Characterization of alignment using measurements of the sine condition

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ABSTRACT

An experimental test for violations of the sine condition is presented. This test is particularly useful for identifying the state of alignment of an imaging system because it provides a direct measurement of the linear astigmatism (astigmatism that varies linearly with field) in a system using only on axis measurements. The concept of the test is explained from the perspective of both geometrical optics, using the sine condition, and wave optics. In addition, the results of an experimental proof of concept are presented. This experiment shows good agreement between the measured and predicted results.

Keywords: sine condition, alignment, linear astigmatism

1. INTRODUCTION

This paper presents the SCTest which is a measurement that uses the sine condition to characterize the alignment of an imaging system. The SCTest could be used to measure a system's alignment, or it could be used as an alignment tool. Because it is based on the sine condition, this test is used to measure the linearly field dependent aberrations, and it is insensitive to others. The key linear aberration is linearly field dependent astigmatism because it is a signature of misalignment for many imaging systems. Because the SCTest is not sensitive to all aberrations, it is likely to be used in conjunction with another test.

One of the key advantages of the SCTest is that it can be used to measure the performance across the field of view of a system when all of the test components are aligned to the optical axis. One class of systems where this can be an advantage is the testing of an uncorrected sub-system. Some sub-systems are not well corrected across the field and the aberrations due to misalignment are small compared to the aberrations that should be present. This makes it difficult to accurately measure the performance with low enough uncertainty to insure that alignment is acceptable. Another example of how this test can be useful is that it gives a useful measurement of the performance across the field without taking many measurements. This can be useful when the measurements are difficult or costly to make.

First, this paper will present a brief overview of the sine condition and linear astigmatism. After that, the concept of the SCTest will be explained. While the sine condition is based on geometrical optics, a wave optics explanation will also be presented to provide additional insight. Finally, an experimental validation of the SCTest is presented. A more detailed derivation of the SCTest is published elsewhere.[1]

2. THE SINE CONDITION

The sine condition has been used for many decades to design imaging systems.[2] The sine condition states the required relationship between angles coming from an object point and angles going to an image point for the system to have no linearly field dependant aberrations.

$$\frac{\sin(u)}{\sin(u')} = m \tag{1}$$

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where u is the angle a ray makes in object space, u' is the angle the same ray makes in image space and m is the magnification.

The sine condition is directly related to the pupil mapping. It is convenient to imagine a grid of rays that is traced into the entrance pupil from an on axis point. The sine condition can be used to tell what the spacing should be for the grid that is formed in the exit pupil. Any departure from the sine condition means that there will be linearly field dependant aberrations in the system. The form of this mapping error can be used to determine the slope of the aberrations versus field position.

While it is convenient to think about the sine condition as the ideal mapping from the entrance pupil to the exit pupil, knowing the location of the ray at the pupils is not required for evaluating the sine condition. The sine condition imposes a requirement on the angles of the rays only, and it can be evaluated at any surface that is convenient as long as it is defined correctly. The significance of this is that for any experimental system that measures a departure from the sine condition, there is no specific location where the measurement must take place. Any approach that measures the angles in object and image space will work. An analysis of the relationship between the mapping of the ray angles and the linear field dependent aberrations is provided elsewhere.[3]

3. LINEAR ASTIGMATISM

It is well known that the symmetry of an optical system limits the forms of aberration that could be present in the image. In a rotationally symmetrical system, the only Seidel aberration that has linear field dependence is coma.[4] This is why systems that satisfy the sine condition are often referred to as coma free. While they do not have coma, they also do not have any other linearly field dependant aberrations.

This becomes significant when one considers non-rotationally symmetrical systems. For these systems, there are many more forms that the aberrations can take. The sine condition, however, does not require rotational symmetry and applies to non-rotationally symmetrical systems. In other words, by measuring the violation of the sine condition, one can determine all of the linearly field dependant aberrations in the system and not just coma.

