Maximum likelihood estimation as a general method of combining sub-aperture data for interferometric testing

Peng Su, Jim Burge, Robert A. Sprowl and Jose Sasian
College of Optical Sciences, University of Arizona
1630 East University Boulevard, Tucson, Arizona 85721-0094

ABSTRACT

Interferometers accurately measure the difference between two wavefronts, one from a reference surface and the other from an unknown surface. If the reference surface is near-perfect or is accurately known from some other test, then the shape of the unknown surface can be determined. We investigate the case where neither the reference surface nor the surface under test is known. By making multiple modulated measurements where both surfaces are translated and rotated, we obtain sufficient information to reconstruct the figure of both surfaces. We have developed software that provides a maximum likelihood estimation of both surfaces, as well as an assessment of the quality of the reconstruction. This was demonstrated for the measurement of a large flat mirror, using a smaller reference mirror that has significant shape errors.

Keywords: Interferometry, Optical testing, Sub-aperture testing, Absolute surface shape metrology, Maximum likelihood estimation, Modulated sub-aperture interferometric testing

1. INTRODUCTION

Sub-aperture testing (SAT) has been primarily proposed for testing large aperture optics with standard interferometers [1]. Two general algorithms [2, 3] have been proposed for sub-aperture data reduction, where polynomials are used to describe the full aperture surface data, and get the polynomial coefficients by least square fitting the data in sub-aperture. The polynomial fitting methods have been typically used with a non-overlapping sub-aperture testing configuration and could not describe local irregularities in a surface well due to the finite polynomial terms. To adequately make use of the sub-aperture testing data, testing configurations with overlapped apertures have been suggested and widely used [4, 5]. Data from sub-aperture measurements have been stitched by calculating the relative pistons and tilts between overlapping sub-apertures.

Sub-aperture testing has also been investigated as a non-null aspheric test method. By translating the reference surface or test surface, the radius curvature of the reference sphere is controlled to best match the local radius curvature of the aspheric surface. In this way, the numbers of the interferogram fringes are reduced to within the dynamic range of an interferometer. The full aspheric surface can then be measured by stitching a number of sub-aperture measurement data with best-fit reference spheres [6, 7, 8, 9]. To reduce the need of precise prior knowledge of fringe nulling or the alignment of sub-apertures, several iterative algorithms have been developed to estimate the positions of each sub-aperture [10, 11].

International Optical Design Conference 2006, edited by G. Groot Gregory,
Joseph M. Howard, R. John Koshel, SPIE Vol. 6342, 63421X,
 · doi: 10.1117/12.692233

In this paper, we investigate the case where neither the reference surface nor the surface under test is known in sub-aperture testing. By designed relative translation and rotation of the reference and test surfaces, we modulate the sub-aperture testing data, so that errors in the reference and test surfaces can be separated. We have applied linear analysis to create a global maximum likelihood solution for combining sub-aperture testing data and reconstructing reference and test surfaces. Our methodology is self-calibrated and has an easily controlled structure for detecting the six degrees of freedom in the sub-aperture positions. This method can be used for interferogram stitching as other methods mentioned above, however its self-calibration property makes it non-restrictive to the stitching application. Our method can also be used to calibrate other testing methods, for instance verifying a null lens system and CGH aspheric testing.

2. DESCRIPTION OF THE METHOD

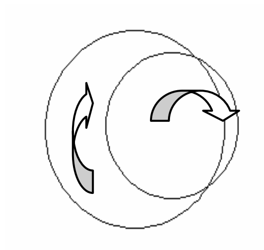


Fig.1. Sub-aperture testing configuration

We applied our method to test a large flat mirror with a Fizeau interferometer set up. A 1-m flat mirror was measured using a 60 cm reference. The position of the mirror under test relative to the reference surface is shown in Figure 1. In the figure the transmitting reference flat, represented by the small circle, was rotated six times around its center during the measurement. The test mirror, represented by the large circle, was rotated eight times around its center. The combination of the rotation of the reference surface and the rotation of the test surface gave information to separate the errors in the test surface from the errors in the reference surface. The numbers and test geometries of sub-aperture tests were designed by eigen analysis of the modulation, so that we could separate the errors in each surface and make test easily performed and robust.

