Use of a flat panel display for measurement of sine condition violations

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ABSTRACT

Previous works have shown the viability of using the Sine Condition Test (SCTest) to verify the alignment of optical systems. The SCTest uses the Abbe sine condition to measure the mapping between the entrance and exit pupils of an optical system. From this pupil mapping, the linearly-field dependent aberrations can be measured and used to verify the alignment. Specifically, the linear astigmatism is used as a metric to determine how well the optical system is aligned. An advantage to using the sine condition to measure the off-axis performance is that the measurement equipment can be placed on-axis. By doing this, the uncertainty of the measurement is reduced, making this test especially useful for verifying systems with large inherent aberrations. In this paper, we expand the design space of the SCTest by exploring the two different source options: a point source with a grating or a flat-panel display. Additionally, we show experimental results of implementing the SCTest using a flat-panel display. Last, we explain how the SCTest can be implemented on more complex systems, such as a three-mirror anastigmat (TMA) and a double Gauss. By exploring the design space, we provide more design options for selecting the SCTest source, increasing the flexibility and utility of the SCTest.

Keywords: Optical system alignment, optical system verification, sine condition, monochromatic aberrations, optical imaging, optical systems, geometrical optics, optics

1. INTRODUCTION

Proposed by Abbe in 1873, the Abbe sine condition gives the relationship between the ray geometry in object space and in image space. Using this relationship, designers can design systems free from linearly field dependent aberrations[1–3]. In this paper we show how the sine condition can be used to measure linearly field dependent aberrations. Specifically, the sine condition violations can be used to measure the pupil mapping error. From the pupil mapping the linearly field dependent aberrations can be measured and used to verify the alignment. Specifically, the linear astigmatism is used as a metric to determine how well the optical system is aligned. Other methods of verifying alignment typically involve measuring the aberrations at different points in the field. For example, the collimation method measures the aberrations at several field points and derives the misalignments of the optical system[4–6].

The advantage of the Sine Condition Test (SCTest) is that it measures the off-axis aberrations using equipment placed on-axis. Because it is more straightforward to align the test equipment on-axis, the uncertainty of the measurement is reduced. This aspect of the SCTest is especially useful for systems like prime focus correctors or uncorrected subsystems that have large amount of inherent aberrations.

In our previous works[7–9] we explained the theoretical background of the SCTest and provided a proof of concept experiment, which showed how to implement the test with a point source and grating. In this paper, we will review the background of the test and then expand on the methods of implementing the test by explaining how a flat-panel display, such as an LCD television or computer monitor, can be used to measure the pupil mapping, as well as some of the advantages and disadvantages between the two approaches. Next, we will show experimental results from using the flat-panel display to measure pupil mapping. Last, we will show how the test can be implemented with more complex systems, such as an afocal three-mirror anastigmat (TMA) and a double Gauss.

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2. BACKGROUND

2.1 Abbe sine condition explanation

The Abbe sine condition is typically thought of as a relationship used by designers to design optical systems that are free from linearly field dependent aberrations. In the SCTest, we come about this from a different perspective by using the sine condition to measure the linearly field dependent aberrations. Figure 1 shows a generic, rotationally symmetric optical system defined by its entrance pupil (EP) and the exit pupil (XP), where the object point *O* is imaged by the optical system onto point *I*. The point $B(x_{EP},y_{EP})$ on the EP is imaged to the XP at point $C(x_{XP},y_{XP})$. Also, ε_o is an off-axis object point a small distance from *O*, and ε_i is its conjugate image point.

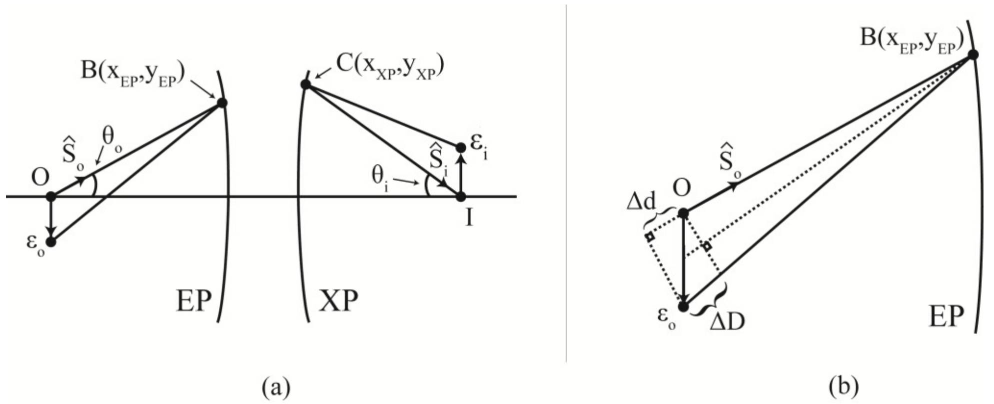

Figure 1. a) General illustration of an axisymmetric optical system with finite conjugates. Annotations are - *O*: object point, *I*: conjugate image point, *B*: point on entrance pupil (EP), *C*: point on exit pupil (XP) conjugate to *B*, ε_o: off-axis object point, ε_i: conjugate off-axis image point, $\hat{\mathbf{S}}_\mathbf{o}$: unit vector pointing from *O* to *B*, θ_o: angle of $\hat{\mathbf{S}}_\mathbf{o}$ with respect to the axis, $\hat{\mathbf{S}}_\mathbf{i}$: unit vector from *I* to *C*, θ_i: angle of $\hat{\mathbf{S}}_\mathbf{i}$ with respect to the axis. b) Close up of object space. ΔD: exact scalar path difference between *O* and ε_o, Δd: approximate scalar path difference. Text and image from Lampen[7].

