Optimal design of computer-generated holograms to minimize sensitivity to fabrication errors

Ping Zhou and James H. Burge

College of Optical Sciences, University of Arizona, Tucson, AZ, 85721
pzhou@optics.arizona.edu; http://www.optics.arizona.edu

Abstract: Aspheric optical surfaces are often tested using computer-generated holograms (CGHs). The etching of the CGH pattern must be highly accurate to create desired wavefronts. Variations of line width, etching depth, and surface roughness cause unwanted wavefront errors. The sensitivity to these manufacturing errors is studied using scalar diffraction analysis. We provide a parametric model that can be used for optimizing the CGH design to give good diffraction efficiency and limited sensitivity to manufacturing errors.

OCIS codes: (050.1380) Binary optics; (090.1760) Computer holography

References and links

1. D. Malacara, *Optical Shop Testing*, 2nd Ed. (Wiley, 1992).
2. S. M. Arnold, "How to test an asphere with a computer generated hologram," Proc. SPIE **1052,** 191–197 (1989).
3. A. G. Poleshchuk, V. P Korolkov, V. V Cherkashin., S. Reichelt, J. H. Burge, "Polar coordinate laser writing system: error analysis of fabricated DOEs," Proc. SPIE 4440, 161-172 (2001).
4. Y. C. Chang and J. H. Burge, "Errors analysis for CGH optical testing," Proc. SPIE **3782,** 358–366 (1999).
5. Y. C. Chang, P. Zhou, and J. H. Burge, "Analysis of phase sensitivity for binary computer generated holograms," Appl. Opt. **45,** 4223–4234 (2006).
6. P. Zhou and J. H. Burge, "Coupling of surface roughness to the performance of computer-generated holograms," Appl. Opt.46, 6572-6576(2007).
7. P. Zhou and J. H. Burge, "Fabrication error analysis and experimental demonstration for computer-generated holograms," Appl. Opt.46, 657-663 (2007).
8. A. F. Fercher, "Computer-generated holograms for testing optical elements: error analysis and error compensation," Opt. Acta **23,** 347–365 (1976).

1. Introduction

An optical element with a large aspheric departure is often tested with computer-generated holograms. The primary role of CGHs is to generate reference wavefronts of any desired shapes [1, 2]. The accurately drawn pattern on the CGH provides exact wavefront control. However, uncertainties from the CGH manufacturing processes introduce errors in hologram and hence in the generated wavefront. Fabrication errors of the CGH can be classified into two basic types: irregularities in the CGH substrate, and the pattern errors. The pattern errors include etching depth errors, duty-cycle errors, surface roughness errors and pattern distortion errors [3].

CGHs for optical testing are usually binary, either chrome-on-glass type or phase type. Chrome-on-glass CGHs have the pattern defined by a thin layer of chrome coating on the glass. This type of CGH is widely used in optical testing, because it is not sensitive to fabrication errors when it is used in transmission. This will be explained in Section 3. However, chrome-on-glass CGHs only have the maximum diffraction efficiency of 10% for the first order of diffraction. When the light intensity is not high enough, phase type CGHs can be used for optical testing. Phase type CGHs have the pattern etched into the glass substrate. The maximum diffraction efficiency for the first order can be up to 40%.

#86633 - $15.00 USD Received 20 Aug 2007; revised 29 Oct 2007; accepted 1 Nov 2007; published 5 Nov 2007

In our previous work, we used the binary diffraction model to study the wavefront sensitivity of binary CGHs on the etching depth and duty-cycle variations [4, 5]. We also used the same model to analyze the effect of small scale surface roughness to the wavefront phase [6]. In optical testing, both the sensitivities of wavefront phase to the fabrication errors and the variation of the fabrication errors are very important for optical engineers to know. We discussed the methods for measuring the fabrication errors in the CGH substrate, duty cycle, etching depth and the effect of surface roughness in our previous paper [7]. This information combined with the knowledge of sensitivity to manufacturing errors can help us design the CGHs, which can give good diffraction efficiency and low sensitivity to manufacturing errors. We now apply the parametric model to optimize the design parameters of CGHs in order to achieve good diffraction efficiency and minimize the manufacturing error coupled into the testing system.

