of imaging systems using on-axis measurements

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ABSTRACT

Imaging aberrations that have linear dependence on field angle are caused by pupil aberrations that can be described using the Abbe sine condition. This well-known relationship is frequently used to guide the design of optical imaging systems. For example, the aberration of coma is eliminated in the design of axisymmetric systems by controlling the pupil distortion, as defined by a standard implementation of the sine condition. An optical system with misalignments of surface irregularities will suffer pupil distortions that are quantified using a more generalized form of the sine condition. Such pupil aberrations create image aberrations that have linear dependence on field angle. While it is possible to infer the state of alignment by measuring multiple field points, it may be more straightforward to perform a single on-axis measurement of the sine condition violations. This paper summarizes the generalized sine condition and relationship between violations of this condition and aberrations with linear field dependence. An application is discussed for measuring sine condition violations of a 4-mirror system, which allows determination of the off-axis aberrations.

Keywords: Optical design, alignment, aberrations

1. INTRODUCTION

The Abbe sine condition is commonly used in optical design to control linear field dependent aberrations such as coma. A generalized form of the sine condition relates the mapping of the entrance pupil to the exit pupil of the imaging system. If this mapping is linear, as represented using direction cosines, then the system will be free from all aberrations that have linear field dependence. Any measured departure from linear mapping can be used to determine the resulting aberrations.

A well-corrected imaging system will achieve good image quality for points over a finite field of view. An optical system that is both fully corrected on axis, and satisfies the sine condition is said to be aplanatic. [1,2,3] The images will be well-corrected near the axis. If a well-corrected system is misaligned, it is possible to achieve good image quality on axis, yet suffer aberrations that increase linearly with the field angle. The effect of the misalignment can be determined using a sequence of measurements that covers multiple points in the field. This information can be used to estimate the state of alignment. Practical solutions for aligning optical telescopes based on field measurements have been described by McLeod[4] and Noethe[5]. It is also possible to determine the state of alignment with a single measurement. The misalignment that creates the off-axis imaging aberrations will cause pupil distortion that is quantified as a violation of a general form of Abbe's sine condition. So the measurement of pupil distortions allows an assessment of off-axis imaging.

While the linear field dependent errors will dominate, higher order effects will also be present. The aplanatic conditions do not ensure that the images are perfect over the full field because aberrations of second order and higher order are not controlled by these conditions.[6,7] The linear field dependent effects from sine condition violations will generally be added with other aberrations that may be constant in the field or have higher order field dependence. The combination creates nodes, which are points in the field where the various contributions sum to zero. The nodal behavior was discovered by Shack and Thompson[8] and has been since discussed extensively. While the nodes are interesting, this paper assigns them no special significance. The first order effects of misalignment can be fully described by the sine condition violation and the nodal behavior follows.

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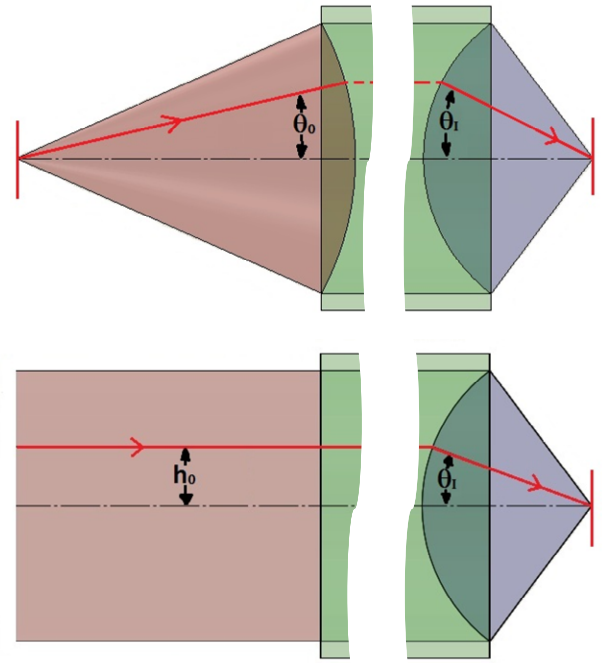
This paper builds on an earlier publication[9] and uses a general form of the Abbe sine condition[10] to show how an offense against this condition can be interpreted as a pupil distortion and how it leads to aberrations with linear field dependence. Some characteristic forms of pupil distortion created from misalignment and shape errors in the optics are presented. An application of measuring the sine condition violation for a 4-m system is presented.