Figure 2 shows the flow diagram of our maximum likelihood estimation algorithm. Distortion corrected sub-aperture test data and the test geometry are used as input information for the program. For the above test scheme, Zernike polynomials are used to represent the reference and test surfaces. Numerical orthogonal basis functions are first created to describe the data within sub-aperture region. The advantage of numerical basis is that it can work for arbitrary sub-aperture geometry, circular, rectangular or other shapes. With the test geometry, an influence matrix is set up. Its structure is shown in figure 3. It describes the influences produced by the reference and test surface to each sub-aperture measurement. Each column in the matrix represents the effect caused by one unit of a certain polynomial term to sub-aperture measurement data. Additional piston and tilt terms are added to include the influences from the unknown piston and tilt motions introduced during the rotations of the two surfaces. So by modifying the structure of the influence matrix, random piston and tilt will not couple to the polynomial terms we are interested in. With sub-aperture testing data and the influence matrix, the maximum likelihood estimation are performed, and both reference and test surface shapes are obtained. By checking the fitting residuals and correlation between the sub-aperture residuals, we can estimate the test accuracy and diagnose the error sources and inconsistencies among sub-aperture data. Setting up a merit function with fitting errors, we can control all six degrees of freedom sub-aperture position uncertainties by optimizing the structures of the influence matrix separately or globally. Detailed mathematical description of the whole algorithm was described in our other paper [10]. As mentioned in the introduction, polynomial methods could not describe local irregularities in a surface well due to the finite polynomial terms. One way to deal with this issue is by checking modulation effects in the residual data, we can find the data which are modulated and are not absorbed by polynomials, and then we put those data to the corresponding regions of the test surface.

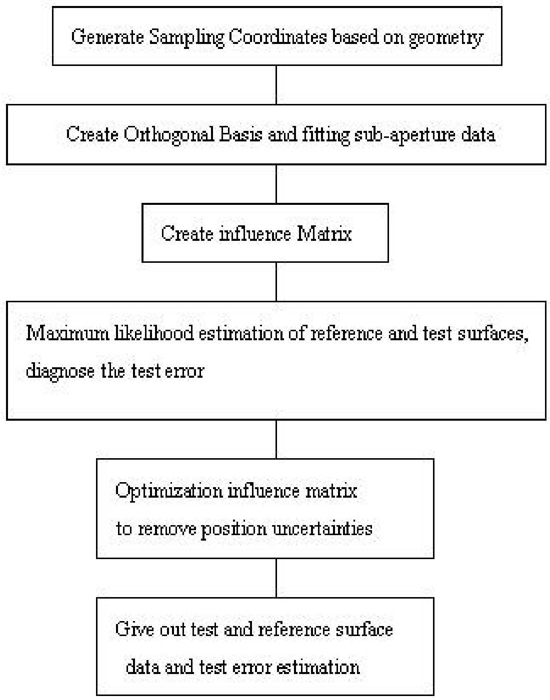


Fig.2. Flow diagram of the Maximum likelihood estimation algorithm

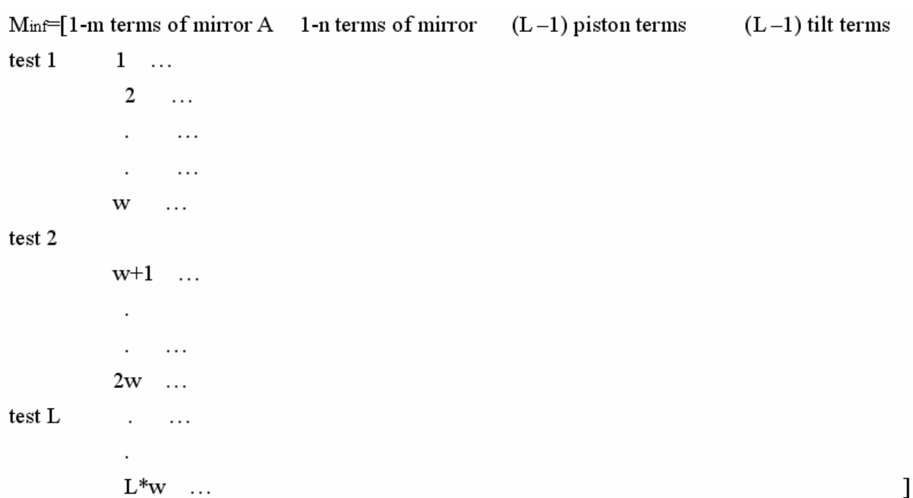


Fig.3. The structure of Influence Matrix

3. COMPUTER SIMULATION AND EXPERIEMT RESULT

In our simulations, each surface was assumed to have an rms error of about 100 nm. We considered the following error sources.

