Effects of birefringence on Fizeau interferometry that uses polarization phase shifting technique

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Abstract: Interferometers that use different states of polarization for the reference and test beams can modulate the relative phase shift using polarization optics in the imaging system. This allows the interferometer to capture simultaneous images that have a fixed phase shift, which can be used for phase shifting interferometry. Since all measurements are made simultaneously, the interferometer is not sensitive to vibration. Fizeau interferometers of this type have advantage over Twyman-Green type systems because the optics are in the common path of both the reference and test wavefronts, therefore errors in these optics affect both wavefronts equally and do not limit the system accuracy. However, this is not strictly true for the polarization interferometer when both wavefronts are transmitted an optic that suffers from birefringence. If some of the components in the common path of the reference and test beams have residual birefringence, the two beams see different phases. Therefore, the interferometer is not strictly common path. As a result, an error can be introduced in the measurement. In this paper, we study the effect of birefringence on measurement accuracy when different polarization techniques are used in Fizeau interferometers. We demonstrate that measurement error is reduced dramatically for small amount of birefringence if the reference and test beams are circularly polarized rather than linearly polarized.

Keywords: interferometry, polarization, birefringence

1. Introduction

In a common path interferometer, e.g. a Fizeau interferometer, both the reference and test beam go through the same optics up to the reference surface, therefore, any defect in the common path affects the phases of reference and test beams equally and has no effect on the measurement accuracy.[1] However, when the reference and test beams have different polarization states, if a component in the common path has residual birefringence, the reference and test beams see different phases and an error is introduced in the measurement. In this case, the interferometer is no longer strictly common path, though it still physically is. There now exist commercial phase-shifting Fizeau interferomters that use this polarization technique to simultaneously take multiple frames to freeze vibration.[3,4] Birefringence is a concern because it is always present, especially for big and thick optics. For example, Schott specifies the residual birefringence of standard BK7 glass at <6nm/cm.[2] If a test plate is 10cm thick, the birefringence is about 60nm which is significant if the required measurement accuracy is high. But by using circular polarized light instead of linearly polarized light, the measurement error caused by the residual birefringence can be dramatically reduced and can be eliminated. In Section 2, we give the maximum measurement error when the reference and test beams are linearly polarized. In Section 3, first we set up a model to study the effect of birefringence when circularly polarized beams are used for the reference and test beams, and show analytically the combined beam before phase shifting is elliptically polarized rather than linearly polarized. In Section 3.2, we

Optical Manufacturing and Testing VI, edited by H. Philip Stahl, Proc. of SPIE Vol. 5869
(SPIE, Bellingham, WA, 2005) · 0277-786X/05/$15 · doi: 10.1117/12.615521

study the beam intensities after a general elliptically polarized beam passes the linear polarizer. We show that the observed intensity is sinusoidal, but a phase retardation is introduced and fringe contrast is reduced. We use this result to analyze the phase measurement error due to birefringence in Section 3.3. We present the simulation result in Section 3.4.

2.Linear polarization case:

To analyze the effect of birefringence we assume a Fizeau interferometer as shown in Figure 1. As in all Fizeau interfermeters, the reference surface is the last surface in the system, so all of the transmissive optics are common path. If some optics have residual birefringence, we model the combined birefringence as a waveplate whose fast axis and retardation vary from point to point in the pupil. The light reflected from the reference surface creates the reference wavefront and light transmitted through this surface, then reflected from the surface under test creates the test wavefront. The interference between the two is used to determine the error in the surface under test. Phase shift interferometry is conventionally performed moving the reference surface to cause a phase shift and capturing successive interferograms. Recently, Fizeau interferometers that simultaneously capture all of the different phase shifts have been developed. These systems are configured so that the reference wavefront and the test wavefront have orthogonal polarization states. Through clever use of geometry or coherence, the system can be configured so that only the desired polarization states are measured.

