# Testing an off-axis parabola with a CGH and a spherical mirror as null lens 

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#### Abstract

: Steward Observatory Mirror Lab is currently polishing an off-axis parabola which will be the primary mirror of the New Solar Telescope. To test this mirror, we built a test equipment to combine a spherical mirror and a Computer Generated Hologram (CGH) as null lens. The spherical mirror is tilted to compensate much of the astigmatism and some coma. And the CGH compensates rest of aberrations. The combination of a spherical mirror and a CGH makes the test system compact. The technology developed here will be used to test the Giant Magellan Telescope's primary mirror segment - a five times larger off-axis parabola.


## 1. Introduction:

The Steward Observatory Mirror Laboratory at the University of Arizona is currently polishing the primary mirror of the New Solar Telescope (NST) for Big Bear Observatory. The mirror is a 1.7 meter off-axis segment of an $\mathrm{f} / 0.7$ parabola. It is a $1 / 5$ scale of Giant Magellen Telescope (GMT) mirror ${ }^{1,2}$. Although the NST mirror can be easily tested with an auto-collimation test, we developed a reflective-diffractive test to get experience testing the GMT mirrors. What we did is to use a tilted fold spherical mirror to correct most of astigmatism and a computer generated hologram (CGH) to correct the rest of aberrations. ${ }^{3}$ In Section 2, we describe the NST mirror and the aberrations it has. The testing requirements are listed in Section 3. We present the design of the testing equipment in Section 4. In Section 5, the tolerance analysis is presented. The design and fabrication of the main CGH is described in Section 6. The alignment of the system and the test mirror is presented in Section 7. In Section 8, we briefly describe the measurement sequence. We summarize the technologies that we developed for testing the NST mirror and can be used to test the GMT mirrors in Section 9.

## 2. The NST mirror:

The NST mirror is an off-axis segment of a parabola. The parent parabola has a vertex radius of 7.7 meters. The segment's center is 1.84 meters from the parent's vertex. The nominal prescription of the mirror is given below in Table 1. The telescope is specified to have a $1.6-\mathrm{m}$ clear aperture. The primary is off axis and tilted relative to the incoming light. This causes the circular telescope pupil to map to a $1.6 \mathrm{~m} \times 1.65$ elliptical pupil on the mirror, as viewed from its center of curvature.

The definition for the off axis geometry is shown in Figure 1.

Table 1. Specifications for the NST mirror

| Parameter | Nominal value | Tolerance |
| :--- | :--- | :--- |
| Vertex Radius of Curvature | 7.700 meters | $\pm 10 \mathrm{~mm}$ |
| Conic Constant | -1.00 | $\pm 0.001$ |
| Diameter of mirror blank | 1.7 meters | - |
| Clear aperture of mirror in telescope | 1.6 meters | - |
| Distance between mirror center and parent vertex | 1.84 meters | $\pm 5 \mathrm{~mm}$ |



Figure 1. Definition of geometry for the NST mirror. This is shown as a projection in the direction of the optical axis of the parent.

It is useful to evaluate the aspheric departure of surface for the optical test. The dominant term for this aspheric departure for the complete 5.28 meter diameter parent is about 3.3 mm P-V or 1 mm rms spherical aberration, adjusted to the best fit sphere. This can be approximated for the off axis part as a combination of low order Zernike polynomials, centered on the circular aperture of the mirror. The coefficients are given below in Table 2 and a plot of the aspheric departure is shown in Figure 2. When adjusted to the best fit power and tilt, the surface has about $2600 \mu \mathrm{mP}-\mathrm{V}$, and $470 \mu \mathrm{~m} \mathrm{rms}$ aspheric departure.

Table 2．Aspheric departure in part－centered coordinates

| Parameter | Zernike coefficient in <br> $\boldsymbol{\mu m}$ | $\boldsymbol{\mu m} \mathbf{~ r m s}$ |
| :--- | :---: | :---: |
| Astigmatism | 1120 | 457 |
| Coma | 324 | 114 |
| Spherical aberration | -18.6 | 8 |
| Trefoil | 0.8 | 0.3 |
| Residual higher order |  | 0.06 |



Figure 2．Aspheric departure of the off axis mirror in units of microns．The parent axis would be above the plot．The color scale is shown in units of micrometers．