1) Random piston and tilt error

They can be well absorbed by the additional piston and tilt terms in the influence matrix.

2) Random rotation errors

A standard deviation (std) (1.6 mm/semi-diameter) of the mirror rotational angular errors was randomly introduced to the sub-aperture measurement data. Here 1.6 mm corresponded to one pixel in our CCD. The resulting average rms errors of each surface were about 0.7 nm.

3) Relative shifts or uncertainties in determining the center of each surface.

With std of 1.6 mm random lateral shifts or uncertainties in determining the center of each surface, it produced about 2 nm average rms errors.

4) Random noise

10 nm rms random errors were introduced, which produced about 0.3nm mean errors.

By optimizing the structures of the influence matrix, the dominant error relative shifts can be well reduced and the total average estimation error can be controlled to less than 1 nm. Figure 4 showed the histogram of the surface rms estimation errors for test and reference mirrors.   Figure 5 and 6 showed an example of the estimation of the reference surface with above error budgets.

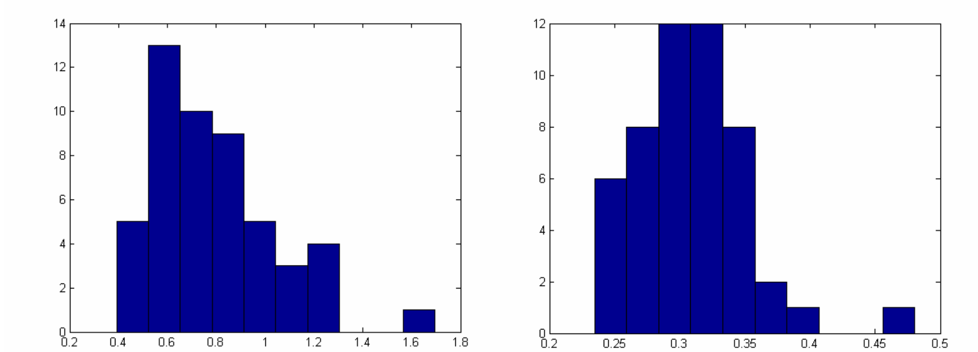
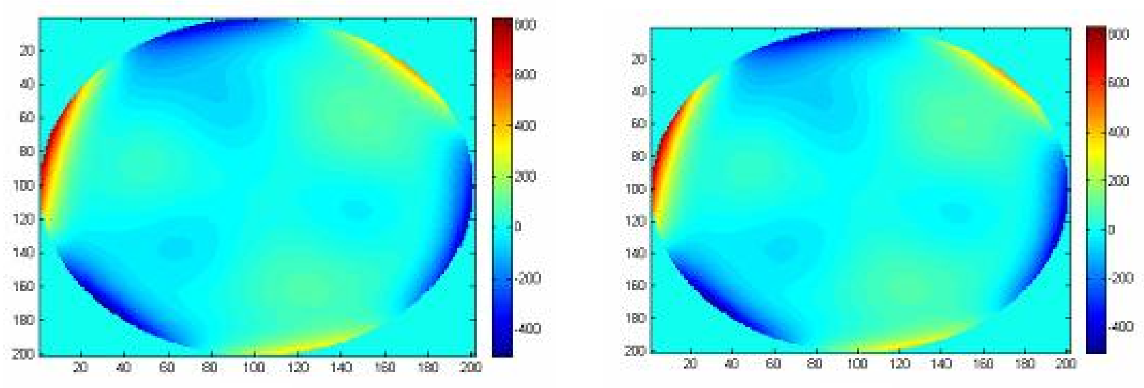


Fig.4. Estimate errors of the test surface: mean=0.79 nm, std=0.25nm; estimate errors of the reference surface: mean=0.3 nm, std =0.04nm

5) Selection of the terms of polynomials to be used

One way to estimate them is fitting polynomials to the sub-aperture data to get a feeling of how many terms need to be used in the algorithms. Surface data can be described better by using more terms, however more random noise will be coupled in the test result simultaneously. So the numbers of polynomials to be used are set by the noise level of the sub-aperture measurements.



