Calibration Limits for Interferometric Measurements

Ping Zhou, James Burge
College of Optical Sciences, University of Arizona, 1630 E. University Blvd, Tucson, AZ 85721
pzhou@optics.arizona.edu

Abstract: This paper presents how to quantify the measurement noises in optical surface testing. We also discussed how to apply a smoothing filter in map registration and subtraction.

OCIS codes: (120.6650) Surface measurements, figure; (220.4840) Optical testing

1. Introduction

When testing an optical surface using an interferometer, it is always important to know how precise and how accurate the measurement is. Precision is defined as repeatability or the noise level in the measurement. Accuracy means the difference between the measured quantity and the true value. In optical surface measurement, the precision can usually be improved by averaging many data sets. However, to know how many data sets are needed to meet the requirement, we need a way to quantify the precision in the data sets. This problem will be addressed in section 2.

In optical testing, it is very common to use a reference optic to test another optic. For instance, a reference sphere is used to test a flat in Ritchey-common test. Errors in the reference optic must be tested and removed from the test map to improve test accuracy. The two maps (reference optic and test optic), which may be offset by translation, rotation, scale, or other complex distortion, must be precisely mapped to each other for subtraction. Fiducials are often used to facilitate accurate map registration. Any error in mapping, denoted as Δr, couples into the slope error $\nabla W(x, y)$ of the reference map and causes the wavefront error shown in the test surface. We propose to apply a smoothing filter to the reference map when we subtract the reference map from the test one. This can reduce the errors due to mis-registration between two maps and improve the measurement accuracy.

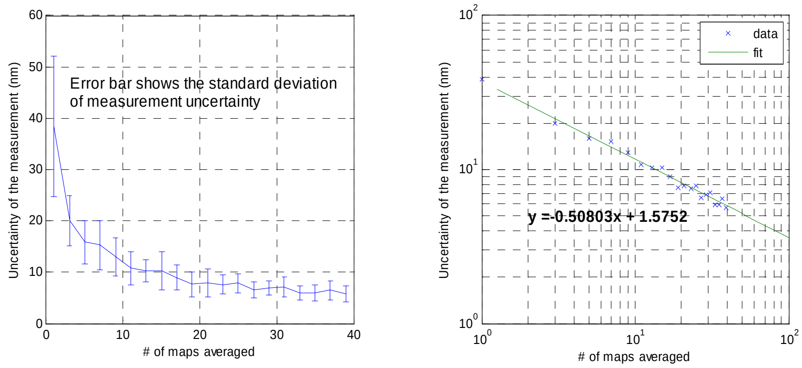
2. Estimate of the noise in measurement

The noises in optical surface measurement are usually random, and can be averaged out. Our method of quantifying noise is similar to the repeatability quantification in random ball test[1]. Since the true surface figure is unknown, instead, an average of N measurements is used as an estimate the surface figure. The difference between a measurement and the average map is regarded as noise in the measurement. A reference sphere with the radius of curvature of 16m was tested to investigate the noises. Since the radius of curvature of the mirror is so long that the air turbulence causes a large amount of noises. Fig.1(a) shows a map of single measurement. The astigmatism shown in the picture is mainly due to the air turbulence. The average of 100 measurements $\overline{W}$ is shown in Fig.1(b), which is more rotationally symmetric.

(a) (b)

Fig.1. Map of a single measurement (a) and the averaged map (b)

From the 100 measurements, N maps are chosen at random and averaged. This average map, denoted as $\overline{W}_N$, is subtracted from $\overline{W}$, and the RMS is calculated as $RMS(\overline{W_N} - \overline{W})$. This process is repeated several times, and the mean and the standard deviation of $RMS(\overline{W_N} - \overline{W})$ are calculated. This is repeated for values of N from 5 to 40, and the results are plotted in Fig.2. The curve follows the mean, and the error bar represents the standard deviation. The mean of $RMS(\overline{W_N} - \overline{W})$ becomes significantly smaller as the number of averaged measurements increases. From this plot, how many measurements are needed to achieve the noise requirement can be determined. For example, to achieve the noise requirement of 10nm, 15 measurements need to be taken. Fig.2(b) shows the same curve in a log-log scale. The data were fitted into a straight line with the slope of -0.508. This means that the noises in the measurements are random, and they almost follow $1/\sqrt{N}$ law. In the log-log scale plot, we can estimate that the noise in the average of 100 maps is about 3.6nm.



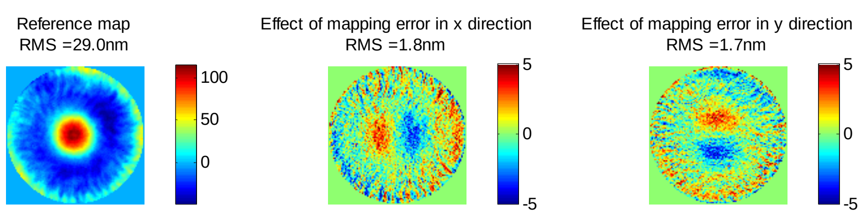
(a) (b)

Fig.2. Measurement noise as a function of measurements averaged (a) normal scale (b) log scale