2. THE ABBE SINE CONDITION

The Abbe sine condition was initially developed for the design of microscope objectives.[11] Abbe shows how a relationship between the rays from an object point and corresponding rays in image space will determine the coma in a system. There are two cases to consider – object at finite distance and object at infinity. For the case with the object at a finite distance, the ray angles in object space are compared with those in image space. The ratio of the sines of these two angles should be constant for all rays. This constant provides the system magnification. For an object at infinity, the ratio of the off-axis distance h_o of the ray to the sine of the angle in image space should be constant. This constant provides the effective focal length of the system. These definitions and relations are shown in Figure 1.



Coma free images as long as

$$\frac{\sin\theta_O}{\sin\theta_I} = \text{Constant}$$

$$\frac{h_O}{\sin\theta_I} = \text{Constant}$$

for all rays

Figure 1. The conventional form of the Abbe sine condition provides rules for mapping of ray angles between object space and image space. For axisymmetric systems, the violation against this sine condition causes coma, which varies linearly with field angle.

Since the exit pupil is considered to be the image of the entrance pupil, the optical length along any path from a point on the entrance pupil to the corresponding point on the exit pupil will be constant. The pupils are not perfect images, so the constant path length condition is valid for a narrow range of angles. This is not a limitation for this analysis because we are evaluating only small angles.

The "sine" dependence can be seen from simple geometry for the axisymmetric case by evaluating the change in wavefront that is equivalent to a small shift in the position of a field point. If the object point shifts off axis by a small amount Δx_o, then the wavefront phase in the entrance pupil will change by

$$OPD_o(\theta_o) = \Delta x_o \sin\theta_o . \qquad \text{(Eq. 1)}$$

This is easily seen from the geometry. Since the exit pupil is at an image of the entrance pupil, Fermat's principle teaches that the OPD at a point in the exit pupil matches that of the corresponding point in the entrance pupil, even for small field angles. We can consider the motion of an object point as being equivalent to an OPD change given by Eq. 1. This is typically referred to as tilt in the wavefront. This wavefront tilt will be transferred to the exit pupil, but the mapping may be distorted by the pupil distortion. The wavefront in the exit pupil will have the value

$$OPD_I(\theta_o) = OPD_o(\theta_o) = \Delta x_o \sin\theta_o . \qquad \text{(Eq. 2)}$$

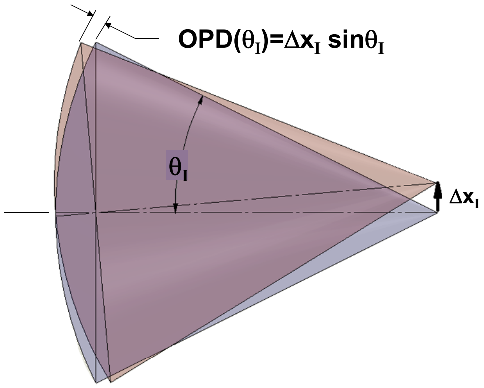


Figure 2. A shifted object or image point is defined as a new reference sphere. In spherical coordinates centered on the axis, the OPD for an off-axis image point, projected in the radial direction, goes as $\Delta x_I \sin(\theta_I)$ as shown above.

For the case where the sine condition is obeyed,

$$\sin(\theta_o) = \frac{1}{m}\sin(\theta_I) , \qquad \text{(Eq. 3)}$$

Combining Eqs. 2 and 3, the OPD in image space can be expressed in terms of image space angle θ_I

$$OPD_I = \frac{\Delta x_o}{m}\sin(\theta_I) . \qquad \text{(Eq. 4)}$$

This image space OPD in the exit pupil for the system that satisfies the sine condition (Eq. 5) corresponds to pure image shift, including the appropriate magnification between image and object space. If the mapping relationship in Eq. 3 is not strictly maintained, then the image will suffer aberrations in addition to the shift.