(a) (b)

Fig.5. (a) Original surface: rms=119.5 nm, (b) The estimated surface: rms=119.46nm

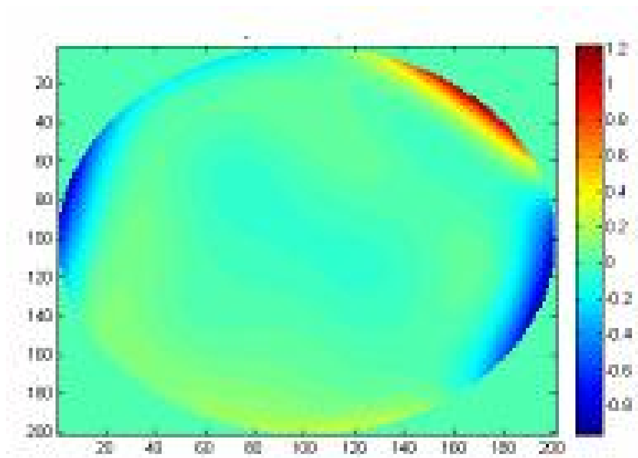


Fig.6. Point-by-point subtracting map between original surface and estimated surface, rms=0.4nm.

An experiment of this double rotation testing configuration was performed recently. The estimated reference surface compared with another independent test method has rms difference around 1nm. Fitting residual of the estimate was at the noise level of the interferometer. We also compared with the result from commercial stitching software; the estimate difference of the test surface was around 1nm.

4. CONCLUSIONS

In our maximum likelihood estimate method we investigate the case where neither the reference surface nor the surface under test is known. By making multiple modulated sub-aperture measurements, we obtain sufficient information to reconstruct the figure of the test and reference surfaces. Our methodology is self-calibrated and has an easily controlled structure for detecting the six degrees of freedom in the sub-aperture positions. The accuracy of the method is at least at the same level, 1 nm rms, of commercial stitching software. Our method is not restricted to stitching applications. It can also be used to calibrate other testing methods. The modulation and optimization characteristics of our method make it a general and easily applied method for multi-interferogram testing.

REFERENCES

1. C. Kim and J. Wyant, “Subaperture test of a large flat on a fast aspheric surface,” *J. Opt. Soc. Am.* **71**, p. 1587, 1981.
2. J. G. Thunen and O. Y. Kwon, “Full aperture testing with subaperture test optics,” in *Wavefront sensing*, N. Bareket and CL Koliopoulos eds., *Proc. SPIE* **351**, pp.19-27, 1982.
3. W. W. Chow and G. N. Lawrence, “Method for subaperture testing interferogram reduction,” Opt. Lett. **8**, 468–470 (1983).
4. T.W. Stuhlinger, “Optical testing of large telescopes using multiple subapertures,” Ph.D. Dissertation, U. Arizona, Tucson (1984)
5. M. Bray, “Stitching interferometer for large plano optics using a standard interferometer,” Proc. SPIE **3134**, 39–50 (1997).
6. Y-M. Liu, G. Lawrence and C. Koliopoulos, "Subaperture testing of aspheres with annular zones," Applied Optics, 27, 4504-4513, 1988.
7. Tronolone, et al., “ Mothod of testing aspherical optical surface with an interferometer,” US patent # 5,416,586; Tropel Corporation.
8. Jon Fleig, Paul Dumas, Paul E. Murphy, Greg W. Forbes, “An automated subapeture stitching interferometer workstation for spherical and aspherical surfaces,” Proc. SPIE Vol 5188 (2003)
9. QED technologies inc, “Software Patent: Simultaneous self-calibrated sub-aperture stitching for surface figure measurement (interferometer), ” US patent #EP1,324,006.
10. . Mikael Sjo¨ dahl , Bozenko F. Oreb, “Stitching interferometric measurement data for inspection of large optical components,” Opt. Eng. **41(2)**, 403–408 (2002).
11. Shanyong Chen, Shengyi Li, and Yifan Dai, “Iterative algorithm for subaperture stitching interferometry for general surfaces,” J. Opt. Soc. Am. A, Vol 22, No, 9 ,1929-1936 (2005).
12. Peng Su, Jim Burge, “Modulated multi-aperture interferometric testing,” to be submitted.