3. Smoothing filter design

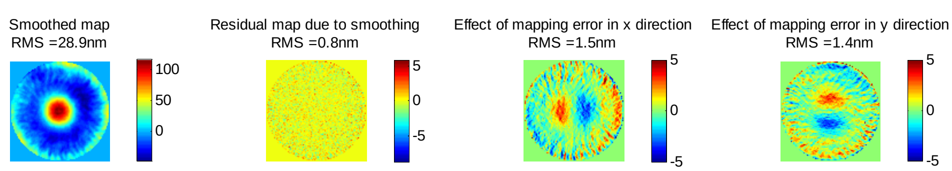
The reference sphere mentioned above is used to test another optic in our shop. In order to know the surface figure of that part accurately, the figure of the reference sphere must be tested separately and removed from the test surface. When we subtract the two maps, the mapping error Δr and the surface slope error of the reference sphere $\nabla W(x, y)$ causes the surface error in the test surface. Therefore, both the slope errors and mapping errors need to be controlled to a reasonable level. To reduce the sensitivity of wavefront errors to mapping, we can smooth the reference sphere to a certain resolution, and then subtract the smoothed map from the test surface [2]. The difference between the smoothed reference map and the full resolution map is the small-scale errors, denoted as $W_{residual}$, which will be left in the test surface. Note that some of these small-scale errors are noise in the measurement, and some of them are real errors due to smoothing. The total wavefront errors due to subtraction consist of these small scale errors and the product of the mapping errors and the slope errors of the reference sphere. Using a smoothed version minimizes the propagation of noise from the measurement of the reference sphere to the measurement of test surface.

The measurement of the reference sphere has a resolution of 640 X 640. Use the reference sphere shown in Fig.1(b) as an example and assume the mapping errors are 2 pixels in both x and y direction. Fig.3 shows the errors without smoothing the reference map. Fig.4 shows the errors with smoothing the reference sphere to a resolution of 7 pixels by 7 pixels.



(a) (b) (c)

Fig.3. Reference map (a); Wavefront error due to 2-pixel mapping error in x direction (b); Wavefront error due to 2-pixel mapping error in y direction (c)



(a) (b) (c) (d)

Fig.4. Smoothed reference map with a 7X7 filter (a) Residual small-scale error after smoothing (b) Wavefront error due to 2-pixel mapping error in x direction (c); Wavefront error due to 2-pixel mapping error in y direction (d)

The total RSS error can be calculated as $RSS^2 = [RMS(W_{residual})]^2 + [RMS\left(\frac{\partial W_x}{\partial x}\cdot\Delta x\right)]^2 + RMS[\left(\frac{\partial W_y}{\partial y}\cdot\Delta y\right)]^2$, where $\frac{\partial W_x}{\partial x}$ and $\frac{\partial W_y}{\partial y}$ are the slope errors of the reference sphere in x and y directions, respectively. Therefore, the RSS error is 2.5nm without smoothing the reference sphere, and is 2.1nm when the reference sphere is smoothed to a resolution of 7X7. Using a smoothed map will decrease the wavefront error more dramatically when the mapping error is large.

The mapping error sometimes is complex and difficult to predict. If the mapping error can be estimated, then the filter size can be optimized to achieve minimum wavefront errors. Fig.5 shows the RSS errors versus the filter sizes with different mapping errors. In an ideal case which has no mapping error, the RSS error would be zero when no smoothing filter is applied. Given a certain mapping error, an optimal filter can be chosen to minimize the RSS error.

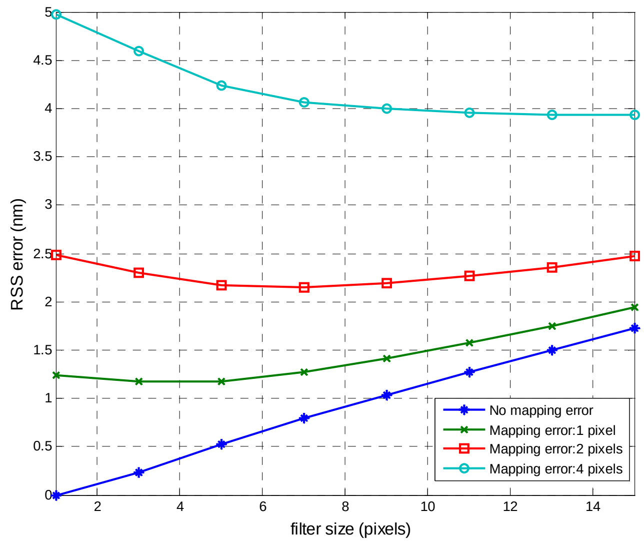


Fig.5 RSS errors as a function of filter size

The total uncertainty in the test surface includes the measurement noises in both reference sphere and test surface, and the wavefront errors due to data registration and subtraction. There are other sources of errors, like the re-trace error and errors from interferometer. We calibrated the interferometer using the transmission sphere to an accuracy of 2nm. The total uncertainty of the test surface is 5.4nm as shown in Table.1, after we add up all sources of errors in a RSS fashion.

Table.1. The total uncertainty in the test surface

Noises in measurement of the reference sphere	3.6nm
Noises in measurement the test surface	3.5nm
Errors due to map registration and subtraction	2.1nm
Errors in the interferometer	2.0nm
Total uncertainty of the test surface (RSS)	**5.8nm**

4. Conclusion

This paper provides a method of quantifying the noises in optical surface measurements, which help us to know the precision in the measurement. We also discussed to use a smoothing filter when two maps need to be registered and subtracted from each other, which can minimize the total wavefront errors due to map registration and subtraction.

5. References

[1] Ulf Griesmann, Quandou Wang etc. "*A simple ball averager for reference sphere calibations*", SPIE **5869**, 58690S1-8 (2005)

[2] Buddy Martin "*Specifications for the 3.75m fold sphere for the GMP principal test*", Steward observatory internal report (2008)