A general form of the sine condition, generated using derivatives of Hamilton's characteristic functions[5], is needed to model the effects of misaligned system. The coordinate system for this analysis is defined by the object and image planes rather than the optical system itself. For systems with anamorphic magnification, we chose a coordinate system aligned to the principal axes. The generalized direction cosines of the object ray are (p_O, q_O, s_O) and the direction cosines for the corresponding ray in image space are (p_I, q_I, s_I).

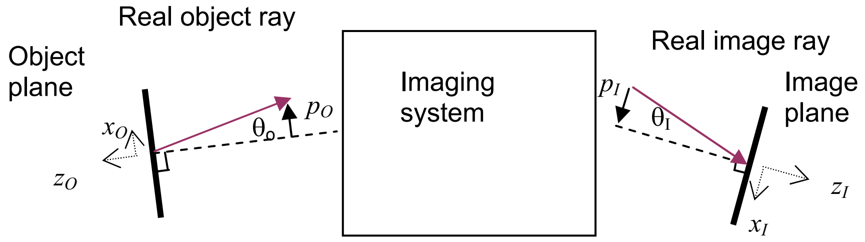


Figure 3. Definition of coordinates for evaluating the general sine condition. The *x* coordinate is shown in the plane here, so the *y* direction and the corresponding direction cosine *q* is out of the plane.

Using Hamiltonian optics, the rays that satisfy the sine condition can be shown to obey the simple relation

$$\begin{aligned} p_I &= m_x p_O + C_x \\ q_I &= m_y q_O + C_y \end{aligned} . \qquad \text{(Eq. 5)}$$

where C_x, C_y are constants and m_x and m_y give the lateral magnification for the principal directions. For axisymmetric systems, this reduces to the familiar

$$\sin\theta_I = m \ \sin\theta_O . \qquad \text{(Eq. 6)}$$

The relations derived from Hamiltonian optics also provide the ability to quantify the effect of a violation against the sine condition. If the ray mapping does not follow the relations in (1), then we can quantify the deviation in terms of pupil distortions. We define the functions d*p* and d*q* as the sine condition violation in direction cosine space:

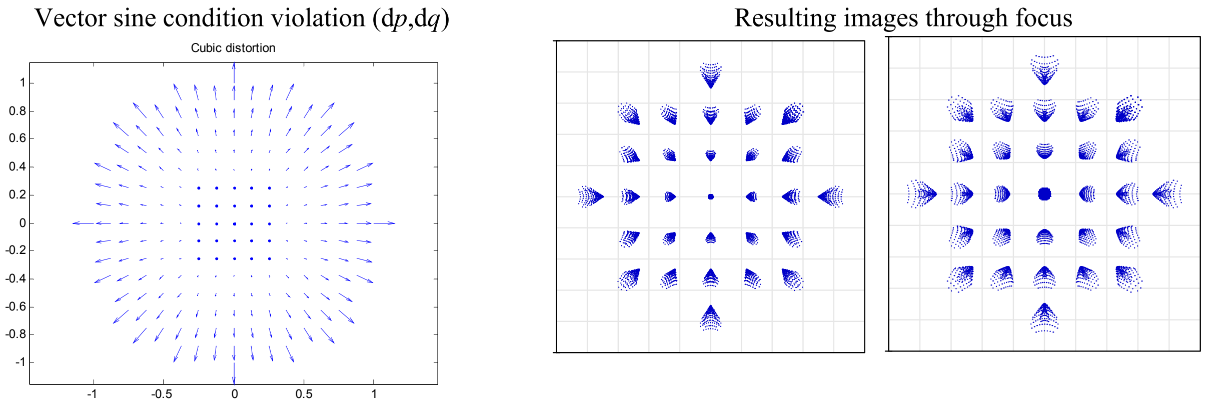
$$\begin{aligned} dp &= p_I - m_x p_O + C_x \\ dq &= q_I - m_y q_O + C_y \end{aligned} . \qquad \text{(Eq. 7)}$$

where m_x, m_y, C_x, and C_y are constants as described above. The linear field dependent wavefront errors follow the simple form

$$\Delta W = dp \cdot \Delta x_I + dq \cdot \Delta y_I . \qquad \text{(Eq. 8)}$$

where (Δx_I, Δy_I) gives the image shift in the same coordinates used to define the direction cosines *p* and *q*.