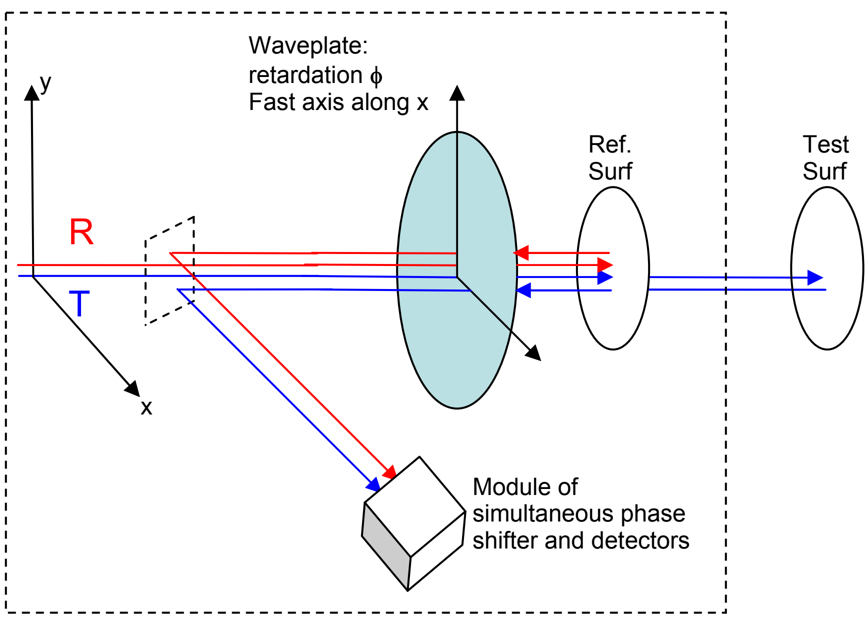


Figure 1. A simultaneous phase-shifting polarization Fizeau interferometer.

If the reference and test beams are linearly polarized, e.g. the reference beam is x-polarized and test beam is y-polarized, and if the residual birefringence is ϕ (in radian) and its fast axis is along x, then the test beam sees an additional phase 2ϕ due to this birefringence. It is mistakenly attributed to the surface figure error:

$$\delta_{\max} = \frac{\phi}{2\pi}\lambda. \qquad (1)$$

This would be the maximum measurement error caused by the residual birefringence in the common path of the reference and test beams.

3. Circular polarization case:

When the reference and test beams are circularly polarized, a linear polarizer can be used as the phase shifter as shown in Figure 2. Rotation of the linear polarizer shifts the relative phase between the two polarizations.[5] 4D Technologies uses the same principle but a pixilated mask to achieve simultaneous phase shifting.[4]

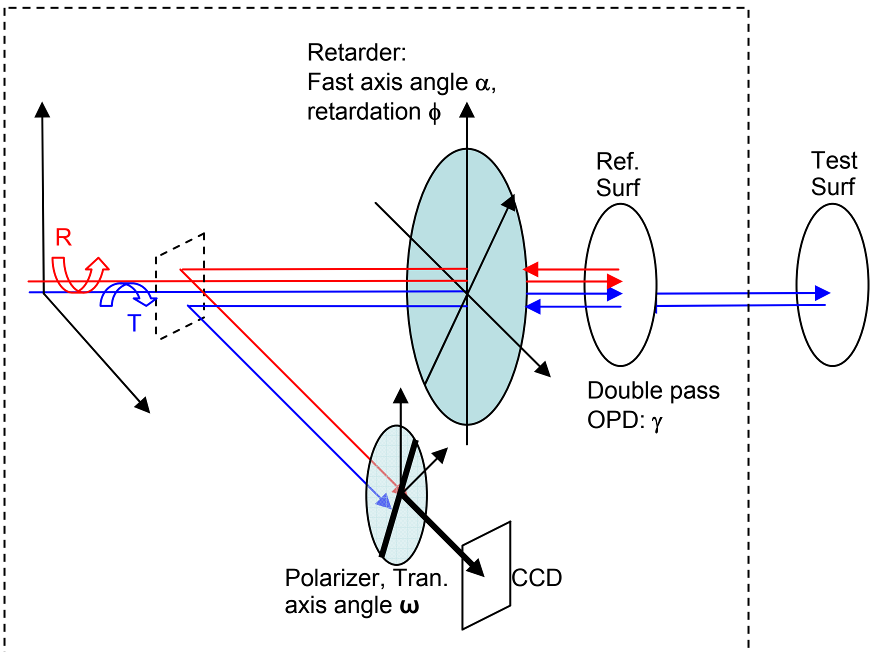


Figure 2. A phase-shifting Fizeau interferometer with circularly polarized reference and test beams uses a linear polarizer as the phase-shifter.