The wavefront phase difference at the EP at $B(x_{EP},y_{EP})$, $W_o(x_{EP},y_{EP})$, is defined as the scalar path difference between point O and ε_o. Assuming $\Delta D \approx \Delta d$,

$$W_o\left(x_{EP}, y_{EP}\right) \approx \Delta d = \hat{\mathbf{S}}_\mathbf{o} \bullet \boldsymbol{\varepsilon}_\mathbf{o} \qquad (1)$$

Through Fermat's principle, we know that the optical path length between point *B* and point *C* is stationary, so $W_o(x_{EP},y_{EP})$ must equal $W_i(x_{XP},y_{XP})$. The pupil mapping error is then

$$W_{PME} = W_o\left(x_{EP}, y_{EP}\right) - W_i\left(x_{XP}, y_{XP}\right) = \hat{\mathbf{S}}_\mathbf{o} \bullet \boldsymbol{\varepsilon}_\mathbf{o} - \frac{1}{m}\hat{\mathbf{S}}_\mathbf{i} \bullet \boldsymbol{\varepsilon}_\mathbf{i} \qquad (2)$$

where $\frac{1}{m}$ is necessary to account for the magnification between the object wavefront radius of curvature and image wavefront radius of curvature. For a more detailed derivation, see Lampen[7]. Because $\hat{\mathbf{S}}_\mathbf{o}$ can be written as $\left(\left|\hat{\mathbf{S}}_\mathbf{o,x}\right| \sin\theta_{o,x}, \left|\hat{\mathbf{S}}_\mathbf{o,y}\right| \sin\theta_{o,y}\right)$, it is easy to see that in the absence of pupil mapping error, Eq. (2) reduces to the classic Abbe sine condition.

There are two additional important considerations to remember with regards to the sine condition. First, while the illustration in Figure 1 is of a rotationally symmetric system, the SCTest is not limited to rotationally symmetric systems.

Second, the selection of the reference geometry will define the pupil mapping. Specifically, the selection of the reference point, as well as the selection of the reference direction, will define a specific pupil mapping. In Figure 1, the reference point is the object point *O*, and the reference direction is the optical axis. If a different reference point or reference direction is chosen, the calculated or measured pupil mapping will be different.

Because the SCTest is only sensitive to linearly field dependent aberrations, in a rotationally symmetric, perfectly aligned system, only coma will be present in the pupil mapping error if the system does not satisfy the sine condition. However, when the rotational symmetry is broken, the field dependency of the astigmatism gains a linear component, called linear astigmatism.

2.2 Linear Astigmatism Explanation

Linear astigmatism is the linear component of binodal astigmatism first proposed by Shack and Thompson[8]. Linear astigmatism comes from the combination of the native quadratic astigmatism of the optical system and a linearly field dependent component that added as a result of the misalignment the system. For a rotationally symmetric system, linear astigmatism is only present if there is a misalignment. Thus, linear astigmatism is a good metric for quantifying the alignment of optical systems. For systems that are not rotationally symmetric, one can use the deviation of the measured linear astigmatism from the expected linear astigmatism to verify the alignment of the optical system.

Figure 2 shows the full field spot diagrams for linear astigmatism. In this case, the linear astigmatism was created by a misalignment in the horizontal direction, such as an element in the optical system rotated about its vertical axis. Misalignments in optical system always create 0° astigmatism and linear focus along the direction of the misalignment and 45° astigmatism in the direction orthogonal to the misalignment. Therefore, in Figure 2a along the horizontal, red line A, there is 0° astigmatism plus the linear focus, and along the vertical, dotted green line, there is 45° astigmatism. Along the yellow dotted line C is the superposition of 0°astigmatism, linear focus, and 45^{o} astigmatism. Stepping through focus from (a) – (c) shows how the spots for the linear astigmatism changes, and the orientation of the astigmatism rotates 90^{o}. Figure 2d shows a close up of a section in Figure 2b to highlight how the aberration increases linearly with field.