This simple relationship between pupil distortions and the functional form of the resulting linear field aberrations is demonstrated for the case of coma. The common third order pupil distortion for axisymmetric optical systems causes Seidel coma, which has linear dependence with field angle. The vector form of the sine condition violation in Eq. 8 and the resulting comatic images obtained by simulation are shown in Figure 4.


Vector sine condition violation (dp,dq)

Resulting images through focus

Figure 4. Sine condition violation with the form of cubic distortion creates the linear field dependent aberration of coma.

The conventional interpretation of the sine condition defines the direction cosines p and q along the actual rays. This interpretation is valid in the absence of other on-axis aberrations. For this special case, it should be noted that the stop in the system is never used for the calculation of the aberration, which indicates that all linear field dependent aberrations are invariant to stop shift. This invariance is only true for the special case where there are no on-axis aberrations. A more general case defines the direction cosines p and q according to the mapping of the rays in the exit pupil. For a particular pupil point, the unit vector (p, q, s) points from the image point at the center of curvature of the exit pupil sphere to the point where the real ray intersects the exit pupil sphere. If the system has aberrations at the point being evaluated, then this vector does not point along any real ray. Note that this same convention is used to determine the OPD or wavefront error for imaging systems. The OPD is calculated with respect to the reference spherical wave in the exit pupil. It does not correspond to the path lengths of rays that go from the exit pupil to the image.

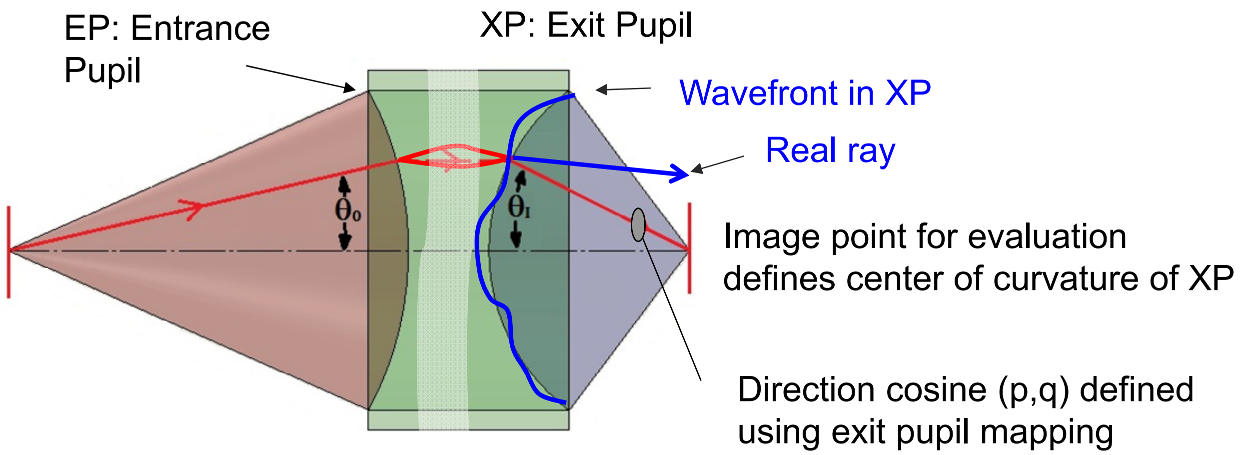


Figure 5. The direction cosines used in Eq. 8 are defined by the ray mapping in the exit pupil and the evaluation point. For the case of on-axis aberrations, this vector does not correspond to any real ray.