3.1 The combined beam before the phase shifter

If, for example, the test beam is right hand circular (RHC) and the reference beam is left hand circular (LHC), there always exists a coordinate system where the reference and test beams are in phase. We define this coordinate system as the global coordinate system X_G-Y_G (see Figure 3). In this coordinate system, the Jones vectors[6] of the test beam (denoted as T_G) and the reference beam (denoted as R_G) are

$$T_G = \begin{pmatrix} 1 \\ -i \end{pmatrix} \text{ and } R_G = \begin{pmatrix} 1 \\ i \end{pmatrix}. \qquad (2)$$

As stated in Section 2, the residual birefringence is modeled as a waveplate. Assume its retardation is ϕ (angle), and its fast axis has an angle α with the global X_G-axis. We choose its fast axis as the local x-axis, denoted as X_T-axis. In the local coordinate X_T-Y_T (see Figure 3), the double pass Jones matrix of the waveplate is

$$B_T = \begin{pmatrix} 1 & 0 \\ 0 & e^{i2\phi} \end{pmatrix}. \tag{3}$$

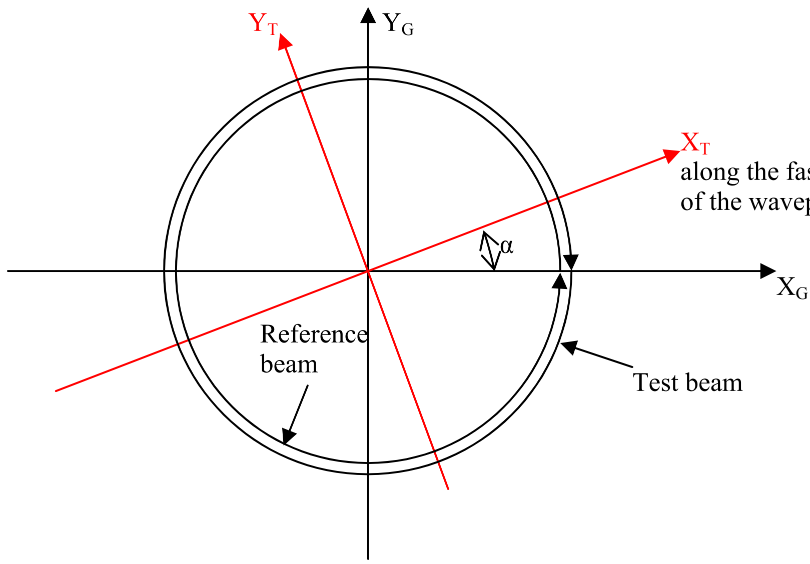


Figure 3. Illustration of the definitions of global coordinate system and the waveplate local coordinate system. X_G-Y_G is the global coordinate system where the incident reference and test beams are in phase. X_T-Y_T is the waveplate local coordinate system with the fast axis along the X_T direction. The test and reference beams are circularly polarized, and in phase in the global coordinate system. They have equal intensity (exaggerated in the figure).

In the waveplate local coordinate system, the reference and test beam see a phase shift,

$$T_T = \begin{pmatrix} 1 \\ -i \end{pmatrix} e^{-i\alpha} \text{ and } R_T = \begin{pmatrix} 1 \\ i \end{pmatrix} e^{i\alpha}. \tag{4}$$

After the light is reflected from the reference and test surfaces, and passes the waveplate a second time, the reference and test beams become

$$T_T' = B_T T_T e^{i2\gamma} = \begin{pmatrix} 1 \\ -ie^{i2\phi} \end{pmatrix} e^{i(2\gamma-\alpha)} \text{ and } R_T' = B_T R_T = \begin{pmatrix} 1 \\ ie^{i2\phi} \end{pmatrix} e^{i\alpha}, \tag{5}$$

where γ is the single pass phase difference between the reference and test beams caused by the physical separation between reference and test surfaces along a ray.