When one misaligns a rotationally symmetrical system, a number of aberrations are added. The lowest order aberrations do not vary with field. For example when one misaligns a system, it is common to see an aberration that is constant with field. A simple on axis test can be used to detect this state. It is possible, however, to have aberration free imaging at one point but still have a misaligned system. A Cassegrain telescope is a good example. By tilting and decentering the secondary mirror, one can achieve aberration free imaging at one point, but the performance across the field will be less than ideal.[5,6]

After aberrations that are constant with field, the next term that one sees will be aberrations that are linear with field. The key one is linearly field dependant astigmatism (linear astigmatism). This aberration has the same pupil dependence as Seidel astigmatism, but the field dependence is linear. In addition, the orientation of the pupil dependence is a function of the orientation of the field point.

$$W_{LA} = W_x h_x \rho^2 \cos(2\theta) + W_y h_y \rho^2 \sin(2\theta) \tag{2}$$

where W_{LA} is the linear astigmatism, W_x is the magnitude of the aberration in the X direction, h_x is the magnitude of the field in the X direction, W_y is the magnitude of the aberration in the Y direction, h_y is the magnitude of the field in the y direction and ρ and θ define the location of the ray in the pupil in polar coordinates. Since this aberration is not rotational symmetrical, the choices for the X and Y axes are not arbitrary. For simplicity, it has been assumed that the axes where aligned to the aberration. Often the linear astigmatism comes from shearing of coma. When this is the case, the magnitudes W_x and W_y are the same and there is also a linear focus term. This focus term is what one would see if the image plane were tilted. Figure 1 shows spot diagrams of this type of linear astigmatism for three different amounts of defocus. When this type of linear astigmatism is added to Seidel astigmatism, one sees binodal astigmatism.

Because linear astigmatism is only present in systems that are not rotationally symmetrical, it can be used as an indication of the state of alignment in a system that is designed to be rotationally symmetric. In addition, because linear astigmatism is linear with field, any system that has it will not satisfy the sine condition. This means that it is possible to assess the state of alignment of a system by measuring the violations of the sine condition that result in linear astigmatism.

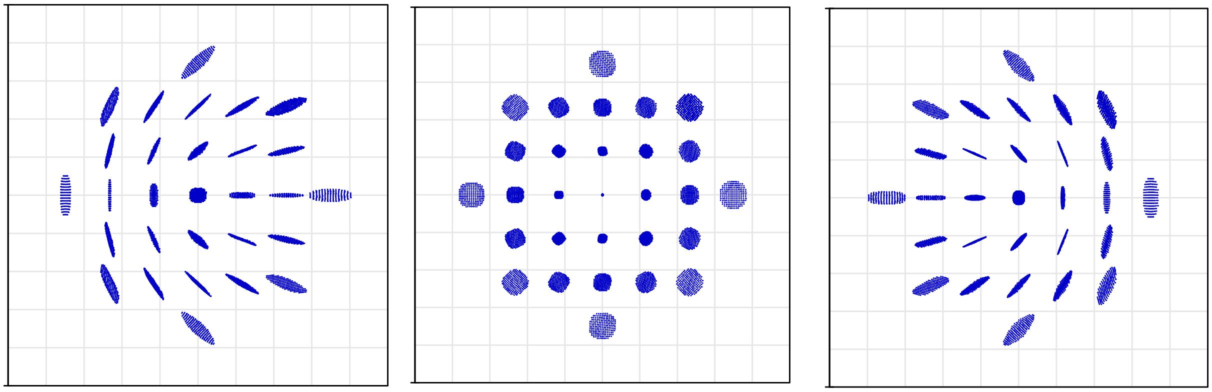


Figure 1. Through focus spot diagrams of linear astigmatism for three different amounts of defocus. Each plot shows the spot diagrams for points across the field of view. The plot in the center has no defocus added while equal and opposite amounts of focus have been added to the plots on the left and right.[3]

4. EXPERIMENTAL CONCEPT

To measure if a system satisfies the sine condition, one simply measures the angle a ray makes in object space and the corresponding angle it makes in image space. There are many ways that one could design a measurement system to do this. First, a generic concept will be presented. After this, an approach using periodic structures will be described.