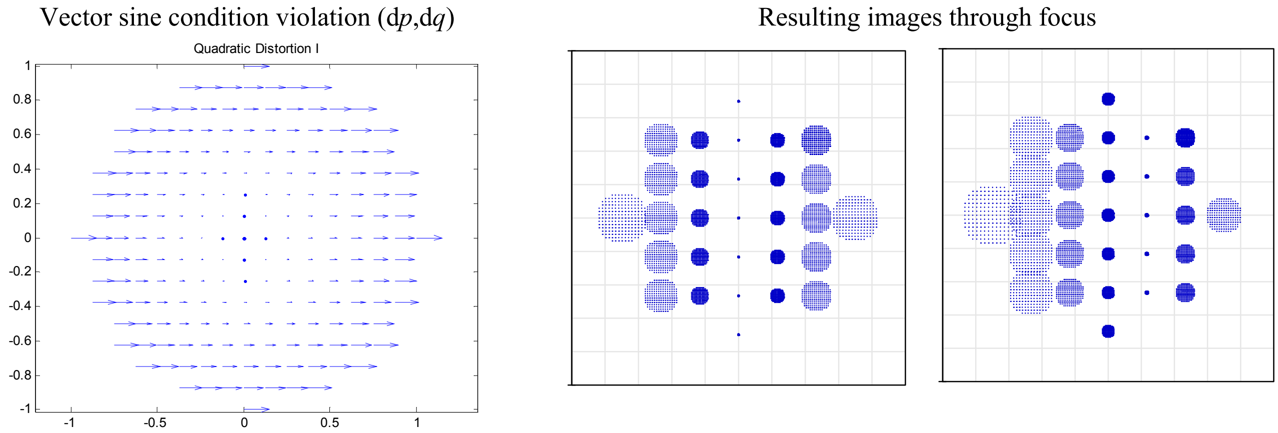
The simple interpretation described above allows Eq. 8 to be fully general. Other researchers have used the conventional definition, and added correction terms to accommodate the coupling of longitudinal aberrations with the linear field dependent terms.[12] Also, the constants m_x, m_y, C_x, and C_y can be considered as arbitrary. Ideally, these would be chosen to minimize the functions dp and dq. But according to Eq. 8, a change in m_x or m_y would only cause a wavefront error of tilt that varies any linearly with field. This is identical to a magnification error. Also, the clocking angle that defines the x and y axes in object space and image space is arbitrary. If the image coordinate axes are rotated with respect to the object plane axes, then a cross-term of wavefront tilt would occur. A field point in the *x* direction would appear to have wavefront with a *y* component of tilt that increases linearly with the *x* field.

3. SINE CONDITION VIOLATIONS FOR MISALIGNED SYSTEMS

The causal relationship between pupil mapping and linear field dependent aberrations extends for mapping errors created my misalignment as well as design. We show specific cases where specific non-axisymmetric sine condition violations are the cause of other linear field dependent aberrations, including linear focus and linear astigmatism.

3.1 Tilt of image plane

A common aberration that has linear field dependence is tilt of the image plane. The images go out of focus by an amount that increases linearly with the field angle. The equivalent sine condition violation can be created by simply rotating the coordinate frame used to define the direction cosines *p* and *q* since these were defined by the normal to the focal plane. A vector plot created by such a transformation, and the resulting images are shown in Figure 6.


Vector sine condition violation (d*p*,d*q*)

Resulting images through focus

Figure 6. Sine condition violation with the form of quadratic distortion is equivalent with linear field dependent focus. This can be created to second order by tilting the focal plane. There is also an anamorphic magnification term which was removed.

3.2 Quadratic pupil distortion

A common form of quadratic distortion in the pupil that is cause by a misaligned system is explored and shown in Figure 7. When the distortion equivalent to focal plane tilt is removed, a particular vector pupil aberration remains, which creates the aberration of astigmatism with linear field dependence, shown in Figure 8. This linear aberration is frequently called “binodal astigmatism” because it frequently combines with the native quadratic axisymmetric astigmatism to create two “nodes” or regions in the field where the two effects cancel.