The combined beam is

$$R_T'+T_T'=\begin{pmatrix} e^{i(2\gamma-\alpha)}+e^{i\alpha} \\ ie^{i(2\phi+\alpha)}-ie^{i(2\phi+2\gamma-\alpha)} \end{pmatrix}=e^{i\gamma}\begin{pmatrix} 2\cos(\gamma-\alpha) \\ 2\sin(\gamma-\alpha)e^{i2\phi} \end{pmatrix}. \qquad (6)$$

So, when there is no birefringence, i.e. $\phi=0$, the combined beam is linearly polarized with the electrical field vector oscillates along a direction which has an angle $\gamma-\alpha$ with the waveplate's local X_T axis and an angle γ with the global X_G axis. When the residual birefringence is non-zero, the combined beam is elliptically polarized with the phase difference 2ϕ between the E-fields in Y_T and X_T directions (see Figure 4).

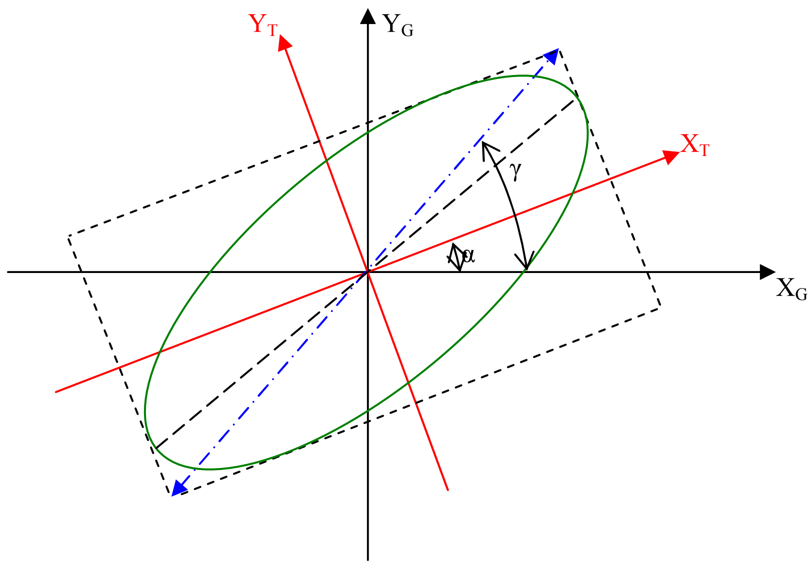


Figure 4. Illustration of the combined beam's polarization. The blue dashed line illustrates the ideal linearly polarized beam when no birefringence exists, while the green solid ellipsis illustrates the elliptically polarized beam when birefringence exists.

3.2. Intensity after an elliptically polarized beam passes the phase shifter

The reference and test beams both pass a linear polarizer before reaching the CCD. The polarizer combines the reference and test beams to obtain interference. It also serves as a phase shifter – when it rotates by angle of ω the phase difference between the reference and test beam will increase by 2ω.[5] The interferometer can make simultaneous measurements with different phase shifts by creating multiple images of the pupil and viewing them through polarizers set at different angles.[4]

When a general elliptically polarized beam passes through a linear polarizer, the transmitted beam intensity is a function of incident beam parameters and the angle of linear polarizer's transmission axis.

The Jones vector for general elliptically polarized light is:

$$E = \begin{pmatrix} E_x \\ E_y \end{pmatrix} = \begin{pmatrix} A_x \\ A_y e^{i\delta} \end{pmatrix}. \tag{7}$$

The electrical field vector at any point forms an ellipsis described by[7]

$$\left(\frac{E_x}{A_x}\right)^2 + \left(\frac{E_y}{A_y}\right)^2 - 2\left(\frac{E_x}{A_x}\right)\left(\frac{E_y}{A_y}\right)\cos(\delta) = \sin^2(\delta). \tag{8}$$

Define an angle θ such that

$$\tan(\theta) = \frac{A_y}{A_x}. \tag{9}$$

Then the axis of the ellipsis has an angle ψ with the x-axis, as shown in Figure 5 where

$$\tan(2\psi) = \tan(2\theta)\cos(\delta). \tag{10}$$

Assume the elliptically polarized beam passes through a linear polarizer whose transmission axis has an angle ω with x-axis. The Jones matrix for the polarizer is

$$P = \begin{pmatrix} \cos^2\omega & \cos\omega\sin\omega \\ \cos\omega\sin\omega & \sin^2\omega \end{pmatrix}.$$

