Vector Polynomials for Gradient Metrology Data Processing

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Abstract: We discuss the advantages of vector polynomials for wavefront / surface reconstruction from gradient data. The Chebyshev-based ***G*** polynomials allow accurate and efficient reconstruction of mid-to-high spatial frequencies particularly where traditional integrations approaches could be problematic. © 2021 M. Aftab, D. Kim

1. Introduction

Since most metrology systems measure the slope or gradient of a surface or wavefront, measured data must be converted back to the form of a surface or wavefront. There are two main ways to do this: (a) zonal methods such as Southwell integration [1], and (b) modal methods, such as Zernike or Chebyshev based polynomials.

Modal methods generally give a higher accuracy than zonal approaches, especially in terms of root mean squared (RMS) value and surface figure [2] but suffer from an inability to efficiently reconstruct mid-to-high spatial frequencies. We developed a modal approach [3] (using ***G*** polynomials) where tens of thousands of polynomials can be used with good accuracy and reasonable computational efficiency, hence they can represent/reconstruct mid-to-high spatial frequencies well. This is important for high resolution and / or freeform surfaces like telescope mirrors. In addition, they are more accurate than zonal integration methods in cases where the surface has defects (e.g., scratches), or when the aperture is obscured e.g., spider webs in telescopes or when measurement fiducials are placed on the optic [3]. They are also very useful for reconstruction of a wavefront or surface when its measured data is unevenly sampled [4] (e.g., different sampling points in x and y directions). ***G*** polynomials have several other attractive properties, for example they are orthogonal in the gradient domain, which is preferred for data fitting and analysis.

2. Chebyshev-based *G* polynomials

Due to several useful properties, such as orthogonality across rectangular apertures, two dimensional (2D) Chebyshev polynomials of the first kind (called *F* polynomials) are chosen as the scalar basis set. ***G*** polynomials are generated as the gradients of *F* polynomials, and can be written mathematically as:

$$\vec{G}_n^m(x,y) = \nabla F_n^m(x,y) = \frac{\partial}{\partial x} F_n^m(x,y)\,\hat{i} + \frac{\partial}{\partial y} F_n^m(x,y)\,\hat{j} \tag{1}$$

F polynomials are composed of two 1D (T) polynomials:

$$F_j(x,y) = F_n^m(x,y) = T_m(x)T_n(y) \tag{2}$$

The *T* polynomials are Chebyshev polynomials and can be described recursively as:

$$\begin{aligned} &T_{m+1}(x) = 2xT_m(x) - T_{m-1}(x) \\ &where\ \ T_0(x)=1,\ T_1(x)=x,\quad for\ -1\le x\le 1 \end{aligned} \tag{3}$$

G polynomials have several noteworthy properties [3] that allow the advantages of accurate and efficient data fitting when they are used, especially for the cases mentioned earlier. The important ones are:

i. Both *F* and ***G*** polynomial sets can be generated recursively.
ii. There is a one-to-one correspondence between the coefficients of the two polynomial sets, i.e., once the ***G*** polynomial coefficients are determined from data fitting, the same values will be used as coefficients of *F* polynomials.
iii. ***G*** polynomials can easily and efficiently be generated since they are already orthogonal when derived from *F* polynomials and, unlike most other gradient polynomial sets, do not need an orthogonalization process.

3. Reconstruction data processing results

Several examples are used to demonstrate the good fitting and advantages of using ***G*** polynomials method for reconstruction of surface / wavefront data.

Example I: **Mid-to-high spatial frequency representation of the measured data, using a subsection of the 4.2 m Daniel K. Inouye Solar Telescope primary mirror data, with a high-pass Gaussian filter applied to it.**

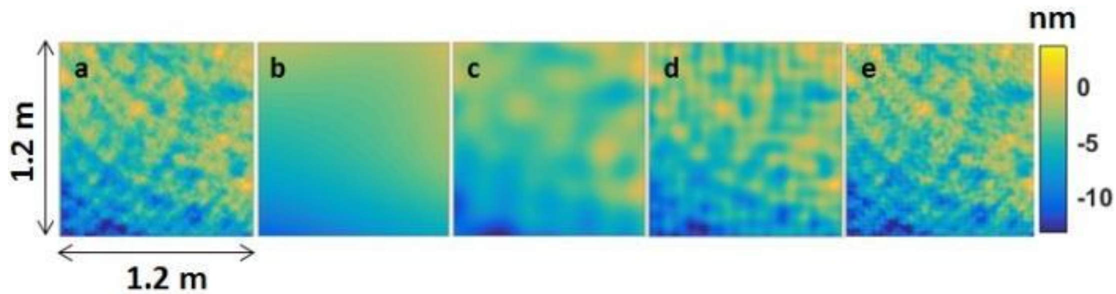
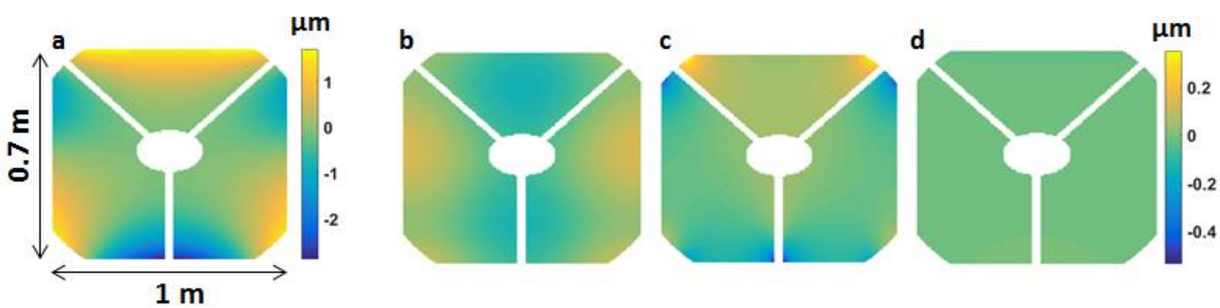


Fig.1. (a) High-pass-filtered Southwell zonal reconstructed map and high-pass-filtered G-polynomial reconstructed maps generated using (b) 37, (c) 750, (d) 3,000, and (e) 20,000 polynomial terms [3].

As the number of ***G*** polynomials increases (left to right in sub-figures 1(b) to 1(e)), surface reconstruction maps become sharper and more high-frequency features are resolvable. The right-most map (1(e)) closely resembles the Southwell zonal reconstructed surface (1(a)).

Example II: **Comparison for blockers using simulated noise-free data modeling an aperture with central obscuration and spiders.**



	Zernike gradients (30 terms)	**Southwell method**	***G* polynomials (30 terms)**
[RMS error map / RMS ideal map] × 100 (%)	10.96 %	11.41 %	2.57 %

Fig.2. (a) Simulated data. The reconstruction residual error maps for (b) Zernike gradients fitting case, (c) Southwell method case, and (d) ***G*** polynomials method case (compared to the simulated ideal map) [3].

The error is lower from ***G*** polynomials. (Note: Specific numbers could vary with other reconstruction algorithm implementations.)

4. References

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