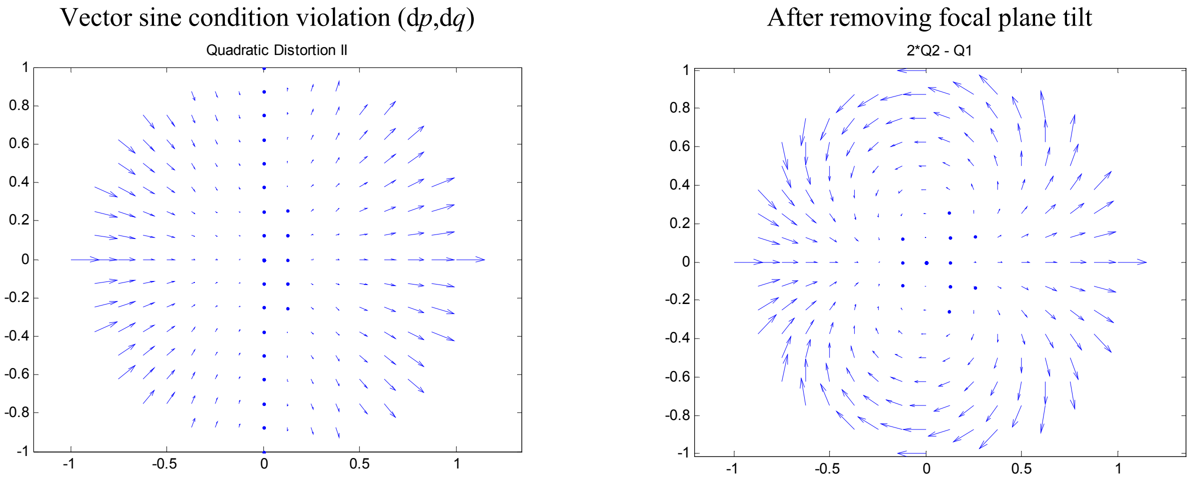


Figure 7. Sine condition violation with the second form of quadratic distortion. After balancing with focal plane, the vector relation on the right remains. This causes linear field dependent astigmatism, below.

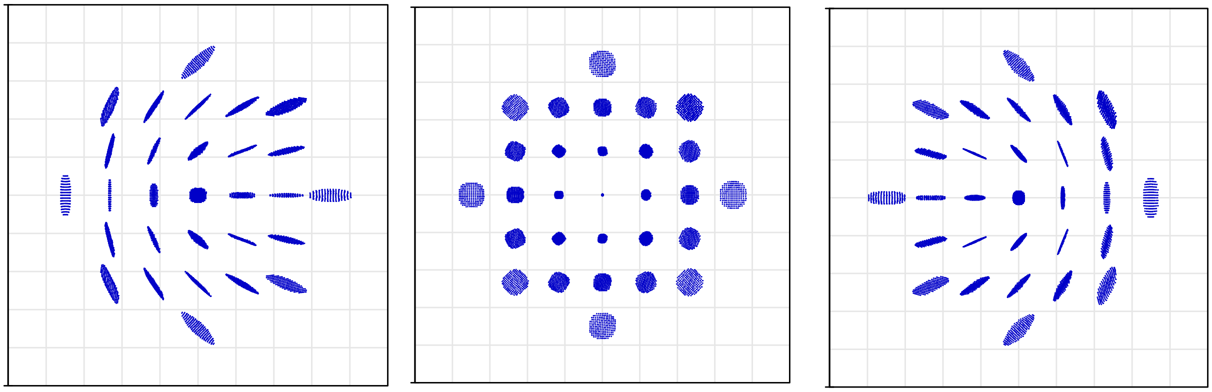


Figure 8. Images through focus showing the characteristic linear field dependent astigmatism. This is often called “binodal” astigmatism because of the appearance when it is combined with Seidel astigmatism.

4. MEASUREMENT OF SINE CONDITION VIOLATIONS

The sine condition violations can be measured by placing a calibrated reference near the entrance pupil and measuring the distortion in the exit pupil. This can be accurately performed by using a grating as test target, then placing a second grating at the image of the target grating. If the magnification of the gratings matches that of the system, then moiré fringes can be used to measure the distortion between the analyzer grating and the image projected by the system. For testing the performance of a known system, the gratings not need to be located at the pupils. The measured distortion is simply compared to the distortion to the ideal value as determined by a ray trace simulation.