2. Parametric model

The parametric model was first developed by Chang and Burge [4, 5]. A binary, linear grating model was used to build the parametric model based on scalar diffraction theory. Figure 1 illustrates a binary, linear grating.

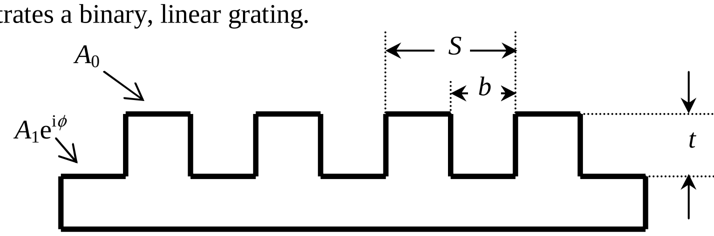

Fig. 1 Binary, linear grating profile

The grating is defined by the period S and the etching depth t. Duty-cycle is defined as $D = b/S$, where b is width of the etched area. A_0 and A_1 are the amplitudes of the output wavefront from the unetched area and etched area of the grating, respectively. The phase function ϕ represents the phase difference between rays from the peaks and valleys of the grating structure. For a chrome-on-glass CGH used in transmission, A_0 is zero, and A_1 is unity. For a phase type CGH used in transmission, A_0 and A_1 are both unity, and $\phi = \frac{2\pi}{\lambda}(n-1)t$, where n is the refractive index of the grating.

Based on Fraunhofer diffraction theory, the wavefront phase Ψ and diffraction efficiency η in the far field can be derived. The wavefront phase sensitivity functions $\partial\Psi/\partial D$, $\partial\Psi/\partial\phi$ and $\partial\Psi/\partial A_1$ are introduced to specify the wavefront error caused by small deviations in duty-cycle ΔD, phase function $\Delta\phi$ and the amplitude ΔA_1. The variation in the amplitude is due to scatter loss from the surface roughness, which varies across the substrate due to the limitations in the etching process [6]. Table 1 summarizes the diffraction efficiencies, wavefront phases and sensitivity functions for the zero and the non-zero diffraction orders [7].

#86633 - $15.00 USD Received 20 Aug 2007; revised 29 Oct 2007; accepted 1 Nov 2007; published 5 Nov 2007

Table 1. Summary of equations for parametric model analysis

	Zero order (m = 0)	Non-zero order (m ±1, ±2,…)
	Diffracted wavefront	
η, diffraction efficiency	$A_0^2(1-D)^2 + A_1^2 D^2 + 2A_0 A_1 D(1-D)\cos\phi$	$\left(A_0^2 + A_1^2 - 2A_0 A_1 \cos\phi\right) D^2 \operatorname{sinc}^2(mD)$
$\tan\Psi$, (Ψ wavefront phase)	$\dfrac{A_1 D\sin\phi}{A_0(1-D) + A_1 D\cos\phi}$	$\dfrac{A_1 \sin\phi\cdot\operatorname{sinc}(mD)}{(-A_0 + A_1\cos\phi)\cdot\operatorname{sinc}(mD)}$
	Sensitivity functions	
$\partial\eta/\partial D$	$-2A_0^2(1-D) + 2A_1^2 D + 2A_0 A_1(1-2D)\cos\phi$	$2\left(A_0^2 + A_1^2 - 2A_0 A_1\cos\phi\right) D\operatorname{sinc}(2mD)$
$\partial\eta/\partial\phi$	$-2A_0 A_1 D(1-D)\sin\phi$	$2A_0 A_1 \sin\phi\, D^2 \operatorname{sinc}^2(mD)$
$\partial\eta/\partial A_1$	$2A_1 D^2 + 2A_0 D(1-D)\cos\phi$	$(2A_1 - 2A_0\cos\phi) D^2 \operatorname{sinc}^2(mD)$
$\partial\Psi/\partial D$	$\dfrac{A_0 A_1 \sin\phi}{A_1^2 D^2 + A_0^2(1-D)^2 + 2A_0 A_1 D(1-D)\cos\phi}$	$\begin{cases}\infty, & \text{for } \operatorname{sinc}(mD) = 0\\ 0, & \text{otherwise}\end{cases}$
$\partial\Psi/\partial\phi$	$\dfrac{A_1^2 D^2 + A_0 A_1 D(1-D)\cos\phi}{A_1^2 D^2 + A_0^2(1-D)^2 + 2A_0 A_1 D(1-D)\cos\phi}$	$\dfrac{A_1^2 - A_0 A_1\cos\phi}{A_1^2 + A_0^2 - 2A_0 A_1\cos\phi}$
$\partial\Psi/\partial A_1$	$\dfrac{A_0 D(1-D)\sin\phi}{A_0^2(1-D)^2 + A_1^2 D^2 + 2A_0 A_1 D(1-D)\cos\phi}$	$\dfrac{-A_0\sin\phi}{A_0^2 + A_1^2 - 2A_0 A_1\cos\phi}$