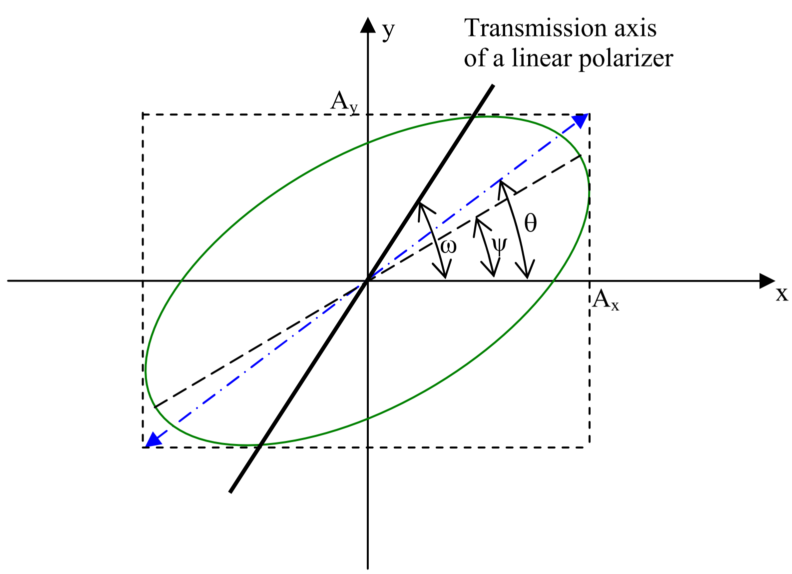


Figure 5. Illustration of definitions of the angles θ and ψ associated with an elliptically polarized light. Also shown is a linear polarizer with transmission axis having an angle ω with x-axis.

After the linear polarizer, the transmitted E-field is

$$E_{trans} = P\begin{pmatrix} A_x \\ A_y e^{i\delta} \end{pmatrix} = (A_x \cos\omega + A_y \sin\omega \cdot e^{i\delta})\begin{pmatrix} \cos\omega \\ \sin\omega \end{pmatrix}. \tag{11}$$

The intensity of the beam is

$$I = \left|E_{trans}\right|^2 = \frac{A_x^{\ 2} + A_y^{\ 2}}{2} + \frac{\sqrt{A_x^{\ 4} + 2A_x^{\ 2}A_y^{\ 2}\cos(2\delta) + A_y^{\ 4}}}{2}\cos(2\omega - 2\psi), \tag{12}$$

where again $\tan(2\psi) = \tan(2\theta)\cos(\delta)$.

Eq. (12) demonstrates that we can measure ψ, which is a characteristic of the incident elliptically polarized beam, by phase shifting interferometry with a linear polarizer as phase shifter. When the incident beam is linearly polarized, i.e. $\delta = 0$, then $\psi = \theta$. When the beam is elliptically polarized, compared to the linear polarization case, the beam intensity as a function of ω sees a phase shift

$$2\Delta = 2(\theta - \psi). \tag{13}$$

From Eq. (12), we also obtain the fringe contrast

$$C = \frac{\sqrt{A_x^{\ 4} + 2A_x^{\ 2}A_y^{\ 2}\cos(2\delta) + A_y^{\ 4}}}{A_x^{\ 2} + A_y^{\ 2}}. \tag{14}$$

With the definition of θ, the fringe contrast can be rewritten:

$$C = \sqrt{1 - \sin^2(2\theta)\sin^2\delta}. \tag{15}$$

Figure 6 plots the transmitted beam intensity as a function of ω for a linearly polarized beam and an elliptically polarized beam.