## 3．Testing Requirement：

The mirror is to be supported by actuators．So its shape can also be adjusted by the supporting actuators．Table 3 lists the amount of aberrations that can be corrected if each actuator＇s force is limited to 15 N ．Half of these correctable aberrations are allocated to mirror fabrication budget，and the rest is allocated to the testing equipment budget．Since little coma can be bent out by the actuators，no budget is allocated to either fabrication or testing equipment．Instead，the solid body motion of the test mirror is used to compensate coma coming from misalignment of the testing equipment．
After the lower order aberrations are subtracted out，the allowable residual aberration is 20nm RMS．

Table 3: Amount of lower order aberrations that can be corrected by actuators and the budget allocated to fabrication and testing equipment.

| Aberrations | Correctable by <br> actuators (in nm <br> RMS) | Budget for mirror <br> fabrication (in nm <br> RMS) | Budget for testing <br> equipment (in nm <br> RMS) |
| :--- | :--- | :--- | :--- |
| Astigmatism | 200 | 100 | 170 |
| Coma | 17 | - | - |
| Trefoil | 50 | 25 | 42 |
| Quatrefoil | 20 | 10 | 17 |
| Spherical | 25 | 12.5 | 20 |

## 4. Design of the testing equipment:

To limit the size of the testing setup, a tilted fold sphere is used as part of the null lens. It corrects most of the astigmatism and some of coma from the NST mirror. A CGH is used to correct the rest of aberrations. The system layout is shown in Figure 3. A picture of the assembled testing system is shown in Figure 4.


Figure 3. The optical layout of NST testing system.


Figure 4. The NST testing equipment in alignment stage.

## 5. Tolerance analysis:

The CGH illuminating wavefront error and the spherical mirror's surface error will be subtracted out from the NST measurement. Then lower order aberrations come only from the alignment of CGH and the spherical mirror, the measurement error of the spherical mirror's radius of curvature and misalignment caused by temperature variation. We did tolerances on these parameters. The results are listed in Table 4. The definition of misalignment parameters is illustrated in Figure 5.


Figure 5. Illustration of definitions of alignment parameters for CGH and the spherical mirror.

Table 4. Tolerance analysis for lower order aberrations: astigmatism, trefoil, spherical and two-theta.

| in nm | CGH |  |  | Spherical mirror |  |  | ROC | Thermal |  | Actuator |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | dz | tilt x | tilt y | dz | tilt x | tilt y |  |  | RSS | Amount Correctable By 15 N | Forces Needed (N) |
| um | 7 | $\begin{array}{r} 7 @ \\ \text { Edge } \end{array}$ | $\begin{array}{r} 7 @ \\ \text { Edge } \end{array}$ | 7 | $\begin{array}{r} 7 @ \\ \text { Edge } \end{array}$ | 7 @ Edge | 20 | 1deg C |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |
| Z5 | 0.2 | 0.0 | -5.6 | 0.0 | 0.0 | -37.6 | 0.0 | 0.0 | 38.1 | 346.0 | 1.6 |
| Z6 | 133.9 | -7.0 | 1.6 | 93.0 | -149.5 | 3.1 | 63.0 | 206.5 | 309 | 346.0 | 13.4 |
| Z9 | -16.1 | 1.7 | -0.3 | -4.3 | 19.1 | -0.5 | -3.0 | -12.2 | 28.4 | 87.0 | 4.9 |
| Z10 | 0.0 | 0.0 | -1.8 | 0.0 | 0.0 | -15.0 | 0.0 | 0.0 | 15.1 | 87.0 | 2.6 |
| Z11 | 5.9 | 1.2 | 0.0 | 1.9 | -4.2 | 0.0 | 1.2 | 4.7 | 9.0 | 43.0 | 3.1 |
| Z14 | 1.1 | 3.1 | 0.0 | 0.0 | -0.6 | 0.0 | -0.1 | 0.2 | 3.3 | 35.0 | 1.4 |
| Z15 | 0.0 | 0.0 | 2.8 | 0.0 | 0.0 | -0.5 | 0.0 | 0.0 | 2.9 | 35.0 | 1.2 |
|  |  |  |  |  |  |  |  |  |  | RSS | 15.0 |

Note that the most sensitive alignment is the tilt of the spherical mirror in $x$ direction and temperature variation of 1 degree Celsius introduces the most astigmatism.

For higher order residual aberrations, the total allowed is 40 nm , of which 20 nm is allocated to the testing system budget. When errors in lens, CGH and spherical mirror are backed out, the residuals are estimated. Table 5 shows the tolerances for the total residual aberrations in the testing system.