The sensitivity functions can be evaluated directly to give the wavefront error due to variations in duty-cycle D, etch depth t or amplitude A_1. These functions are shown in Eqs.(1-3) respectively,

$$\Delta W_D = \frac{1}{2\pi}\frac{\partial\Psi}{\partial D}\cdot\Delta D = \frac{1}{2\pi}\frac{\partial\Psi}{\partial D}\cdot\left(\frac{\Delta D}{D}\right)\cdot D\,, \tag{1}$$

$$\Delta W_\phi = \frac{\partial\Psi}{\partial\phi}\cdot\Delta\phi = \frac{\partial\Psi}{\partial\phi}\cdot\left(\frac{\Delta\phi}{\phi}\right)\cdot\phi\,, \tag{2}$$

$$\Delta W_{A_1} = \frac{1}{2\pi}\frac{\partial\Psi}{\partial A_1}\cdot\Delta A_1 = \frac{1}{2\pi}\frac{\partial\Psi}{\partial A_1}\cdot\left(\frac{\Delta A_1}{A_1}\right)\cdot A_1\,, \tag{3}$$

where

ΔD : duty-cycle variation across the grating,
ΔW_D : wavefront variation in waves due to duty-cycle variation,
$\Delta\phi$: etching depth variation in radians across the grating,
ΔW_ϕ : wavefront variation in waves due to etch depth variation,
ΔA_1 : variation in amplitude A_1,
ΔW_{A_1} : wavefront variation in waves due to amplitude variation.

#86633 - $15.00 USD Received 20 Aug 2007; revised 29 Oct 2007; accepted 1 Nov 2007; published 5 Nov 2007

As long as the duty-cycle, etching depth and amplitude vary over spatial scales that are large compared to the grating spacing, Eqs.(1-3) can be used to determine the coupling between fabrication errors and system performance. The wavefront sensitivity functions provide a way of calculating the wavefront phase changes resulted from the fabrication non-uniformities. They can be used to identify which hologram structures are the most or the least sensitive to those fabrication uncertainties.

Another wavefront error, known as CGH pattern position error, or pattern distortion, results from the displacement of the etched pattern in a CGH from its ideal position. The amount of wavefront errors introduced by the CGH pattern distortions can be expressed as

$$\Delta W(x, y) = -m\lambda \frac{\varepsilon}{S}, \tag{4}$$

where ε is the grating position error in the direction perpendicular to the pattern. The produced wavefront phase errors due to pattern distortions are linearly proportional to the diffraction order number and inversely proportional to the local fringe spacing [8]. CGH pattern distortion errors do not affect the zero-order diffracted beam. The pattern distortion ε of 0.1 μm is achievable based on the current CGH fabrication technique. For a CGH with an average line spacing of 20 μm, the pattern distortion gives a wavefront error of 0.005λ for the first diffraction order. This is small enough that we can usually ignore it in optical testing.

Irregularities of CGH substrates are typically of low spatial frequencies. The surface figure of a custom CGH substrate, with the size less than 100mm diameter, can be as good as 0.02λ RMS. For the system requires higher measurement accuracy or when the CGH substrate is large, the substrate irregularities need to be removed by calibration. Since the substrate errors influence all the diffraction orders equally, we can measure the effect of irregularities using the zero order diffraction from the CGH, and then subtract it from the first order surface measurement to remove this error.