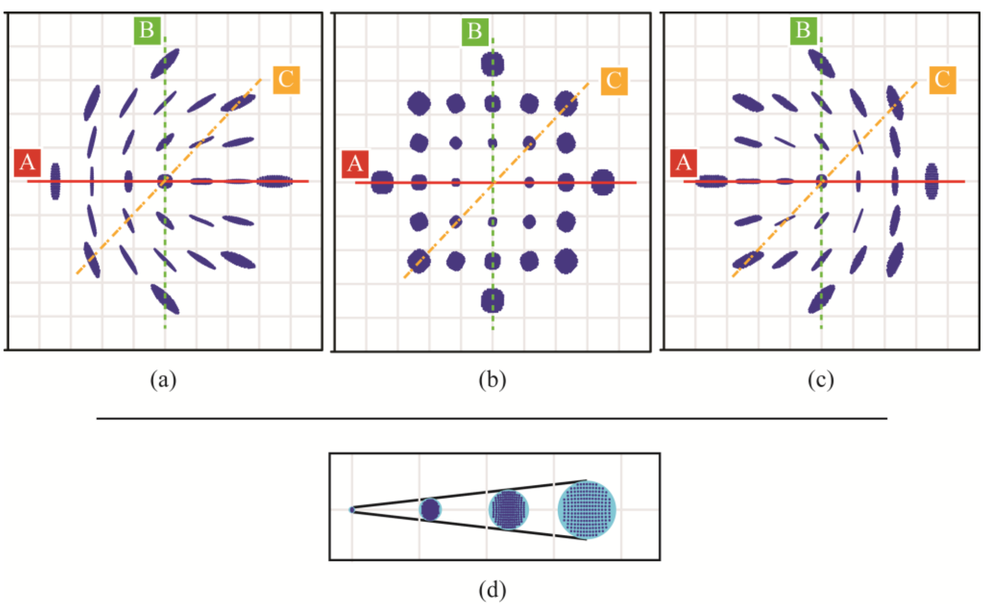

Figure 2. Full field spot diagram of the linear astigmatism created by a misalignment in the horizontal direction. a) – c) Full field spot diagram at three different positions of the image plane. d) Close up of fields in (b) to illustrate how the astigmatism increases linearly with field. Image from Lampen[7].

3. GENERAL IMPLEMENTATION

To use the sine condition to measure the mapping between the entrance pupil and the exit pupil, the SCTest must measure the mapping between the angles θ_o and θ_i. To do this, the angular mapping is converted to a spatial mapping. To illustrate this, Figure 3 shows an optical system defined by its entrance and exit pupil. Again, the SCTest can be used to test the alignment of a non-rotationally symmetric system. In this example, the point source is the reference point, and the test target and point source define the reference direction. Thus, the point source and features on the test target define the angles θ_o into entrance pupil. The test target is imaged onto the analyzer target by the optical system, where features on the analyzer target aid the measurement of the angles θ_i coming out of the XP.

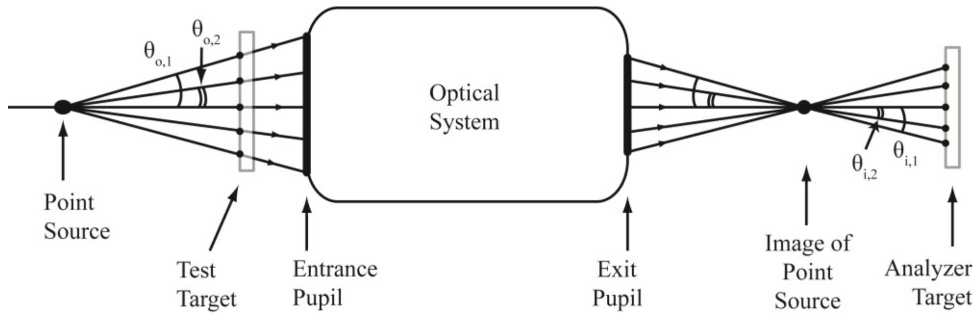

Figure 3. General implementation of SCTest on rotationally symmetric system.Text and image from Lampen[7].

However, the SCTest can be implemented more easily and with higher accuracy by using gratings as the test target and analyzer target. By using gratings as the targets, a Moiré pattern is produced when the test target is imaged onto the analyzer target. For rotationally symmetric system that satisfies the sine condition, the combination of the image of the test target and the analyzer target produces a null fringe if the system was aligned. On the other hand, in a system that does not satisfy the sine condition, the curved lines of the test grating image will form a Moiré pattern with the analyzer target. This forms the basis for the display approach.

To understand the other method of implementing the SCTest, it is useful to explain the test in terms of wave optics. In Figure 4 a collimated beams is incident on the test grating, which diffracts the light into multiple orders. The orders passed through the UUT and order selection aperture, and interfere the analyzer grating, which diffracts the light back along its original direction. The order selection aperture selects which orders are interfered. For this example, the +/- 1 orders were selected.