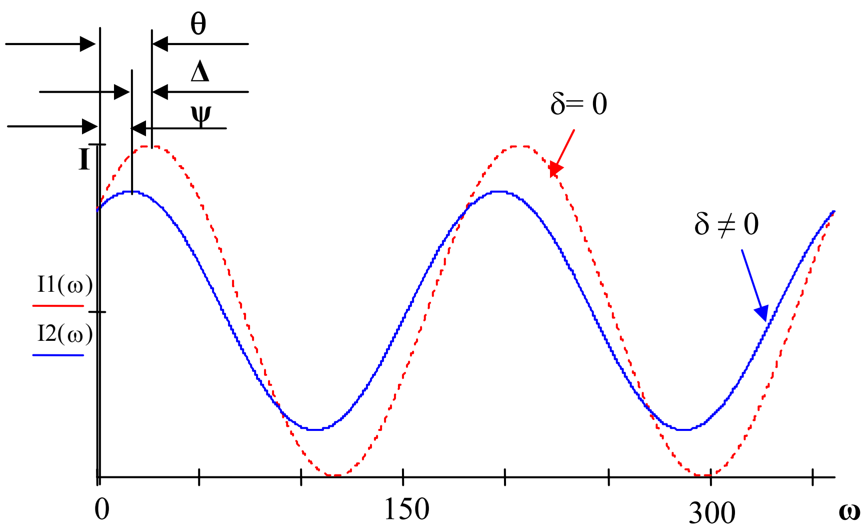


Figure 6. The transmitted light intensity as a function of the linear polarizer's rotation angle, after the waveplate the converts circular to linear polarization. The red dash line represents the case when there is no error and the light is linearly polarized, while the blue solid line represents the case when birefringence has caused the light to be elliptically polarized. Note the intensity has a phase shift and a reduced contrast when the incident beam is elliptically polarized compared to the linearly polarized.

3.3 Analysis

In Section 3.1, we show that, when birefringence exists, the combined beam from reference and test surfaces is elliptically polarized (see Eq. (6)). In Section 3.2, we show that we still measure a phase using phase shifting interferometry but with an error (see Eqs. (12) and (13)). The error depends on the characteristic of the elliptically polarized beam. From Eqs. (6), (10) and (13), we get

$$\tan(2\Delta) = \tan(2\theta - 2\psi) = \frac{2\tan[2(\gamma-\alpha)]\sin^2\phi}{1+\tan^2[2(\gamma-\alpha)]\cos 2\phi}, \qquad (16)$$

Which indicates that the surface measurement error Δ is a function of the retardation ϕ, the birefringence angle α and the actual phase γ. Since this is a function of the phase γ, the error will vary when this phase is changed by making slight adjustments to the system.

Assume

$$-\frac{\pi}{4} < \phi < \frac{\pi}{4},$$

then the maximum phase measurement error is

$$\tan(2\Delta_{max}) = \frac{\sin^2\phi}{\sqrt{\cos 2\phi}}. \tag{17}$$

For small ϕ approximation,

$$2\Delta_{max} \approx \phi^2. \tag{18}$$

The maximum surface figure measurement error is then

$$\delta_{max} = \frac{\Delta_{max}}{2\pi}\lambda \approx \frac{\phi^2}{4\pi}\lambda, \tag{19}$$

which indicates the maximum measurement error has quadratic dependence on the amount of birefringence.

For comparison, we plot the maximum surface measurement errors as a function of birefringence when the reference and test beams are linearly and circularly polarized, respectively, for small amount of residual birefringence.

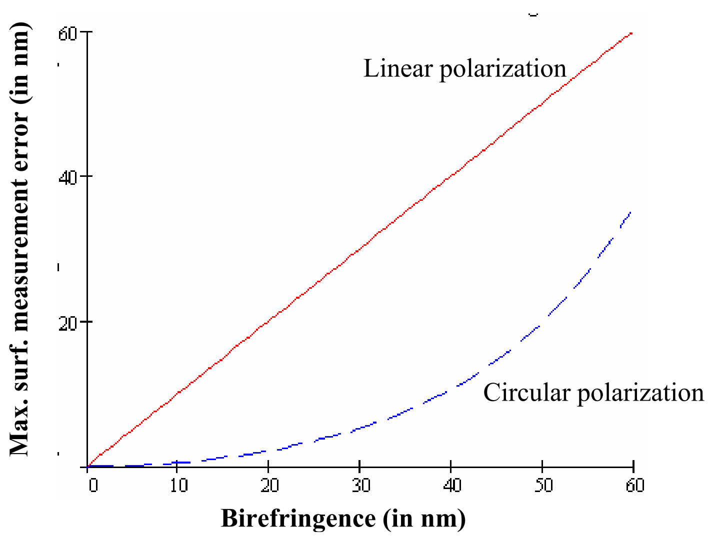
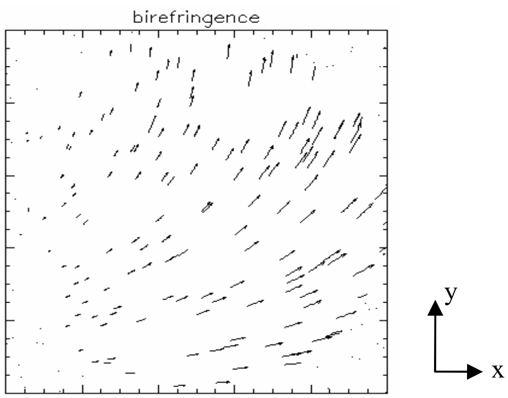