Table 5. Tolerance analysis for higher order residual aberrations.

| in nm | CGH |  |  | Spherical mirror |  |  | ROC | Thermal | RSS |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | dz | tilt x | tilt y | dz | tilt x | tilt y |  |  |  |
| um/md eg | 7 | 7 | 7 | 7 | 1.4 | 1.4 | 20 | 1degree |  |
| RMS Fit error | 2.0 | 2.6 | 1.7 | 1.2 | 2.6 | 2.5 | 0.7 | 3.5 | 6.4 |
| Z12 | -6.3 | -2.0 | -0.1 | -0.9 | 4.4 | -0.2 | -0.6 | -3.2 | 8.6 |
| Z13 | 0.0 | 0.0 | -0.5 | 0.0 | 0.0 | 3.2 | 0.0 | 0.0 | 3.2 |
|  |  |  |  |  |  |  |  | RSS subtotal | 11.2 |
|  |  |  |  |  |  |  |  | Lens+CGH | 8 |
|  |  |  |  |  |  |  |  | Mirror | 8 |
|  |  |  |  |  |  |  |  | RSS Total | 15.9 |

## 6. The CGHs:

There are total 8 CGHs written on the same substrate. Besides the main CGH which serves as part of the null lens, one CGH is for substrate aligning purpose, two of CGHs create a cross hair and a clocking line for roughly aligning the NST mirror, and four of CGHs create reference beams for aligning the spherical mirror. All these CGHs are chrome-on-glass amplitude type. There were made with a laser writer at the Institute of Automation and Electrometry in Sibivirsk, Russia. The substrate is 110 mm in diameter and 9.5 mm thick fused silica. The anatomy of the CGHs is shown in Figure 6.

The main CGH is defined by phase data at a grid of $80 \times 80$ points across $80 \times 80 \mathrm{~mm}$ square area. ${ }^{4}$ The exact phase value at any other point is calculated by bicubic spline fitting ${ }^{5}$ when the CGH was modeled in the design phase and when it was being fabricated. All other CGHs are defined with Zernike Fringe Polynomials.


Figure 6. A 1:350 rendition of the CGHs.

## 7. System alignment:

### 7.1 Alignment of Lenses and CGH:

The CGHs were made on the back surface of the substrate, i.e. the laser beam is shining on them from the substrate side. To make the design of the main CGH easy, we designed a meniscus diverger lens to crorrect spherical aberrations introduced by the substrate so that the converging beam after the compound diverger lens and the CGH substrate has a well-corrected wavefront. When aligning these oiptics, we put a reference sphere after the focus to get fringes from reflection of the $0^{\text {th }}$ diffraction order. The compound lens is adjusted to get best possible wavefront and the CGH is adjusted to get null reflection fringes from the ring type alignment patch.

### 7.2 Alignment of the Spherical Mirror

We put four balls on the surface of the spherical mirror. The four beams created by the four CGHs are directed to the balls. We adjust the ball positions so that the beams are retroreflected. Another ball is placed at the focus of the $0^{\text {th }}$ order beam. Its position is adjusted such that reflection from the ball forms null fringes. Then we use the four metering rods to measure the distances from the balls at the mirror surface to the ball at focus. We adjust the mirror to get the four distances right. Then we adjust the balls on the
mirror surface again to get the beams retroreflected, and we measured the distances and adjust the mirror position again. See Figure 7 for the illustrations of the alignment scheme. After a few iterations, the spherical mirror is aligned to within tolerances.


Figure 7. Illustration of the spherical mirror alignment scheme. (a) Side view sketch. (b) Four beams are created by holograms to focus on the center of curvature of the balls at the mirror surface.

### 7.3 Alignment of the NST mirror

Two CGHs create a clocking line and a cross hair which are projected onto the mirror under test. They provide the references for aligning the NST mirror relative to the testing system (see Figure 8). Fringes can be seen after the NST mirror is roughly aligned, then we use the fine adjustment provided by the mirror support stage to get fringes across the clear aperture of the mirror.


Figure 8. Illustration of the NST mirror alignment relative to the testing equipment -a crosshair and a clocking line are generated for positioning the off-axis parabola.

## 8. Measurement Procedure:

Two wavefront errors are subtracted from the measurement: one is surface figure error of the fold spherical mirror, the other is error in CGH illuminating wavefront, i.e. the $0^{\text {th }}$ order wavefront after the main CGH. These two wavefronts were measured and saved. Because of the mapping distortion, they were morphed before subtracted out from the NST measurement.