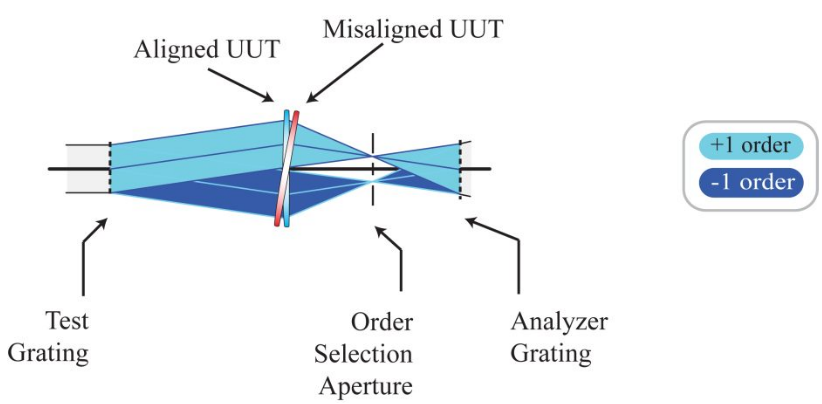

Figure 4. Explanation of SCTest in terms of wave optics, where the test grating and order selection aperture create two beams that interfere at the analyzer, similar to the two paths of an interferometer. Blue lens – an aligned UUT, red lens – a misaligned UUT.

In a well aligned system, the two orders will pick up the equal and opposite amount of linearly field dependent aberrations, such as Seidel coma. When the two orders interfere at the analyzer grating, the amount of linearly field dependent aberrations add. However, the two orders will acquire the same amount of quadratic field dependent aberrations, such as Seidel astigmatism, which will cancel at the analyzer. For misaligned system, like the red lens shown in Figure 4, the amount of quadratic field dependent aberrations acquired by the +/- 1 orders as they pass through the UUT is no longer the same and does not cancel when the interfere the analyzer. This leaves a residual amount of astigmatism that increases linearly with field.

In the context of the reference geometry discussion, the beam incident on the test grating defines the reference direction, and the point source at infinity defines the reference point. Choosing different reference geometry will change the pupil mapping. Additionally, misalignment of the reference geometry or the other SCTest equipment will add linear astigmatism, which increases the measurement uncertainty. For example, if the UUT was aligned and the incident angle of the beam of light on the test grating was changed, the SCTest would measure linear astigmatism even though the UUT was aligned because the reference direction was different.

For systems that are not corrected for coma, it is possible that the measurement of a small amount of linear astigmatism would have to be made in the presence of a larger amount of coma, which creates some practical problems. To remove this problem, either the test or analyzer gratings could be made into a computer generated hologram (CGH). In addition to diffracting in the light into two different orders, the lines of the CGH are curved to add an opposite amount of coma to both orders to cancel out to the native coma of the system. Thus, for a system that is well aligned, the interference of the two orders would produce a null fringe.

4. SELECTION OF THE SOURCE

In this section, we will show how to implement the coherent, point source approach and the display approach, which uses a large, extended source. Note, these two source options are at the opposite ends of the source size spectrum. For some applications, such as interferometry, it is useful to utilize a small extended source to reduce the effect of coherent noise, which can degrade the measurement of higher order aberrations[9]. However, the SCTest only uses low order aberrations to verify the alignment of optical systems. Therefore, there is no advantage to using a small, extended source when implementing the SCTest[10].

To implement the coherent approach, a point source is placed before the test grating as shown in Figure 5a. A collimating lens was also added so that the UUT could be tested at its design conjugates. Additionally, there is an imaging system placed after the analyzer grating that consists of a field lens to converge the light, and an aperture to block the unwanted orders created by the analyzer grating. An imaging lens then images the analyzer grating onto the detector.

In this configuration, the point source defines the reference point. Together, the point source and the collimating lens define the angle of the light incident on the test grating. This defines the reference direction of the SCTest. If the collimating lens was not present, test grating and point source would define the reference direction.

In display approach, shown in Figure 5b, the illumination optics and test grating are combined into the electronic display which is placed at the same plane as the test grating. The UUT then images the electronic display onto the analyzer grating, and a similar imaging system is used to image the analyzer grating onto the detector. In the display approach, the test aperture serves two purposes. First, the size of the test aperture limits the bundle of light that pass through the system in order to ensure that the fringes of the Moiré pattern have a high enough contrast for good measurements. Second, it also defines the reference point of the SCTest. If the test aperture position were shifted, the aberrations of the UUT would change and its pupil mapping would change. Additionally, the electronic display and the test aperture define the reference direction.