3. Optimization design

An optimal design of CGHs has the generated wavefront the least sensitive to the fabrication errors. It also has the maximum diffraction efficiency such that there is enough light back into the interferometer. Here we discuss the examples of both chrome-on-glass and phase CGHs, and provide an optimal design based on their fabrication uncertainties.

3.1 Chrome-on-glass CGH

For a chrome-on-glass CGH used in transmission, there is no transmitted beam from the coated portion of the CGH, because the chrome coating either reflects or absorbs all the light. So this chrome-on-glass hologram acts as a pure amplitude CGH. Therefore, chrome thickness error does not affect the wavefront. CGH duty-cycle and amplitude errors have no effect on wavefront for either the zero or the first diffraction orders. The wavefront sensitivity function to duty-cycle and amplitude is zero for both zero order and non-zero orders according to the equations listed in Table 1.

The pure amplitude type CGHs are not sensitive to variations in duty-cycle, chrome thickness and amplitude, so the irregularities in CGH substrate and pattern distortion are the two main error sources. Since substrate irregularities can be removed by calibration, only pattern distortion error remains. Therefore, the overall wavefront accuracy using an amplitude type CGH can be as good as 0.005λ.

As for the diffraction efficiency, since the fabrication errors on duty-cycle do not affect the wavefront phase for amplitude CGHs, we can choose a duty-cycle that maximizes the first order diffraction efficiency. A duty-cycle of 50% has the maximum diffraction efficiency of 10% at the first order of diffraction. The zero order has 25% diffraction efficiency. These results can be easily achieved by applying the equations in Table.1.

When the chrome-on-glass CGH is used in reflection, A_0 and A_1 are both non-zero. The phase difference between the chrome and glass must be considered. There is a phase shift

#86633 - $15.00 USD Received 20 Aug 2007; revised 29 Oct 2007; accepted 1 Nov 2007; published 5 Nov 2007

introduced between the interfaces of the metal and the air due to metal thickness and complex Fresnel reflections. In this case, we consider both amplitude and phase effects to analyze how the fabrication errors affect the wavefront phase. In the following section, an example of phase CGHs is discussed.

3.2 Phase CGHs

When testing a bare glass surface using chrome-on-glass CGHs in transmission, light passes through CGH twice and only 0.04% of light returns to the interferometer, which may not be enough in optical testing. Phase CGHs can have up to 40% diffraction efficiency at the first order, so the light back into the interferometer increases by a factor of 16, which makes phase CGHs useful for low-light-intensity optical testing.

For phase CGHs, the zero and non-zero order diffractions have different sensitivities to duty-cycle, etching depth and amplitude. When we measure the CGH using zero order diffraction and then subtract the result from the non-zero order surface measurement, there are residual wavefront errors left from the fabrication non-uniformities in duty-cycle, etching depth and amplitude. Furthermore, all these fabrication errors are coupled together to affect the wavefront phases for both zero and non-zero orders of diffraction. Our parametric model relates each fabrication error to wavefront performance. To evaluate the overall effect of all the fabrication errors, we can combine the wavefront errors from each fabrication error by a root-sum-square (RSS).

To verify the amount of wavefront errors caused by duty-cycle, etching depth and amplitude variation, we assume the P-V variations in the etching depth, duty-cycle and amplitude are 2%, 1% and 0.5%, respectively. We also assume both A_1 and A_0 are unity. The P-V wavefront errors from CGH fabrication non-uniformities after subtracting the zero order wavefront can be calculated using the parametric model. It is shown in Table 2.