4.1 Generic measurement concept

To determine if a system satisfies the sine condition, one must define and measure ray angles. For this generic concept, angles will be defined by placing alignment features at a known location from a point source. The vector from the point source to any feature will define an angle in two directions. Figure 2 shows a conceptual layout for the SCTest. One places a point source at the ideal object point location. After this, one places a test object that consists of an array of alignment features. They could be as simple as an array of dots. Because the SCTest is based on angles, the location of the test object is arbitrary. Once the position has been selected, however, the test object must be positioned properly for the alignment features to define the desired angles. The point source and test object create an array of angles in object space

An analyzer object is placed in image space so that it is conjugate to the test object. Alignment features, such as crosshairs, on the analyzer object will define angles with respect to the location of the image point. This creates an array

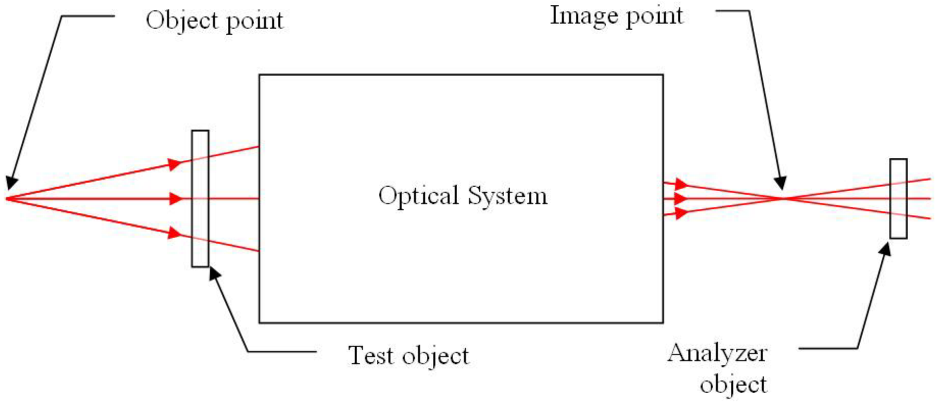


Figure 2. Conceptual layout of test configuration. The optical system images the object point to the image point. The test and analyzer objects are used to measure the angles of rays.

of angles in image space. One can design the analyzer object so that the angles it defines are the angles the rays in object space should make if the system satisfies the sine condition. If this is the case, then one measures the distance between the image of a test object dot and its corresponding crosshair on the analyzer. This error is a measurement of the departure from the sine condition. By plotting the error as a function of pupil coordinate, one can calculate the linearly field dependant aberrations.

If the system being tested does not satisfy the sine condition when it is properly aligned, the errors due to a misalignment could be small compared to the amount of coma. Any noise that is a function of the mapping error could make this measurement difficult. If this is the case the SCTest can be modified to give a null test even with large amounts of coma. This can be done because the analyzer object does not have to represent the angles that satisfy the sine condition. If the ideal system has coma, then this departure from the sine condition can be designed into the analyzer. This is one of the features that makes the SCTest useful for measuring sub-systems that are not well corrected across their field of view.

4.2 Use of periodic test and analyzer objects

While the concept described above would work, it requires multiple, high accuracy measurements to evaluate the angles across the pupil. By using a periodic structure, it is possible to collect all of the data simultaneously. If the test and analyzer objects are amplitude gratings, then one will see a Moiré pattern when the test object is imaged onto the analyzer object. This Moiré pattern will show how the image of the test object differs from the analyzer object. This is a direct measurement of mapping error from the test grating to the analyzer grating. By controlling the positions of the gratings with respect to the optical system, this mapping error is a measure of how the system departs from the sine condition. The significant advantage of this approach is that one needs precision gratings instead of performing multiple precision measurements. Given the current state of computer generated hologram manufacturing, obtaining high quality gratings is straightforward.