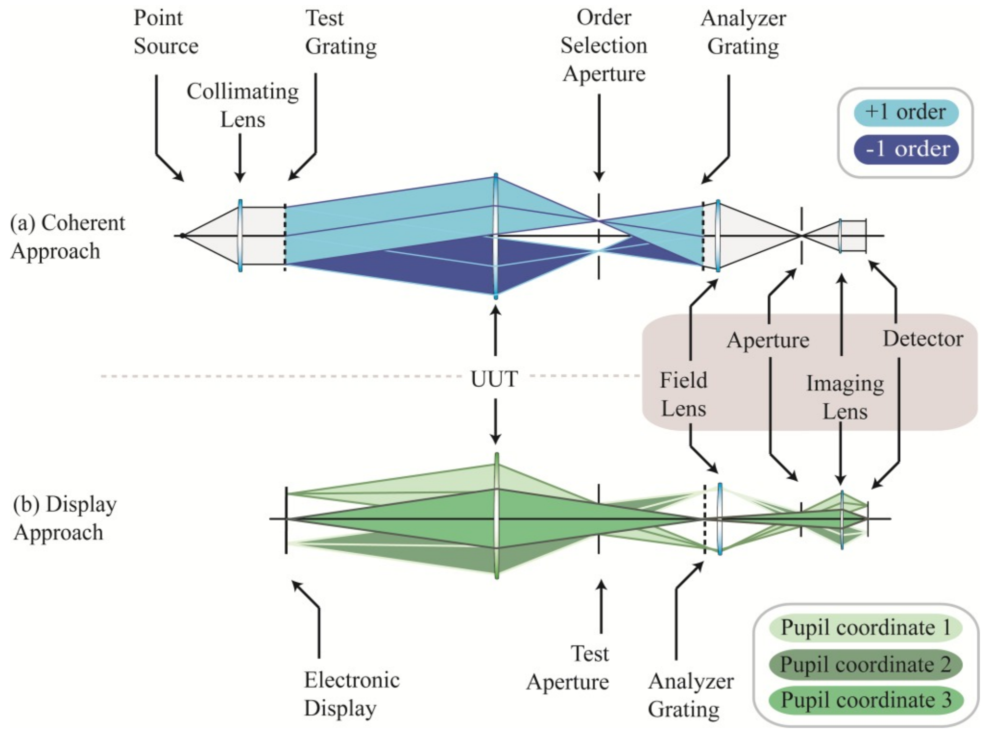

Figure 5. Illustration of the two approaches. a) Coherent approach: Point source is used to illuminate the test grating, diffracting the light into orders, which are selectively interfered. Text and image from Lampen[7]. b) Display approach: Electronic display is imaged by the UUT to the analyzer, where the image of the display pattern forms a Moiré pattern with the analyzer grating. Ray bundle shown are examples of the many ray bundles that propagate through the system.

.One of the advantages of the display approach is it can be implemented in systems that have large entrance pupils. While the test grating and illumination optics in the coherent case can cause some practical problems, it is relatively easy to find a large flat-panel display, such as LCD television or computer monitor. Additionally, the flat-panel display can be used to electronically phase shift the pattern which allows for a simpler optomechanical design of the test. However, the aberrations of the UUT limit the spatial frequency of pattern that can be used on the electronic display and still have good contrast for measurements. On the other hand, in the coherent approach, one can take advantage of the coherent properties of the point source and produce high contrast fringes by selecting two orders diffracted from the test grating to be interfered. Higher contrast fringes mean that one can use a higher spatial frequency test grating. This results in a higher test sensitivity, which lowers the measurement uncertainty. Additionally, the coherent approach has a more diverse set of spatial frequencies to choose from as the designer is not limited by the pixel size of the display.

5. EXPERIMENTAL RESULTS

5.1 General measurement results

Figure 6 shows the proof of concept, benchtop system alongside the illustration of the SCTest. In this experiment, a 10 lp/mm analyzer grating was chosen, along with a micro-display that had 15 µm pixels. Then, the straight line pattern on the micro-display was chosen such that the image of the pattern on the micro-display at the analyzer grating would be close to 10 lp/mm. Because of the pixel size, the closest spatial frequency that could be achieved was 11.11 lp/mm. As a result of the difference of 0.11 lp/mm, we expected approximately 3.5 waves RMS of tilt.

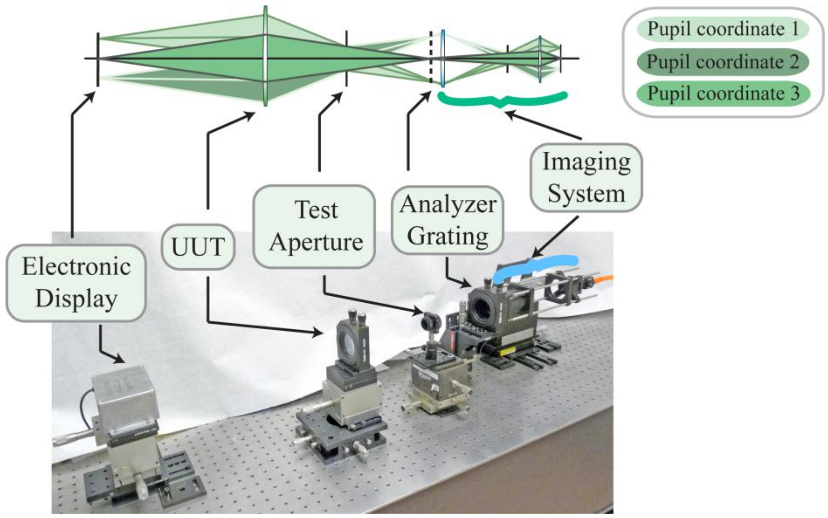

Figure 6. Picture of the experimental setup implemented with the flat-panel display alongside the illustration of the display approach shown in Figure 5b.