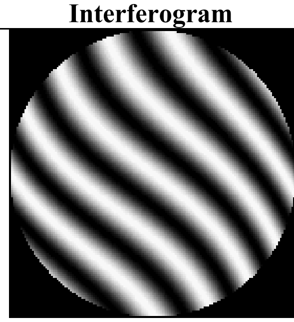
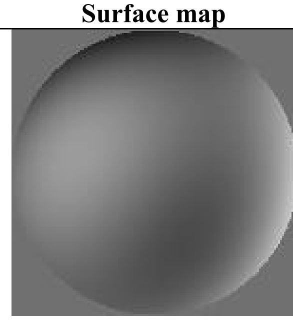
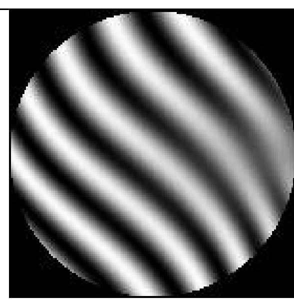
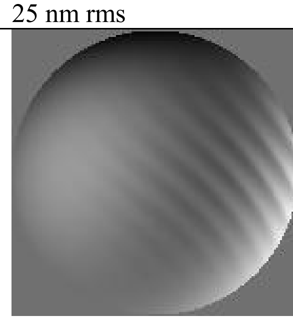
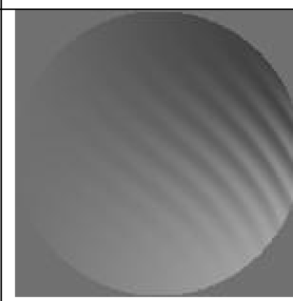
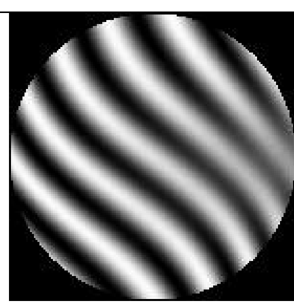
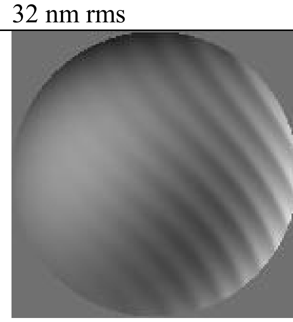
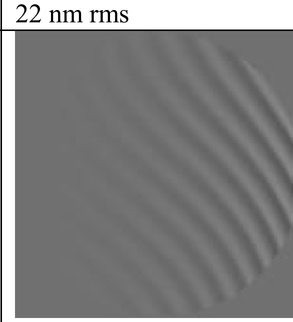
Figure 7. Plot of the maximum surface measurement error vs. birefringence for both linear and circular polarization cases.

3.4. Simulation of surface measurements for a system with birefringence

The effect of birefringence is illustrated using a set of simulations. We perform the analysis for an interferometer that has 10 cm thick transmissive optic with birefringence of 6nm/cm. A map of the birefringence is shown in Figure 8(a). We evaluate the performance of this system as it measures a mirror that has 25 nm rms surface irregularity. We also assume the measurements to be made with 5 fringes of tilt due to alignment. The ideal and measured surface maps and measurement error maps are shown in Figure 8(b). The results verify that the measurement error is significantly smaller for small residual birefringence when circularly polarized beams, rather than linearly polarized beams, are used.