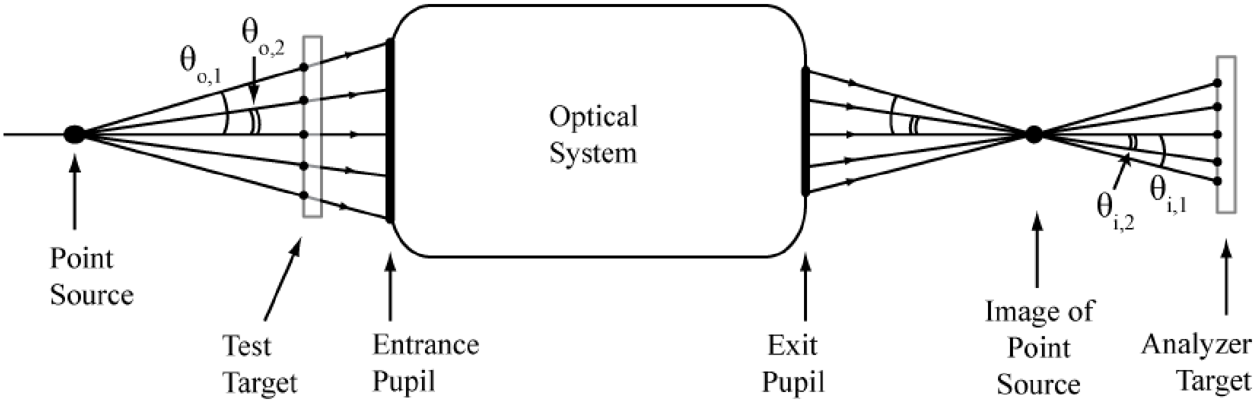


Figure 9. To measure the mapping between object and image space, a test target such as a grating is illuminated, and the distortion is measured by comparing with a master analyzer.

A demonstration of this type of measurement was performed using the layout shown below.[13] The unit under test UUT consisted of a tilted lens. An aperture was inserted into the system to isolate particular orders of diffraction from the test grating.

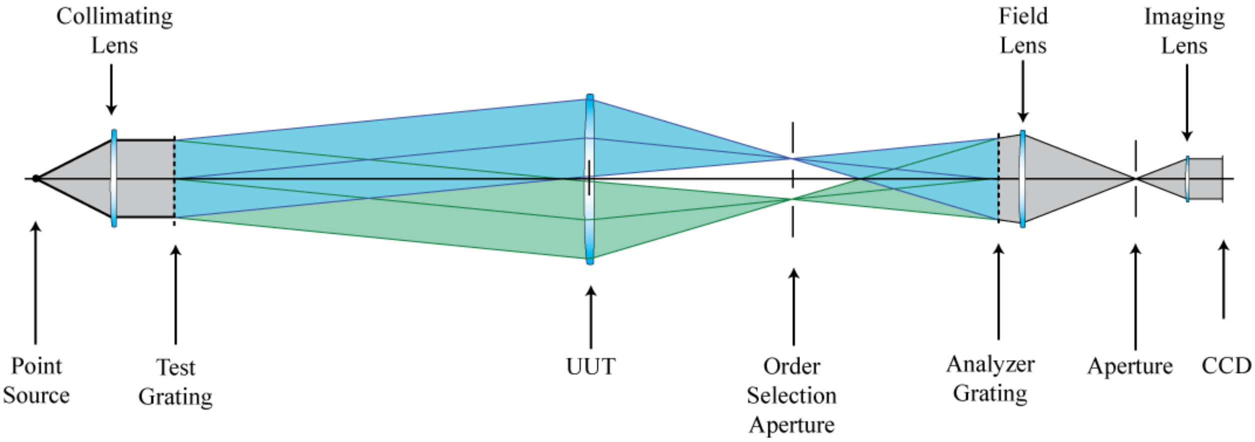


Figure 10. Layout of the test system used by Lampen *et al* to verify the concept of measuring the sine condition violations.

The performance of the test showed the behavior that was expected. High contrast moiré fringe patterns were seen and were measured accurately by shifting the analyzer grating and capturing data every 90°. The intensity maps from 5 successive frames were processed to give the distortion with sensitivity of a few microns. The resulting aberrations matched the expected values to a few percent.