## 9. Road to GMT testing:

The technologies we developed for testing the NST mirror, and experiences we acquired in this project will be applied to the more challenging task of testing the 5 times bigger GMT mirror. Following is a list of the technologies and experiences that are critical to the success of GMT testing:

- We demonstrated that tilted spherical mirrors can be used as part of null lens to measure the surface figure of an off-axis parabola. Using fold spheres makes testing setup compact, therefore, it is the method of choice when the test surface has a long radius of curvature and available space is limited. In the GMT testing system, two fold spheres together with a CGH make the null lens.
- We developed a systematic way of performing tolerance analysis on the testing system, which can be readily applied to GMT testing system.
- We acquired the experiences of measuring and morphing ${ }^{6}$ the surface figure of the fold sphere, and then subtracting it from the measurement of the test surface figure. Both the big and small fold spheres used in GMT testing equipment would be measured and backed out in the same way.
- A simultaneous phase-shifting interferometer with high power laser is used to test the bare glass NST mirror and proved to give fringes with sufficiently good contrast even when the light loss in the test arm is significant (less than $0.04 \%$ of light comes back to interferometer).
- We developed a novel method of designing the heavy-duty main CGH. Computer programs were written to implement the design method and to calculate phase at an arbitrary point on the hologram. The CGH was defined by a grid of phase data. We demonstrated that so defined and designed CGH can be made with high accuracy. The main CGH for GMT testing can be designed and fabricated using the same technology we used here.
- We developed a new way to align a tilted spherical mirror to high accuracy within $7 \mu \mathrm{~m}$ of piston and tilts at the edge. The method combines use of CGHs to direct beam at the targets mounted in touch with the mirror surface and metering rods to measure and control distances from those targets to a fixed and known point. The mirror is adjusted to get these targets to the designated positions, thus the mirror itself is aligned. We wrote a program to measure the influence functions of each adjustment degree of freedom on the mirror, which allow the alignment to converge quickly - after about three or four iterations. The same technique can be used to align the small fold spheres in the GMT testing system.
- The metering rods were made of a low thermal expansion carbon fiber tubes with invar tips at both ends. The carbon fiber tube has a CTE of $1 \mu \mathrm{~m} / \mathrm{m} /{ }^{\circ} \mathrm{C}$. The metering rods were mounted on a stage made of low thermal expansion ULE glass blocks and calibrated with a laser tracker to within an accuracy of $2 \mu \mathrm{~m}$. The extreme accuracy of the metering rods allows us to align the spherical mirror to $7 \mu \mathrm{~m}$ accuracy.
- We developed a method to measure the long radius of curvature of a spherical mirror with high accuracy ${ }^{7}$ A phase measuring interferometer and a laser tracker was used to measure the radius of curvature of an $\mathrm{R} / 5$ spherical mirror of 0.5 meter in diameter accurate to $20 \mu \mathrm{~m}$. The same technique can be used to accurately measure the radius of curvature of the small fold sphere.
- We created a convenient model for off-axis conic surfaces.


## 10. Summary:

Equipment was designed and built to test an off-axis parabolic mirror which is the primary mirror of the New Solar Telescope (NST). The null lens consists of a tilted spherical mirror and a computer generated hologram. Several new techniques were developed along the way which will be applied to development of testing equipment for the off-axis parabola mirror segments of the Giant Magellan Telescope.

## References:

1. L. Didkovsky, J Kahn, P. Goode, "Optical Design for a New Off-axis 1.7 m Solar Telescope (NST) at Big Bear", Proc. SPIE Vol. 5171, p. 333-343. Telescope and Instrumentation for Solar Astrophysics, S. Fineschi, M. A Gummin Eds, 2004.
2. H. M. Martin, J. H. Burge, B. Cuerden, S. Miller, B. Smith, C. Zhao "Manufacture of $8.4-\mathrm{m}$ off-axis segments: a $1 / 5$ scale demonstration", Proc. SPIE Vol. 5494 p. xxx-xxx, Optical Fabrication, Metrology, and Material Advancements for Telescopes. Eli Atad-Ettedgui, Philippe Dierickx Eds, 2004.
3. J. C. Wyant and V. P. Bennett, "Using computer generated holograms to test aspheric wavefronts", Appl. Opt. V11, 2833-2839 (1972).
4. For example, ZEMAX supports a surface type called "grid phase". See ZEMAX manual, www.zemax.com.
5. W. H. Press, B. P. Flannery, S. A. Teukolsky and W. T. Vetterling, Numerical Recipes, Chap. 2, Cambridge University Press (1987).
6. We used Durango Interferometry software for morphing. Durango is a product of Diffraction International Inc. See www.diffraction.com.
7. C. Zhao, R. Zehnder, J. H. Burge, "Measuring the radius of curvature of a spherical mirror with an interferometer and a laser tracker", submitted to Optical Engineering (2005).