A general picture of the Moiré pattern formed at the analyzer grating can be seen in Figure 7 for the two orientations of the grating. While the tilt from the frequency mismatch between the test and analyzer gratings dominates the images, it is possible to see a slight curving of the fringes due to pupil aberrations.

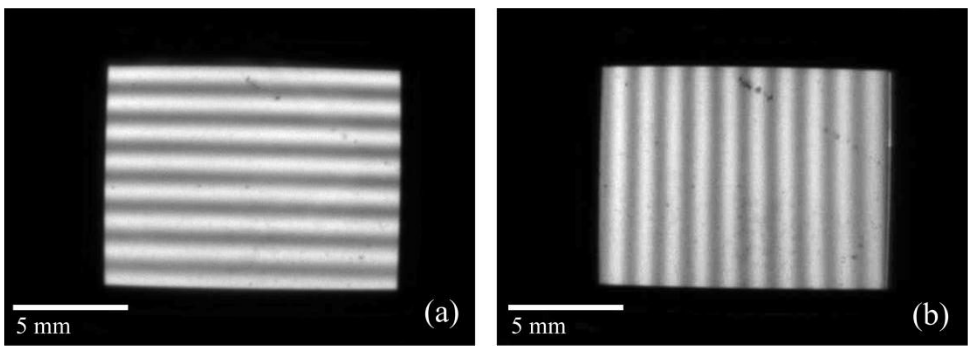

Figure 7. Images of the Moiré pattern formed when the display was imaged onto the analyzer grating by the UUT. (a) Moiré pattern formed when display lines and analyzer are in horizontal orientation to measures the linear astigmatism in the vertical direction. At analyzer plane, the image of display consists of 105.8 line pairs across height of display, versus the 95.3 line pairs across the same section of the analyzer grating. (b) Display lines and analyzer in vertical orientations. The image of the display consists of 141.1 line pairs across the width of the image, while the analyzer has 126.9 line pairs over that same distance.

5.2 UUT tilt sensitivity experiment

To further test our proof of concept experiment, we measured the tilt sensitivity of the UUT. Additionally, the experiment was modeled in Zemax. To collect the data, the analyzer grating was translated to five locations with a 90° phase shift between them. The Hariharan algorithm, or the five bucket algorithm, was used to calculate the phase[11]. Then the phase was unwrapped and fit with Zernike polynomials.

Figure 8 shows the results of the experiment. The blue data points are the measured astigmatism from the benchtop system and the red line is the predicted astigmatism from the Zemax model. In order to account for the spacing errors in the experimental system, the spacings in the model were adjusted. To match the experimental data, the spacings before and after the UUT were adjusted 25 µm and 347 µm from their nominal measured values of 378 mm and 208 mm. Because the experiment is mounted in off-the-shelf parts and aligned with the ruler, these adjustments are within the expected tolerances for the setup. While the measured astigmatism from the experiment has some higher-order dependency, the experimental results match the predicted astigmatism well.

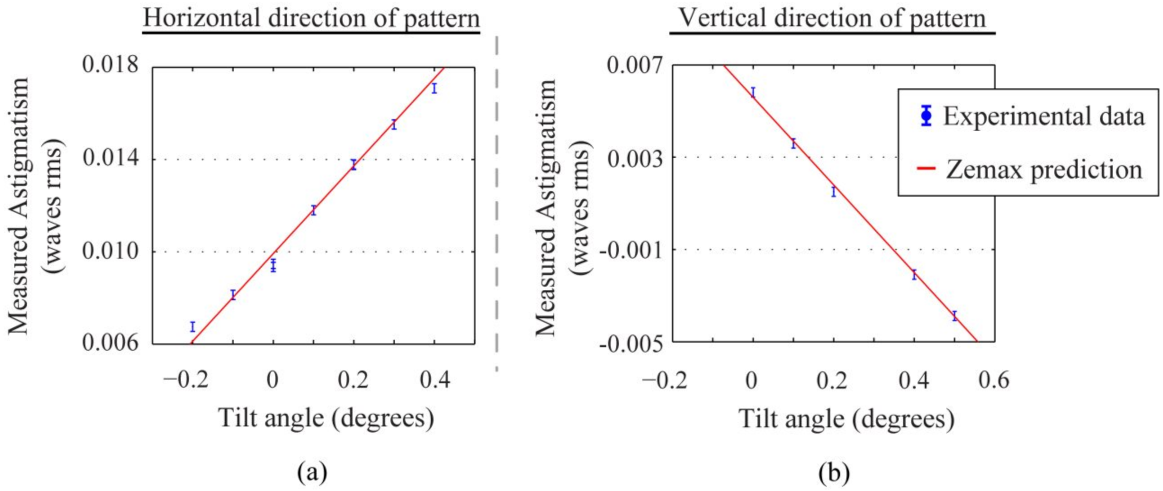

Figure 8. Tilt sensitivities of the UUT shown in Figure 6. Blue data points: the experimental data, red line: Zemax prediction. a) Sensitivity measured when display pattern/analyzer grating were in the horizontal orientation, b) Sensitivity measured when display pattern/analyzer grating were in the vertical orientation.