An additional feature of using periodic objects is that one can move either the test or analyzer object and the Moiré pattern will phase shift. By applying phase shifting algorithms from interferometry, it is possible to measure the error across the pupil by capturing a few pictures and processing them. This means that it is possible to measure the performance across the field with one experimental setup using on-axis measurements.

5. WAVE OPTICS EXPLINATION

While some systems may not be suited to tests using gratings, the advantages of transferring the precision to the grating manufacturing and being able to phase shift make this version of the SCTest attractive. Because it is likely that most applications of this test will use gratings, it is useful to explain what is happening from a wave optics perspective. Figure 3 shows a revised schematic of the test concept that includes gratings for the test and analyzer objects.

When the light from the object point passes through the test grating, multiple diffracted orders are created. Each of these orders creates a virtual object point. Two virtual object points are shown because the rest of the diffracted orders are blocked by an aperture. For simplicity, it will be assumed that the $\pm 1^{st}$ orders are passed. If the object point is on the axis of the optical system and the test grating is perpendicular to it, the virtual points created by the $\pm 1^{st}$ orders will be equidistant from the optical axis. While the magnitudes are the same, they are on opposite sides of the field, so they will have different signs for the value of h when it comes to calculating the aberrations.

When the light from these orders passes through the optical system, each wavefront will be aberrated. Any aberration that has an even field dependence (h^0, h^2, h^4 …) will be added in equal amounts to both wavefronts. Any aberration with odd field dependence (h^1, h^3, h^5 …) will be added in equal but opposite amounts to both wavefronts. When the light exits the system, an aperture is used to block all but a matched pair of orders. It could pass the $\pm 1^{st}$, $\pm 3^{rd}$, $\pm 5^{th}$, etc. orders.

The light is then incident on the analyzer grating. Each of the wavefronts incident on the analyzer will produce a set of diffracted orders. In the system shown in Figure 3, each of these diffracted orders will create a virtual source located in the image plane. For each incident wavefront (i.e. each virtual object point), one of the diffracted orders from the analyzer will create a virtual source that is located on the optical axis. Since only two orders are allowed to pass the order selection aperture, there will be only two virtual sources created on axis by the analyzer grating. When these two

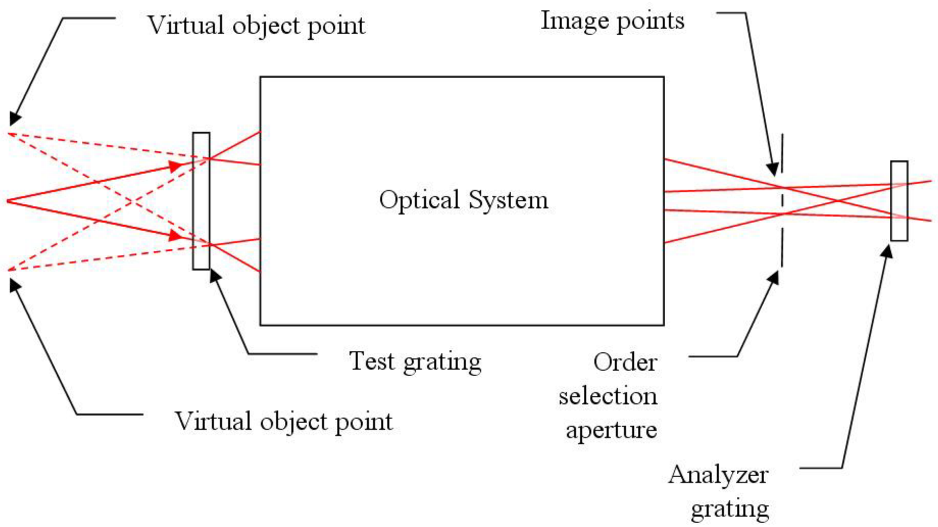


Figure 3. Conceptual layout of test using gratings. Only two orders are shown for the test and analyzer gratings. The test grating forms a pair of virtual object points. In the plane of the image of the points an aperture is positioned that passes only the two desired orders from the test grating.

orders interfere, the pattern that is created is identical to the Moiré pattern described above. The other orders from the analyzer grating will create the higher harmonics in the Moiré pattern and can be filtered out optically or electronically.