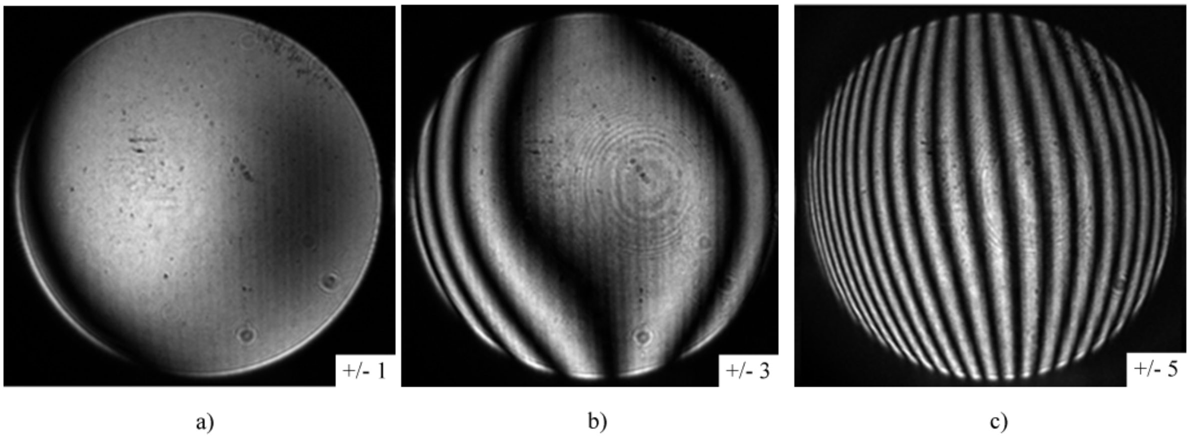


Figure 11. The aperture was adjusted to allow different orders of diffraction to pass. The resulting moiré images show high contrast, and were phase shifted to give good measurements of the pupil distortion.

We are now proposing to measure the pupil distortions for the wide field corrector for the 10-m Hobby Eberly Telescope.[14] The pupil alignment test is interesting for this application because of the difficult of measuring off axis field performance. The wide field corrector relays and corrects images created from the spherical primary mirror. As such, the system by itself has very large aberrations. It is not practical to measure small errors in these aberrations that would come from misalignment in the presence of such large aberrations that are intentionally introduced to balance the effects of the spherical primary mirror.

One implementation of this test uses a video monitor as the source. A fringe pattern is created and viewed through an analyzer. Phase shifting is readily performed with the video image.

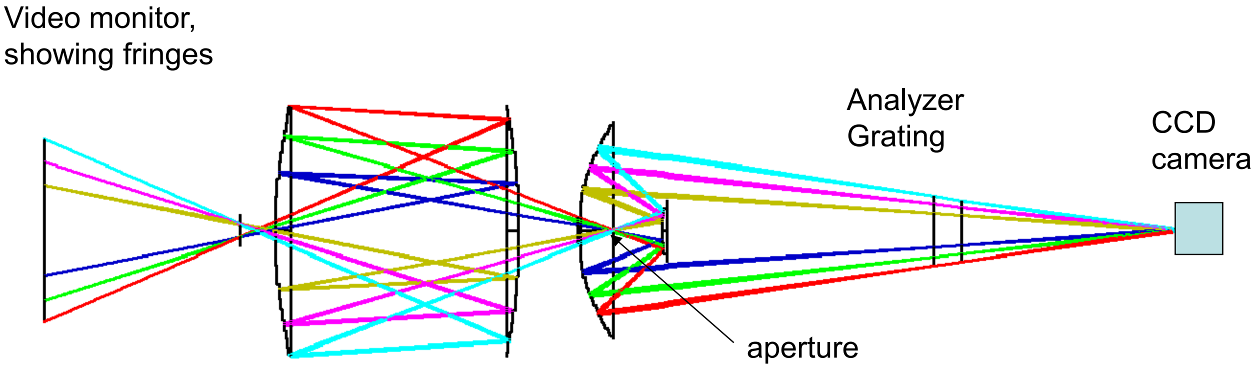


Figure 12. Layout for a possible sine condition test of the 4-mirror wide field corrector for the Hobby Eberly Telescope.

The choice of the video monitor as the source is made based on very successful application of video monitors for another metrology system that measures surface slopes.[15] The accuracy of the test depends on the ability to control systematic errors. Assuming control to 0.02 pixels, accuracy in the mapping can be accurate to 5 µm across the 80 cm screen. This distortion couples with wavefront tilt. At 10 arcmin field for the telescope with 10 m aperture, the resulting wavefront tilt has magnitude ± 15mm. The mapping error of 6 ppm couples with the 15 mm tilt to cause a wavefront aberration that is zero on axis, increasing to a maximum value of 0.09 µm at 10 arcmin field.

5. DISCUSSION

Sine condition violations and the ensuing linear field dependent aberrations can be simply described for general optical systems that lack symmetry. Since a one-to-one mapping between sine condition violations and aberrations with linear field dependence exists, we can use a single on-axis measurement of the sine condition violations to determine distortions that would cause aberrations that have linear field dependence.

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