6. APPLICATIONS OF SCTEST

So far we have only shown SCTest implemented on simple systems. In this section, we will show how to use the test to verify the alignment of more complex optical systems. In both the coherent and display approaches, the optimal placement of the display/test grating and the analyzer grating is at the EP and XP. Additionally, both approaches require the placement of hardware at an intermediate image or at the image plane. As a result, when implementing the SCTest, optical systems fall into two classes: systems where the optimal locations of the test equipment are accessible and systems where this is not the case. Here, we will show how to implement the SCTest on an afocal three-mirror anastigmat (TMA), where the pupils and the intermediate image are accessible. Then we will use the double Gauss system to demonstrate the implementation of the test on a system without accessible pupils.

6.1 Afocal three-mirror anastigmat

Figure 9 shows the implementation of the SCTest on an afocal TMA. In this TMA design the stop is in front of M1 and the XP is after M3 with an intermediate image between M2 and M3. Because the location of the EP and XP in this design, the placement of the SCTest equipment is relatively straightforward, where the test grating or display is placed at the EP and the analyzer grating is placed at XP. Additionally, the order selection aperture or the test aperture is placed at the intermediate image between M2 and M3. While locations of the test equipment are relatively straightforward, it is important to remember that the misalignment of the test equipment will add uncertainty to the measurement.

6.2 SCTest implementation on systems with inaccessible locations

For systems that do not have accessible pupils and an intermediate image, such as the double Gauss system shown in Figure 10a (Zemax stock lens, 28^{o} field), where both the EP and XP are inside the optical elements, additional steps are needed to implement the SCTest. There are two methods to implementing the SCTest on such systems. First, the stop of the system can be shifted so that the EP and XP are accessible. Second, the pupils can be re-imaged to locations that are more accessible.

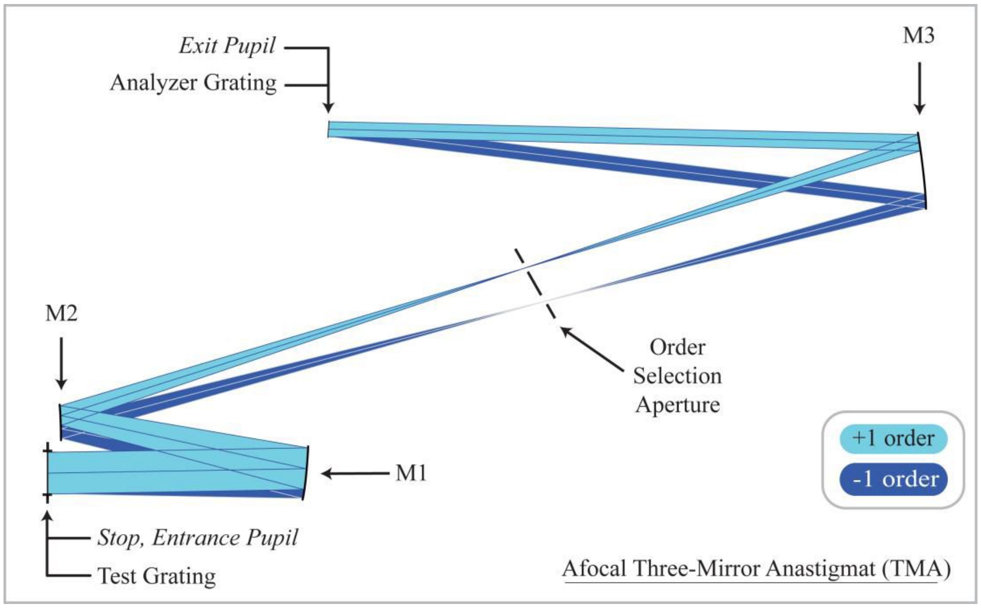

Figure 9. Implementation of the coherent approach of the SCTest to verify the alignment of a TMA. For the display approach, the display is placed at the EP, the analyzer grating at the XP, and the test aperture at the same plane as the order selection aperture.

6.2.1 SCTest implementation on Double Gauss – stop shift

With the first method, a limiting aperture was placed in front of the double Gauss system as shown in Figure 10b to shift the stop location. The test grating or the display can now be placed at the location of the EP, and the analyzer grating can be placed at the XP. While this approach can be implemented with no additional optics, it does have the disadvantage that it alters the aberrations of the system, changing the expected pupil mapping. This means that the amount of coma and astigmatism measured by the SCTest will change, and that the amount of linear astigmatism added for a given set of misalignments will be different. Therefore, before using this approach, it is important that the designer redefine the level of acceptable astigmatism the modified UUT must have to pass the SCTest. Additionally, some optical systems have a stop physically built into the system, which will limit the size of the test grating or display.