Because the aberrations with even field dependence are added in equal amounts to both wavefronts, they cancel in the interferogram. The aberrations with odd dependence, however, are added in equal and opposite amounts, so they will show up in the pattern. One will see coma or linear astigmatism if they are present in the optical system. There could be a significant amount of coma in a sub-system which would add uncertainty to the measurement of linear astigmatism. One advantage of the SCTest is that the gratings themselves can add coma to the wavefronts. One can subtract this coma with either the test or analyzer gratings by adding coma to the grating. Now, instead of being a straight line grating, one will be a computer generated hologram that adds tilt and coma to the wavefronts.

The wave optics explanation of this test highlights one of the potential sources of error. If the object point is not on the optical axis, then the two virtual object points will not have the same magnitude for their field position. This means that each wavefront will not get equal amounts of even aberrations with quadratic and higher field dependency. If there is any Seidel astigmatism in the optical system, one would see linear astigmatism in the measurement of an aligned optical system.

6. EXPERIMENTAL SYSTEM

To test this concept, an experimental system was built with off the shelf parts. In addition, a computer model was developed to simulate the expected results. In the simulation, three degrees of freedom were varied to account for alignment uncertainties in the measurements. The system and results are presented below.

6.1 System layout

Figure 4 shows a schematic layout of the experimental system. The optical system that was tested is a 200 mm focal length cemented doublet with a 50 mm diameter. Because the lens being tested works at infinite conjugates, the object point should be located at infinity. To create this, light from single mode fiber was collimated. A 10 line pair per millimeter Ronchi ruling was used for the test object which was placed approximately two focal lengths in front of the optical system. At the rear focal plane of the optical system the different diffracted orders come to focus. An adjustable aperture was placed at this location. This aperture always blocked the zero order, and it could be adjusted to pass pairs of other orders.

Because the test grating was placed two focal lengths in front of the optical system, its image was two focal lengths behind and the magnification was minus one. This means that the analyzer grating was also a 10 line pair per millimeter