Table 2 Wavefront errors from the fabrication variations

Source of Errors	**Fabrication variation**	**Wavefront error**
Etching depth error	2%	$\left(\frac{\partial\Psi_{m=1}}{\partial\phi} - \frac{\partial\Psi_{m=0}}{\partial\phi}\right)\cdot\left(\frac{\Delta\phi}{\phi}\right)\cdot\phi$
Duty-cycle error	1%	$-\frac{1}{2\pi}\frac{\partial\Psi_{m=0}}{\partial D}\cdot\left(\frac{\Delta D}{D}\right)\cdot D$
Amplitudes Error	0.5%	$\frac{1}{2\pi}\left(\frac{\partial\Psi_{m=1}}{\partial A_1} - \frac{\partial\Psi_{m=0}}{\partial A_1}\right)\cdot\left(\frac{\Delta A_1}{A_1}\right)\cdot A_1$

The plots from Fig. 2 to Fig 4 give the wavefront errors caused by the variations we assumed above for different duty-cycles and etching depths. Figure 2 shows that the wavefront sensitivities to etching depth are the same for the zero and the first orders at 50% duty-cycle. Therefore, the wavefront error is zero at 50% duty-cycle, after subtracting the zero order wavefront from the first order. The variation of duty-cycle only affects the wavefront for the zero order of diffraction. Figure 5 shows the RSS wavefront errors. It shows that when the etching depth is close to 0.5λ, the wavefront error increases dramatically. When we design a CGH, we should avoid choosing the etching depth close to 0.5λ.

#86633 - $15.00 USD Received 20 Aug 2007; revised 29 Oct 2007; accepted 1 Nov 2007; published 5 Nov 2007