(a)

		Interferogram	Surface map	Error in measurement
(b) Real Surface			 25 nm rms	
(c) Simulated Measure-ments	linear polarization case		 32 nm rms	 22 nm rms
	circular polarization case		 26 nm rms	 6 nm rms

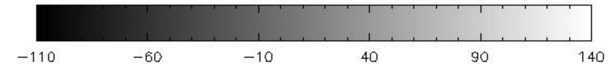
Scale bar for all maps (in nm)	

Figure 8. Simulated Fizeau measurements for a system with birefringence in the common part of the system (a) Birefringence map. The fast axis angle has a linear distribution along y-axis from 0 to 90 degrees. And the retardation has a linear distribution along x-axis from 0 to 60nm. (b) Results of simulation for an ideal system with 25 nm rms surface irregularity (c) Results for simulated phase shift interferometry for the case with 60 nm birefringence with the spatial distribution shown in (a). The measurement error is calculated by subtracting the ideal surface error from the simulated measurement. Note the reduction in fringe contrast as well as the phase error for both cases.

If we maintain the distribution of birefringence and vary the magnitude of retardation, we expect to see the measurement error increases as a function of the maximum retardation. Figure 9 shows the RMS measurement error as a function of the maximum retardation for the linear and circular polarization cases. It is obvious that measurement error is linear to birefringence when linearly polarized beams are used. In contrast, the measurement error is quadratic to birefringence when circularly polarized beams are used instead.

There is an important distinction between the form of the measurement errors for the two cases. When linear polarization is used, an error is created that will be proportional to the birefringence and constant for all interferograms. Superimposed is an error that depends on the alignment and shows up as ripples in the surface with two times the frequency of the interferogram fringes. This component of the error will change as the alignment is adjusted, thus can be reduced by averaging, but the larger, fixed component would remain as a real error in the test. For the case of the circular polarizarion, there is NO fixed error, and only the ripple type that depends on the interferogram alignment. Therefore it is possible to reduce the effect of the birefringence for the case of circular polarization by averaging multiple maps with different alignment.

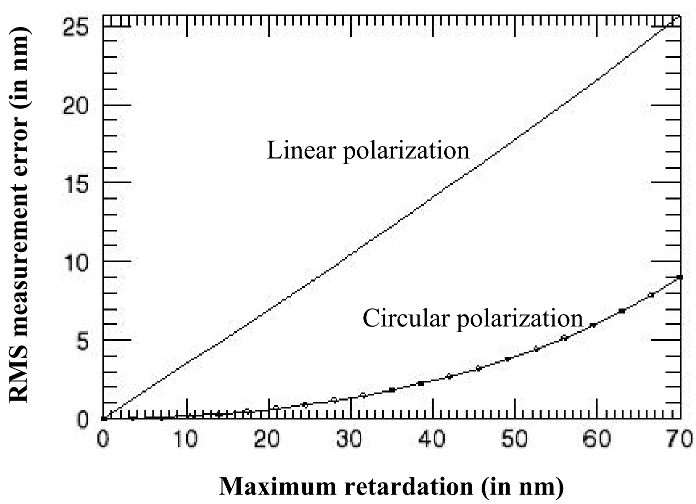


Figure 9. Plot of RMS surface measurement error as a function of maximum retardation of the birefringence distribution shown in Figure 8(a), for both linear and circular polarization cases. The dots are theoretical calculation results using Eq. (16), which agree with the interferometric simulation results for the circular polarization case.

4. Summary

Fizeau interferometers can use polarization techniques to create a phase shift between the reference and test beams. If some element in the common path exhibits residual birefringence, it can limit measurement accuracy. We model the residual birefringence as a waveplate whose fast axis orientation and retardation vary from point to point in the pupil. If the reference and test beams are linearly polarized and orthogonal, the measurement phase error can be as large as the amount of birefringence. We studied the case when the two beams are circularly polarized and orthogonal, and we derived a set of relations to calculate the measurement error and the fringe contrast when birefringence is present. For small amount of birefringence, we showed that the error is quadratic to the amount of birefringence. So, in the case of small birefringence, the measurement error is significantly smaller if circular polarization rather than linear polarization is used to differentiate reference and test beams. In addition, this error is a function of the phase difference between the reference and the test, so the error can be further reduced by averaging multiple measurements with slight phase shifts.

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