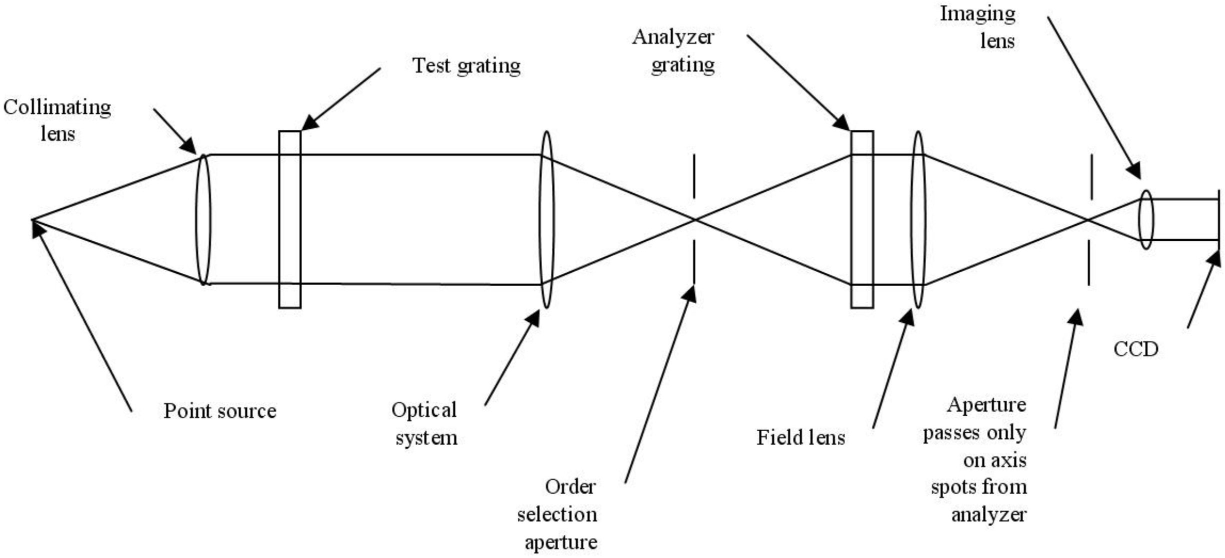


Figure 4. Schematic layout of experimental test.

Ronchi ruling. The analyzer grating forms one diffracted wavefront from each virtual object that should be on the optical axis. This will only occur if the spacing between the images of the point source at the order selection aperture and the analyzer grating is correct. If this spacing is not correct, the diffracted wavefronts will come from different angles and the virtual point sources will not be on the optical axis. This will result in tilt in the interference pattern. When the experiment was set up, the position of the analyzer grating was adjusted to minimize the tilt present in the interference pattern formed by the ±1st orders. In addition, the analyzer grating was mounted on a translation stage that moved perpendicularly to the optical axis for phase shifting.

To record the interferograms, the analyzer grating was imaged onto a camera. This was done with a field lens and an imaging lens. The field lens was placed behind the grating. This is necessary to redirect the light into the imaging lens. In addition, it creates an image of the source where the different diffracted orders from the analyzer grating are separated spatially. An aperture placed here only passes the diffracted orders that form on axis spots. This removes all of the higher harmonics and produces a fringe pattern with a sinusoidal variation with phase. This allows for the direct application of phase shifting algorithms. The imaging lens is positioned to image the analyzer onto the camera. Figure 5 is a photograph of the experimental system.

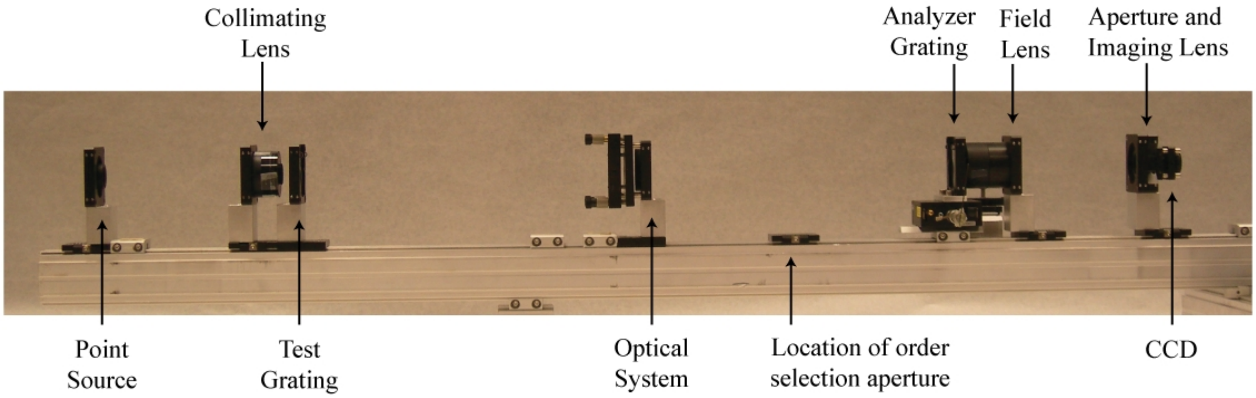
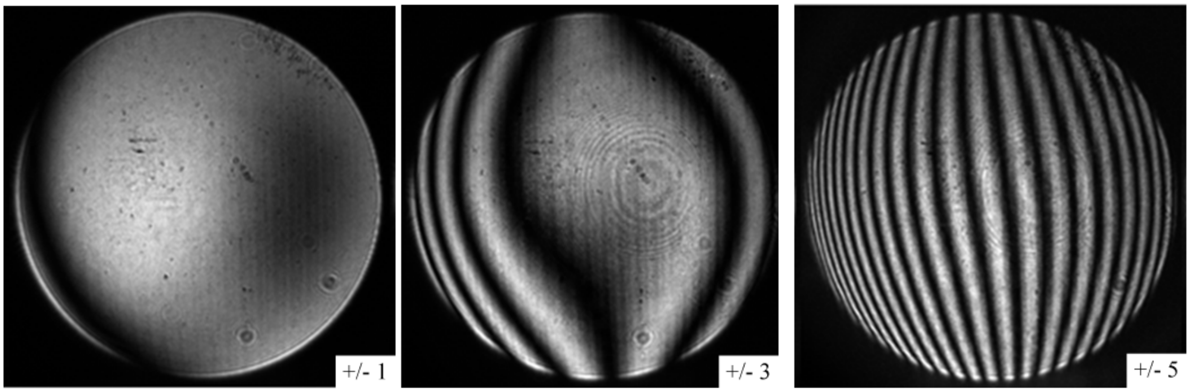