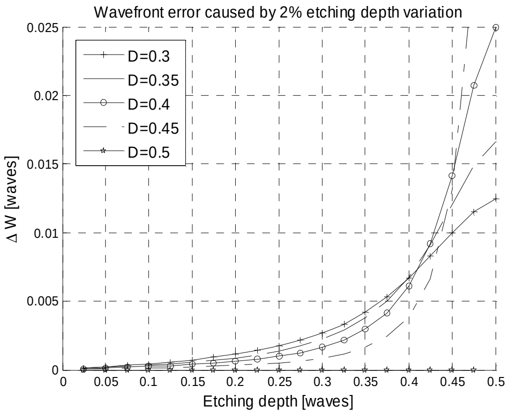

Fig. 2. Wavefront error caused by 2% etching depth variation

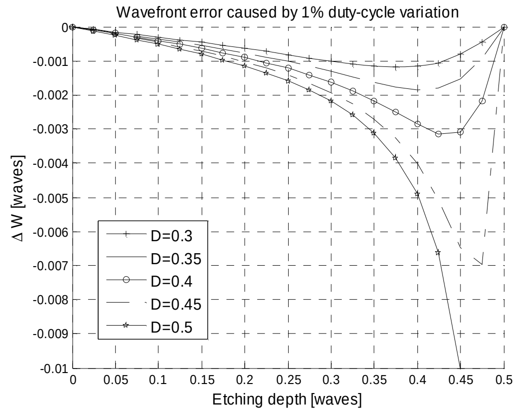

Fig. 3. Wavefront error caused by 1% duty-cycle variation

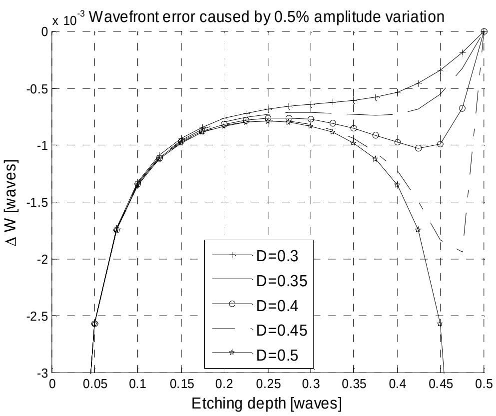

Fig. 4. Wavefront error caused by 0.5% amplitude variation

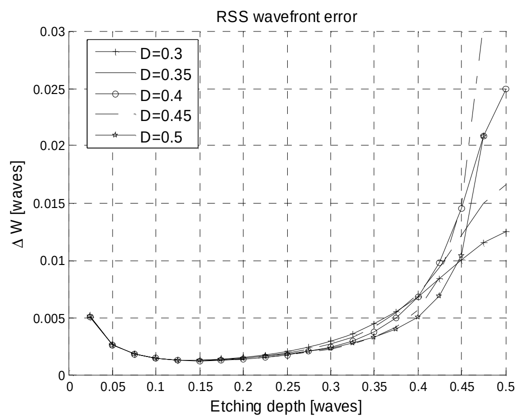

Fig. 5. RSS wavefront error

Assuming that the first order of diffraction is used for testing the surface, Fig. 6 gives the first order diffraction efficiency at different duty-cycles and etching depths. The diffraction efficiency monotonically increases. Larger etching depth has higher diffraction efficiency and larger wavefront errors. The maximum diffraction efficiency of the first order occurs at 0.5λ etching depth. Therefore, there is a tradeoff between the RSS wavefront error and diffraction efficiency.

Figure 7 shows the relationship between the first order diffraction efficiency and the RSS wavefront error in the case of 2% etching depth error, 1% duty-cycle error and 0.5% amplitude error. The RSS wavefront error is plotted on log scale. Each curve represents a duty-cycle. The points on each curve show the RSS wavefront and diffraction efficiency at a certain etching depth. It can be shown that when the etching depth is less than 0.4λ, the diffraction efficiency increases rapidly as the etching depth increases, and the RSS wavefront error does not change much. When the etching depth is greater than 0.4λ, the RSS wavefront error increases significantly but the diffraction efficiency remains almost unchanged. Based on our assumption on the CGH fabrication errors, the duty-cycle of 50% and etching depth between 0.3λ and 0.4λ are a reasonable design, because their diffraction efficiencies are greater than 25% and the wavefront errors are relatively small.

#86633 - $15.00 USD Received 20 Aug 2007; revised 29 Oct 2007; accepted 1 Nov 2007; published 5 Nov 2007

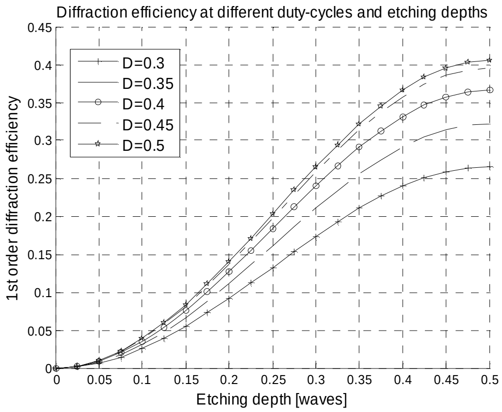

Fig. 6. Different efficiency at different duty-cycle and etching depth

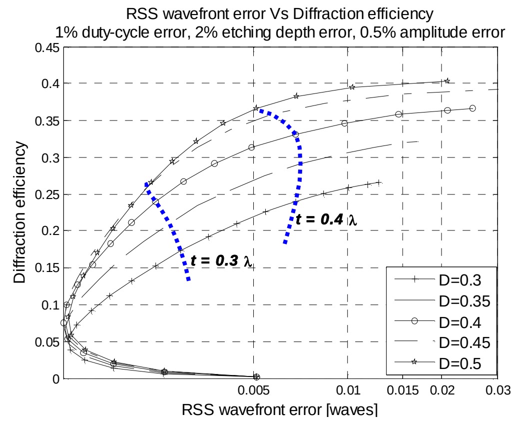

Fig. 7 Relationship between RSS wavefront error and the first order diffraction efficiency (Each point on the curve represents different etching depth)

When we design a CGH, it is important to know the fabrication uncertainties during the CGH writing process. Based on those fabrication uncertainties, a plot like Fig. 7, showing the relationship between diffraction efficiency and RSS wavefront error, can be obtained. Depending on the application, there is a tradeoff between the diffraction efficiency and wavefront errors. This plot helps optical engineers to determine the CGH parameters that are optimal for their applications.

The amplitude errors due to scatter are usually small in CGH. Etching depth and duty-cycle are mostly the main errors. In the case that duty-cycle error dominates, for example, consider a case with 5% duty-cycle variation and 1% etching depth variation. The relationships between the RSS wavefront error and diffraction efficiency are shown in Fig. 8. For this case, CGHs with a larger duty-cycle have larger wavefront errors. This can be predicted from Fig. 3. In Fig. 3, the larger duty-cycles always have more wavefront errors. Therefore, when the duty-cycle error is dominant in the CGHs, the duty-cycle of 40% with the etching depth of 0.5λ is preferable. However, all the curves in Fig. 8 are very close to each other, which means the wavefront error and diffraction efficiency are not very sensitive to duty-cycles.