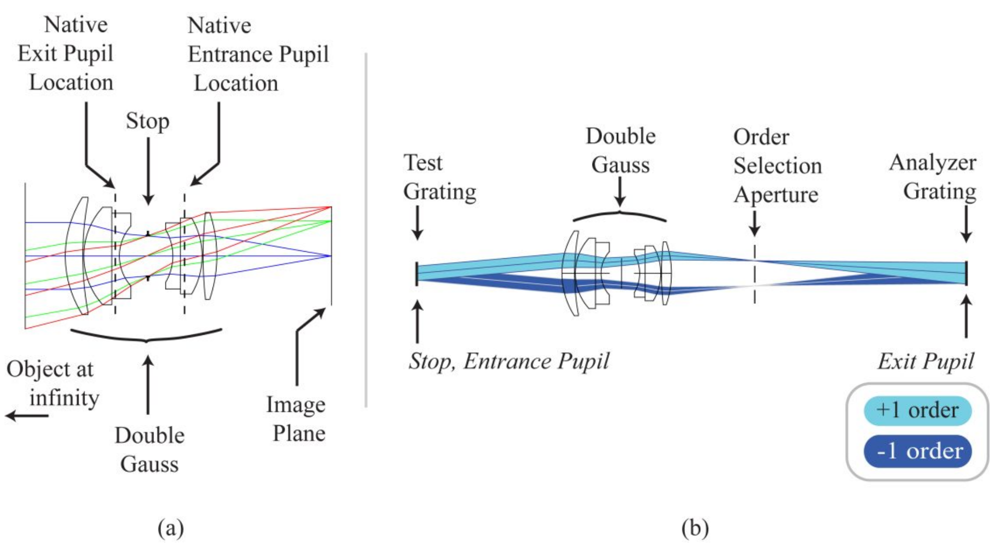

Figure 10. a) Generic double Gauss, Zemax stock lens, 28° field, b) SCTest on double Gauss, where stop is shifted outside the system. For the display approach, the display is placed at the EP, the analyzer grating at the XP, and the test aperture at the same plane as the order selection aperture.

6.2.2 SCTest implementation on double Gauss – re-imaged pupils

The other approach to implementing the SCTest on systems that do not have accessible pupils is to re-imaged the pupil to a more accessible location. Figure 11a shows how to verify the alignment of a double Gauss using the SCTest implemented in this manner. In this case, L2 images the test grating onto the EP of the double Gauss, and L3 images the XP of the double Gauss onto the analyzer grating. L1 was added so that the light entering the double Gauss would be collimated, allowing the double Gauss to be tested at the same conjugates that it was designed to be used. The test grating and L1 could also be combined into a single grating by combining the test grating with a Fresnel zone plate during fabrication. While this method of implementing the SCTest does allow the UUT to be measured in the same way that it was designed, the additional optics do add measurement uncertainty as the misalignment of L1, L2, or L3 will also add linear astigmatism to the measurement, increasing the measurement uncertainty. To reduce the risk of misalignment of the test equipment, it may be possible to align the auxiliary optics separately by using the SCTest, as shown in Figure 11b. In this case the same test grating could be used; however, the frequency and location of the analyzer grating would change.

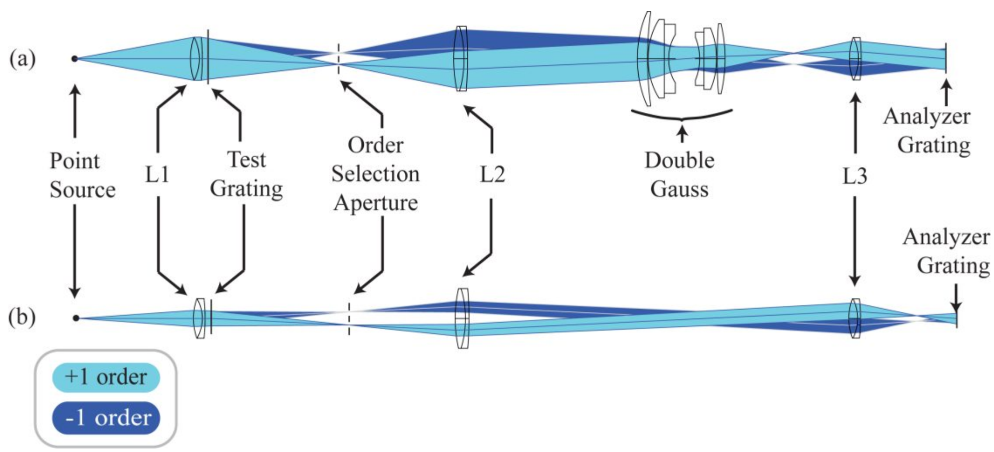

Figure 11. a) SCTest on double Gauss where pupils are re-imaged to a more accessible location. b) SCTest on auxiliary optics of the test to verify the alignment of the test equipment before testing the double Gauss.

7. CONCLUSIONS

In this paper, we have expanded the design space for the implementation of the SCTest by elaborating on different design options. One of the main design decisions to choose between these options is the choice of the source: a point source with a grating or a flat-panel display. In the course of this paper, we explained the background of the two choices, how to implement them, and what the pros and cons of the two approaches are. Next, we showed experimental results from the implementation of the flat-panel display in the SCTest. Last, we explained how to verify the alignment of more complex optical systems, such as the TMA and double Gauss.

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