Figure 5. Photograph of experimental proof of concept SCTest. The length of the entire test is approximately 1.5 m. The order selection aperture has been removed in this picture.

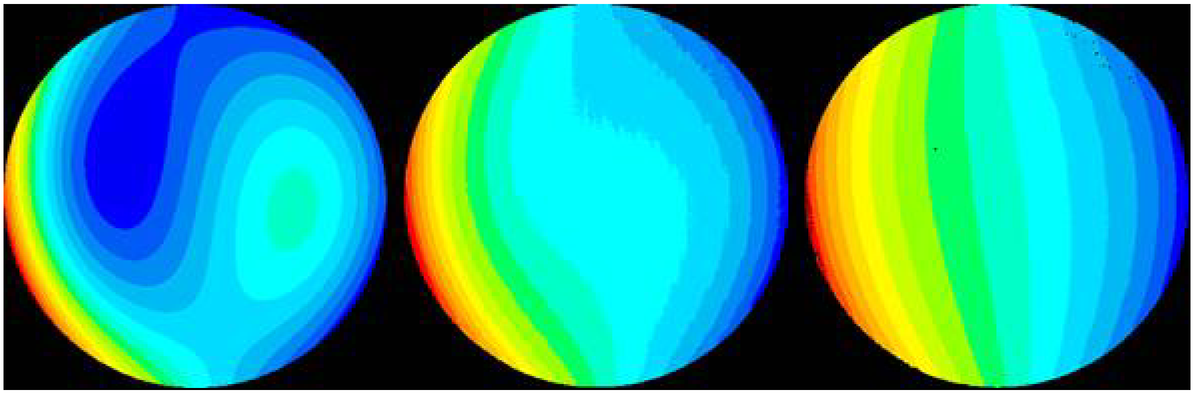
6.2 Measured data

Data was collected for the first, third and fifth orders. For each pair of orders, five phase shifted images were captured and processed. Figure 6 shows one fringe pattern from each set of data. Figure 7 shows the processed phase data.



a) b) c)

Figure 6. Example interferograms for the different orders. a) shows the first orders b) shows the third orders and c) shows the fifth orders.



a) b) c)

Figure 7. Processed phase data for each set of orders. Each plot uses the full scale of the color bar. a) the first order data (0.166 waves RMS) b) the third order data (1.02 waves RMS) c) the third order data (4.18 waves RMS)

6.3 Simulation

A computer simulation was performed to generate the expected values for the aberrations in each pair of orders. The aperture stop for the optical system was located at the test grating. Because of this stop shift, any spherical aberration in the lens would couple into coma in the measurement. The spherical in the lens was measured (-0.11 waves RMS difference from the ideal lens), and it was added to the model.