When etching depth error dominates, consider a case with 1% duty-cycle variation and 5% etching depth variation. The relationships between the RSS wavefront error and diffraction efficiency are shown in Fig. 9. CGHs with 50% duty-cycle have less wavefront error and higher diffraction efficiency. This is because the difference of wavefront errors caused by etching depth between the zero order and first order is zero at 50% duty-cycle, as shown in Fig. 2. Therefore, the duty-cycle of 50% with the etching depth ranging from 0.3λ to 0.4λ is preferable.

#86633 - $15.00 USD Received 20 Aug 2007; revised 29 Oct 2007; accepted 1 Nov 2007; published 5 Nov 2007

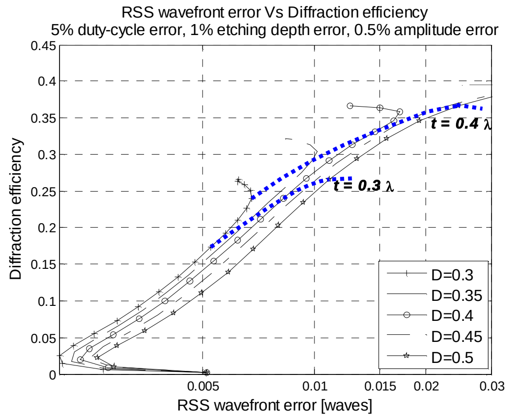

Fig. 8. RSS wavefront error vs. the first order diffraction efficiency (duty-cycle error dominant) Each point on the curve represents different etching depth

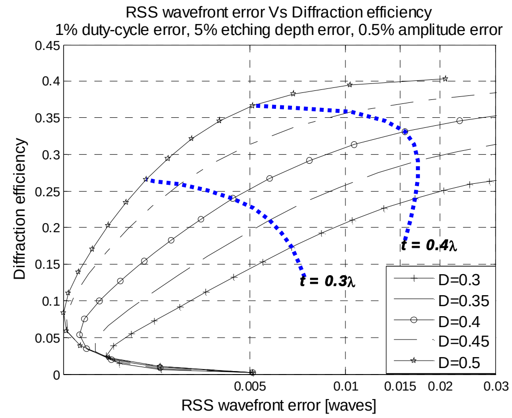

Fig. 9. RSS wavefront error vs. the first order diffraction efficiency (etching depth error dominant) Each point on the curve represents different etching depth

In conclusion, given the fabrication errors, we can optimize the duty-cycle and etching depth to achieve best performance. If the fabrication errors are unknown, we can choose 50% duty-cycle and 0.35λ etching depth. In this way, we will not suffer large wavefront error and low diffraction efficiency, no matter what fabrication error is dominant.

4. Conclusion

This paper introduced the parametric model that relates the wavefront performance to the fabrication errors of CGHs, and also discussed a method for optimizing the CGH design using this parametric model. To calibrate the CGH substrate error, wavefront errors from both the zero and first orders of diffraction must be considered. The examples of both chrome-on-glass and phase CGH are discussed in detail. For chrome-on-glass CGHs used in transmission, fabrication errors on duty-cycle and chrome thickness have no effect on the wavefront. For phase CGHs, fabrication errors affect the wavefront phase for the zero and non-zero orders differently. There is a balance between the wavefront performance and diffraction efficiency. The procedure to find the optimal CGH parameter can be described as below.

- Estimate or measure each fabrication errors of CGHs
- Calculate the wavefront errors caused by each fabrication error using this parametric model
- Plot the relationship between the RSS wavefront error and diffraction efficiency to find the optimal solution for CGH parameters

#86633 - $15.00 USD Received 20 Aug 2007; revised 29 Oct 2007; accepted 1 Nov 2007; published 5 Nov 2007