Because the system was built with off the shelf parts, it was not practical to align all of the optics with a high degree of precision. This means that the beam coming out of the collimating lens was not likely to be parallel to the axis of the optical system. In addition, a tilt of the analyzer grating could add some aberration. Finally, the spacing between the optical system and the analyzer grating was set experimentally.

In the simulation, the tilt of the optical system and the tilt of the analyzer were adjusted until the amount of power and astigmatism predicted by the model matched the measured results for the first order data. The spacing between the optical system and the analyzer grating was also adjusted to match the amount of tilt seen in the measured first order data. While three degrees of freedom can give a model too much flexibility to fit experimental data, it is reasonable to use this many for this experiment. Most importantly, none of the degrees of freedom have a significant impact on the measured coma. If all of them are set to their nominal values, the change in the predicted coma is less than 0.3%. In addition to this, the spacing between the analyzer and the focal plane has virtually no impact on the predicted astigmatism and power. It is only used to adjust the tilt. Finally, the amount of tilt in the interferogram does not depend on the angles of the optical system or analyzer grating.

6.4 Results

The measured and simulated data were fit to Zernike polynomials. The data are shown in Table 1. These data show that the experimentally measured coma agrees very well with the prediction for the 1st and 3rd order data. The coma in the fifth order was measured in the presence of over 15 waves of tilt. Based on this, one would expect higher uncertainty in the measured coma in the 5th order data.

The measured data also matches the predictions for astigmatism and power. In the simulation, the optical system was tilted 0.094° and the analyzer grating was tilted 1.14°. Given the types of mounts used and how they were assembled, these errors are reasonable.

There is also good agreement between the simulated and measured tilt. The ideal spacing between the intermediate image and the analyzer grating is 200 mm. This spacing was adjusted in the simulation in order to match the tilt in the 1st order data. The spacing used in the simulation was 200.94 mm.

Aberration	Orders	measured (waves RMS)	predicted (waves RMS)	delta (waves RMS)	% error
coma	1st	0.120	0.114	0.006	4.7%
	3rd	0.360	0.340	0.020	5.5%
	5th	0.669	0.554	0.115	17.2%
astigmatism	1st	0.066	Used for fitting		
	3rd	0.194	0.202	-0.008	-3.9%
	5th	0.355	0.348	0.007	1.9%
power	1st	-0.051	used for fitting		
	3rd	-0.150	-0.156	0.006	-3.9%
	5th	-0.271	-0.269	-0.002	0.6%
tilt	1st	0.300	used for fitting		
	3rd	0.925	0.820	0.105	11.3%
	5th	4.150	4.081	0.069	1.7%

Table 1. Measured and predicted values for coma, astigmatism, power and tilt. No fitting was done for the coma terms. One degree of freedom was used to fit the tilt and two degrees of freedom were used to fit the astigmatism and power.

7. CONCLUSION

In this paper we have presented the SCTest as a method for using the sine condition to measure the state of alignment of an optical system. This test is based on the fact that the mapping error between two surfaces can be used to determine the linearly field dependent aberrations. The key indication of alignment is the linear astigmatism. To test the concept, an experimental system was built that implemented the key features of the SCTest. In this experiment, the measured coma agrees with the predicted coma well in the first and third order data. The error is larger in the fifth order data, but the large amount of tilt increases the uncertainty. In addition to the coma, the experimental data match the predictions for astigmatism, power and tilt. This was accomplished by setting three degrees of freedom based on three measured values.

While the material presented here has been directed at the measurement of misalignment in rotationally symmetrical systems, the SCTest is not limited to these systems. The sine condition is valid for non-rotationally symmetrical systems, so the SCTest can be applied to them. This may be where the greatest benefit of the SCTest can be realized. Instead of using it to characterize the alignment of a system, it could be used during the alignment if it is coupled with a test at a single field point. This would provide nearly real time information on the state of alignment as the system is adjusted. By switching between different pairs of orders from the test grating, redundant data can be collected. This would greatly reduce the chances that an alignment error in the SCTest equipment would provide false or misleading data.

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